A Monte Carlo Simulation of a Phase Doppler Interferometer

James F. Meyers\(^{(1)}\) and Graham Wigley\(^{(2)}\)

1: JF Meyers Consulting, LLC, Newport News, VA 23602, USA
2: Loughborough University, LE11 3TU, UK
* Correspondent author: bsshome@cox.net

Keywords: PDI simulation, PDA simulation, Particle size, Sprays

ABSTRACT

Analytic and Monte Carlo simulations were developed to investigate the characteristics and measurement capabilities of the Phase Doppler Interferometry (PDI) technique for measuring particle size and velocity. The analytic simulation provided a platform for obtaining insight into the overall PDI concept and to determine how optical configuration changes affected the system characteristics. The Monte Carlo simulation was developed to better model nature by its use of random events to control each subdivision of the PDI components, from the generation of known particle size, velocity and trajectory, to converting the Mie scattered light into streams of photons, to the functions within the photo detectors. Additionally, with the generation of realistic signal bursts, signal processing techniques were developed and tested for measurement accuracy since the processed results could be compared particle-by-particle with the known input particle size, velocity and trajectory. Three signal processing schemes were developed: 1) exact cycle zero crossing location traces shifted for minimum error, 2) binomial curve fit of peak and minimum cycle locations (high-pass filtered signal burst), and 3) histograms of positive and negative voltages of the high-pass filtered signal burst. The average uncertainty (standard deviation) in the measurement of time shifts for particle sizes from 1 to 24 microns were 0.0209, 0.0026, and 0.0016 microseconds, respectively, from a range of time shifts from 0.003 to 0.223 microseconds.

1. Introduction

The Phase Doppler Interferometer (PDI) is a hybrid instrumentation system. It behaves like a fringe-type laser velocimeter in that light is scattered from a single particle passing through the fringe pattern formed when two coherent laser beams are crossed. The Mie scattered light from the particle is collected by a photo detector that converts the oscillating scattered light intensity, that was imposed by the particle passage through the fringe pattern, into an electronic signal. This signal burst is captured by electronics, e.g., a specially designed signal processor, or even a high-speed electronic digitizer (analog-to-digital converter). That captured signal is then interrogated to determine the oscillation frequency. Once known, along with the optical characteristics of the laser velocimeter, the particle velocity could be determined.

The second characteristic of the laser velocimeter that provides the particle size measurement by the PDI is the timing of the refraction characteristics of the particle as it passes through the fringe pattern. The laser light is refracted through an arc of approximately ±70 degrees beginning with a refraction angle of
-70 degrees (approximately opposite of the particle trajectory) sweeping toward the particle's direction of travel as it passes through each fringe. Thus by adding two more detectors placed adjacent to the LV detector in a plane where the refracted light would sweep from one detector to another, the detectors would be illuminated in sequence as the particle passes through each fringe. Measuring the delay times between the detector that is first illuminated and the second and then the third, a relationship is exposed that is proportional to the diameter of the scattering particle. Thus the size and velocity can be measured simultaneously for each particle passing through the measurement volume.

The earliest velocity and particle sizing measurements were reported in the early 1980s by Saffman et al., Bachalo and Houser, and Bauckhage and Flogel. Detailed analyses of the light scattering phenomena from crossed coherent laser beams by particles and droplets were written in the early to mid-1990s by Naqwi and Durst, and Buchhave and von Benzon. These publications introduced computer programs to quantify the light scattering in terms of both geometric optics and Mie theory. These programs could then be used to explore the potential for drop size measurements with a variety of optical configurations. A general discussion on signal processing methods by Lehmann, et al., was published at the same time. An up to date and comprehensive review and discussion of the phase Doppler technique can be found in Albrecht, et al.

This powerful technique can be used to directly determine particle size / velocity relationships in accelerating/decelerating flows, boundary layers, along with its most used application, sprays and fuel injectors. One potentially interesting and important application would be the investigation of shear layers developed between two adjacent flows by seeding each flow with a different sized particle, which would completely remove statistical sampling bias from these flow measurements.

2. Optical Configurations

The analytic investigation concentrated on two optical configurations. The first was the classic configuration where the two crossing laser beams were placed in the horizontal axis and the three detectors arranged to view the fringe pattern from above, symmetrically placed about a vertical plane aligned with the optical axis of the transmitted laser beams, Figure 1. The flow would be in the horizontal plane aligned orthogonal to the optical axis. In this configuration, the refracted light would first illuminate the detector on the left, assuming the particle was also arriving from the left, as viewed from the laser beam focusing lens. Then the refraction angle would sweep to the right to illuminate the center detector when a given fringe is passing through the center of the particle, then finally the detector on the right would be illuminated. With the detectors being symmetrical about the vertical axis, the elevation angle of the
receiver above the horizontal plane does not affect the timing of the illumination sequence, but it does control the viewed length of the measurement volume. The selected scattering angle was set to 70 degrees for the simulations.

![Fig. 1 PDI – Classic configuration](image1)

![Fig. 2 PDI – In-plane configuration](image2)

The second configuration maintained the orientation of the crossing laser beams along with the direction of the particle travel, but the receiver was rotated about the optical axis to lie in the horizontal plane on the left, as viewed from the laser beam focusing lens. The detectors were also rotated about the central detector so that all three detectors were in the horizontal axis, Figure 2. This in-plane asymmetric configuration was selected to determine if a different configuration would have any advantages/disadvantages over the classic configuration. In this case, the refracted light would originate from the leading edge of the particle and impact the detector closest to the origin of the particle's trajectory. As the particle moves through a given fringe, the refraction would sweep to the second and then to the third detector. Since these detectors are not symmetric regarding the refraction angles, the collected scattered light by each detector may not be similar, because of Mie scattering characteristics. It is also noted, that typically the three (or two) detectors are viewing the measurement volume through a single collecting lens that is masked into three (or two) apertures. In this case, both configurations use a mask that is aligned vertically, with the apertures labeled as A (first illuminated), B, and C.

3. Analytic Simulations

The analytic simulations were fairly simplistic using only ray tracing techniques to determine the timing of the detector illumination and expected signal burst visibility (percent of the AC amplitude modulation of the oscillating signal riding on the DC Gaussian intensity profile imposed by the crossed laser beams) as a function of particle size. Variables such as detector location, particle size, particle velocity and Bragg cell frequency were adjusted to estimate the effects on signal quality. The same sequence of parameter changes were made for both configurations. The simulations were conducted using a 1.0 GHz analog-to-digital converter (ADC). Thus if a particle velocity was set to 100 m/s, the refracted light would be “measured”
every 0.1 micron of particle movement through a thin vertical plane used to simulate a single fringe. Each ray that passed through a given detector would add to the signal strength obtained for that “fringe” during each nanosecond. By expanding and including more planes into a sine wave along with an overall Gaussian envelope, the entire fringe pattern was modeled and the acquired signals from each aperture investigated.

As part of the simulation was the investigation into methods for signal processing. Various approaches were investigated from the simplistic timing when the peak voltage occurs in each of the three successive signal bursts to the more accepted cross correlation of the signal bursts. The method that consistently yielded the greatest accuracy was a modification of the zero crossing technique as used in high-speed burst counters for LV applications. First the signal burst is high pass filtered to remove the pedestal, then the symmetric signal is interrogated to determine the location of each zero crossing. Since the captured signal burst is a digitized version, the exact location of each zero crossing was determined by interpolation between the last negative sample and the first positive sample for each cycle. Once obtained, the results from the three signal bursts were adjusted to minimize the time delay errors between signals A and B, and signals A and C knowing that the shift time differences would be 2:1 between A:C and A:B. This approach was used to process the signal bursts generated in the following investigations.

The first parameter selected was the receiver viewing angle. The receiver was rotated in the classic configuration from 30 degrees to 70 degrees in elevation. Correspondingly the receiver for the in-plane configuration was rotated in the horizontal plane at the same angles. Three characteristics were studied for particle sizes of 1 to 50 microns, in one micron steps. The first was signal visibility which affects the ability of the signal processor to determine the timing of the particle passage along with the particle velocity. The results for rotation angles of 30, 50 and 70 degrees are shown in Figure 3.a. As expected the visibility results are the same regardless of viewing angle for the classic configuration, with the visibility dropping by 20 percent (aperture B) and 25 percent (apertures A and C). Also as expected the results for the in-plane configuration yielded different visibilities for the three apertures, however the visibilities all increased with increase in viewing angle.

The second characteristic is the temporal shift of the three signal bursts shown in Figure 3.b. The shift is linear with particle size for both configurations as expected. There was no variation in the classic configuration with viewing angle, but the in-plane configuration showed a drop in slope with viewing angle. These plots were considered the calibration plots to determine particle size based on temporal shift. It is noted that plots of temporal shift versus particle size will be referred to as calibration plots because the actual plots may differ from theoretical as is the case with the in-plane configuration here and results
from the Monte Carlo simulation later. Again the classic configuration shows no variation with rotation angle, but the in-plane configuration has an increase in noise with increased rotation angle due to the reduced slope at large angles coupled with the resulting amplification of digital noise. The particle size, as

![Graphs showing signal visibility as a function of receiver viewing angle for both classic and in-plane configurations with different rotation angles and particle sizes.](image)

**Fig. 3.a** Signal visibility as a function of receiver viewing angle.
determined from the slopes in Figure 3.b and show in Figure 3.c are as expected, all identical except for the increased noise in the in-plane configuration because of digitizing uncertainties with the small slopes.

Fig. 3.b Temporal shift as a function of receiver viewing angle.
Fig. 3.c Measured particle size as a function of receiver viewing angle.

Since there was no change in the results with elevation angle with the classic configuration, and a large viewing angle would reduce the length of the measurement volume viewed, the receiver viewing angle was set to 70 degrees for the remaining tests. The next parameter was to vary the particle velocity. However,
velocity only enters the results if spatial shift is determined, and the spatial shift responds as expected with the temporal shifts versus particle size shown in Figure 3.b.

The particle velocity was set to 100 m/s to determine if the inclusion of a Bragg cell affected the measurements. Comparing the results from Figure 4.a. to those shown in Figure 3.a, it becomes clear that adding a Bragg cell to the classic configuration has a marked effect on the ability to make unbiased measurements, especially at the normal operating shift frequency of 40 MHz. However, there is virtually no change in the in-plane results. Moving to the calibration plots in Figure 4.b, a major shift occurs at the point where the signal visibility went to zero for the classic configuration, which also translates to the ability to measure the particle size, Figure 4.c.

Since adding a Bragg cell created issues with the classic configuration, the question whether velocity variations would accentuate the issues which led to the final analytic test. As shown in Figure 5.a, the lower the velocity, the lower the maximum size particle when a visibility null occurs. The visibility characteristics carry through to the calibration and the ability to measure particle size, Figures 5.b and 5.c, respectively. However, the in-plane configuration showed little change with velocity, and yielded “acceptable” measurements though with noticeable uncertainties from the digital noise.

It is noted that this simplistic analytic simulation was designed to point out potential issues that may exist with PDI technology, but not a definitive listing. It did show that signal visibility does have a direct effect on the ability of the instrument to make unbiased measurements of particle size, and that different configurations beyond the classic configuration should be considered. Also the investigation found that cross correlation was not necessarily the most accurate and repeatable signal processing technique. Ideally the best method to investigate the technique with emphasis on the results found in the analytic simulation would be to send a series of monodisperse glass spheres through a PDI system and investigate the results obtained from each sized particles. This would yield precise characteristics since the particle size would be known and any measurement errors could be determined particle by particle. The alternative is to use Monte Carlo techniques to simulate the PDI, again looking for issues that surfaced from the analytic simulation.
Fig. 4.a Signal burst visibility as a function of Bragg cell frequency.
Fig. 4.b Temporal shift as a function of Bragg cell frequency.
Fig. 4.c Particle size measurements as a function of Bragg cell frequency.
Fig. 5.a Signal burst visibility as a function of particle velocity with a 40 MHz Bragg cell installed.
Fig. 5.b Temporal shift as a function of particle velocity with a 40 MHz Bragg cell installed.
Fig. 5c Particle size measurements as a function of particle velocity with a 40 MHz Bragg cell installed.
4. Monte Carlo Simulations

A Monte Carlo simulation is based on subdividing the desired task, in this case the PDI, into small physical functions that could be easily modeled without assumptions to perform in the manner of the real function. Since most things in nature have a random property, the use of random number generators with various characteristics are used to develop the function based on the random characteristics of the function. For example, the scattered laser light is typically described as a continuous illumination of so many nano Watts of power. However, in actuality light is composed of individual photons which arrive in time in a poisson distribution, as reported by Mayo in 1975, whose shape is determined by the arrival rate of the photons. When the arrival rate is sufficient for the photons to overlap each other within the bandwidth of the detector, the single photon energies add within the rise/fall time of the detector leading to a continuous analog signal. The effect is called photon pileup and can be easily seen on a high speed oscilloscope viewing the output from the detector when the light levels are very low. This random process is easily replicated by the Monte Carlo technique, and has been implemented for investigations in laser velocimetry along with aiding in the design of dedicated wind tunnel systems and the development of new signal processing systems, Meyers and Clemmons, Baker et. al., Meyers and Stoughton, and Meyers and Murphy. An updated and comprehensive description of the Monte Carlo simulation of a fringe-type laser velocimeter can be found in Meyers et. al.

<table>
<thead>
<tr>
<th>Transmitter</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser wavelength (microns)</td>
<td>0.5145</td>
</tr>
<tr>
<td>Laser power (W)</td>
<td>0.00001 → 4.0</td>
</tr>
<tr>
<td>Bragg frequency (MHz)</td>
<td>0 → 40</td>
</tr>
<tr>
<td>Beam 1 Location (x,y) (m)</td>
<td>-0.0250, 0.0</td>
</tr>
<tr>
<td>Beam 2 Location (x,y) (m)</td>
<td>0.0254, 0.0</td>
</tr>
<tr>
<td>Transmission (Beam 1, 2)</td>
<td>0.5, 0.5</td>
</tr>
<tr>
<td>Polarization (Beam 1, 2) (deg)</td>
<td>90.0, 90.0</td>
</tr>
<tr>
<td>Laser Beam Diameter (m)</td>
<td>0.005</td>
</tr>
<tr>
<td>Focal Distance (m)</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Phase Doppler Interferometer Characteristics
This simulation is geared only toward the investigation of the classic configuration whose physical characteristics are given in Table 1. However, a few processes remain analytic because of their complexity, e.g., Mie scattering, light refraction characteristics, and measurement volume characteristics. The simulation begins by selecting a particle size (or a series of randomly selected sized particles statistically based on the selected distribution), a velocity magnitude randomly selected based on a Gaussian distribution established by the input mean and standard deviation, a vertical flow angle randomly selected based on a Gaussian distribution established by the input mean and standard deviation, and a horizontal flow angle randomly selected based on a Gaussian distribution established by the input mean and standard deviation. Each selected particle with its velocity and flow angle characteristics is then launched through the measurement volume starting in a vertical plane located upstream at the 1/e^3 laser power point from the center of the measurement volume. The location within that plane is determined by a uniform random number generator for the horizontal ±1/e^3 extents and a second uniform random number generator for the vertical ±1/e^3 extents. If a slit is used in the receiving optics, as is the case in the simulation, any particle that does not pass through the viewing volume as limited by the slit is counted, but ignored. For example if a 100 micron slit is used and 5,000 particles selected with a mean velocity magnitude of 100 m/s with no standard deviation, flow angles of zero degrees with no standard deviation, only about 10 percent of the particles are actually captured and measured in the simulation.

This approach is used for every element within the PDI system, from building the signal bursts, to the operation of the photo detectors, to the characteristics needed to acquire and process these signals that could range from photon limited to almost continuous signals. For the arrangement matching an existing PDI system, a typical signal burst that yields an output signal from 1.0 to 2.5 volts is built from approximately a million photo electrons. The photo detector saturation point was set to 2.2 volts (measured prior to the ADC amplifier which was set to 6 dB).

In order to investigate the basic characteristics of the PDI technology, the velocity magnitude standard deviations along with the mean flow angles and their respective standard deviations were set to zero. Thus each particle will have the same velocity with a trajectory orthogonal to the fringe plane, but allowed to enter that plane at random locations within the 1/e^3 measurement volume intensity limits. The particle size limits were set from 1 to 24 microns. While this does not cover the often wide particle size ranges observed in practical sprays, it is
sufficient to observe any issues that may arise in the investigation. That investigation begins with the light scattering process.

4.1 Scattered Light Characteristics

There are two processes incorporated in the simulation regarding the scattering of light from a particle passing through the PDI measurement volume. The direction of the scattered light as a function of particle location is based on refractive optics computations, and the intensity distribution received by each of the three masked areas on the receiving lens is based on Mie scattering. Thus the timing of the signal burst arrival at each detector is computed using refractive optics equations to determine when a ray reaches the center of each aperture established by the mask covering the receiving lens. Each signal burst is then analytically developed based on the characteristics of the measurement volume along the particle's trajectory where the value at the center of the measurement volume is set to unity. Since the three signal bursts are developed from a single trajectory, they are identical, but delayed in time based on the delays determined from the refractive optics results. The signal bursts are then amplified by the integrated results from the analytic Mie scattering prediction that was modified to account for phase interactions imposed by the mixing of light scattered by each of the two laser beams as developed by Adrian and Earley.

![Fig. 6.a Mie scattering at 70 degrees.](image)

![Fig. 6.b Mie scattering polar plots for 1 and 24 μm](image)

The standard Mie scattering profile for particle sizes from 1 to 24 microns is shown in Figure 6.a. for a viewing angle of 70 degrees elevation above the optical axis of the focusing lens from a single laser beam aligned with the optical axis. An overall view of the complexity of Mie scattering is shown in the logarithmic polar plots of 1 and 24 microns in Figure 6.b. Expanding it further, color contour maps of the
scattered light intensity and power at the collecting lens with the mask in place are shown in Figure 7 for 1 and 24 micron sized particles. The maps were computed from a standard Mie scattering code with modifications as suggested by Adrian and Earley. In order to obtain the desired signal strength, 1.0 to 2.5 volts, the input laser beam power for a 1 micron particle must be 60 times that for a 10 micron particle. That dynamic range is greater than the response capability of the photo detector. Thus, setting the input laser power has to be a compromise between potentially saturating the PMT for the large particles and the detectability of the smallest particles. Since the laser beam intensity profile is Gaussian, then this means that the measurement volume for the smallest particles would be severely limited or non-existent, and the

Fig. 7 Mie scattering contour maps at 70 degrees receiving angle with three PDI apertures shown.
larger particles would have measurement volumes greater than the $1/e^2$ laser beam intensity limits normally used to define the measurement volume. Thus the probability of measurement of any particle size between 1 and 10 microns is far from uniform. A similar condition exists between 20 and 24 micron particles. However, the probability of measurement of particles from 10 to 20 microns is fairly uniform.

4.2 Photo Detector and High-Speed Digitizer

After determining the strength of the collected scattered light entering each aperture, the “power” is converted to a photon arrival rate as a function of time. The process was developed by Mayo and yields a series of single photons of the same energy arriving at a random rate that obeys poisson statistics. The signal burst is integrated and the integral used to control the rate of a poisson random number generator to release a photon. For example, the peak of a signal burst cycle has a high rate that would generate photons close together in time, whereas a half a cycle later the rate is quite low expanding the distance between photon generations.

The streams of photons entering each of the three apertures are converted to electrical energy by the photo detectors. These detectors are modeled in the program as photomultipliers (PMT). When a photon strikes the photocathode surface a photo electron may or may not be generated. That is governed by the quantum efficiency of the PMT. If the quantum efficiency of the PMT is 21 percent, a uniform random number generator is used to generate a number. If the number is 0.21 or less, the photon is converted to a photoelectron, otherwise it goes no further. The photoelectron stream is next convolved with the transfer function of the PMT, typically modeled as a triangle set to the rise/fall time of the PMT. This function is what develops the photon pileup that makes the stream of equal energy photoelectrons into an analog signal.

The input power supply settings for the modeled PDI system are lowered below normal because of the high scattering intensities obtained. The gain used in the system is typically two orders of magnitude below normal settings for laser velocimeter applications, in the present case 10,000. The number of photo electrons needed to obtain desired signal strength are such that a great deal of normal photon noise is averaged out by the PMT along with the Gaussian noise in the dynode chain being greatly reduced by the low gain. The Gaussian noise is not modeled because under these conditions, it has a negligible effect. The now analog signal, then passes to the high-speed digitizer for conversion into a digital signal. Examples of these signals for a 10 micron water particle are shown in Figure 8 from photon resolved to the optimum signal strength. The signal bursts in Figure 8.a. are the analytic signals based on the particle trajectory, which in this case is orthogonal to the fringe plane, 1.2 percent of the measurement volume half
length to the right of center and 69.6 percent of the radius above center. Thus the peak amplitude would be approximately 75 percent less than produced by a particle passing through the center of the measurement volume. Figure 8.b. illustrates photon resolved signal bursts for the three components, where aperture C actually received no photons. Increasing the input laser energy by a factor of ten yields the signals shown in Figure 8.c. They are structured well enough to be acceptable for laser velocimetry applications should the normal one million PMT gain be applied. However, even these low noise levels would yield a fairly large uncertainty in the particle size measurement. The signals found to be ideal are shown in Figure 8.d., with minimal photon noise and amplitudes sufficient where digitizing noise from the ADC is negligible. Thus, during the calibration process the laser power was adjusted to produce peak signal amplitudes from 1.0 to 2.5 volts for each particle size from 1 to 24 microns.

Fig. 8 Typical simulated signal bursts as a function of laser power.
4.3 Data Acquisition

The high-speed digitizer was designed to allow the continuous passage of an analog signal through it, typically referred to as a first-in-first-out shift register. When a trigger pulse is activated, the digitizer freezes whatever is currently in memory and transfers the captured segment to temporary memory, where it then can be transferred to a desktop computer for processing. The digitizer in the simulation acquires samples at a 1.0 GHz rate with a memory length of 10,000 samples. It also has the capabilities of pre-triggering which allows a portion of the signal that has passed the trigger point to still be captured. These capabilities can be found in a typical digital oscilloscope. An oscilloscope could even be used as the digitizer except they are quite slow in the transfer of the acquired data to other devices.

The capability to pre-trigger the digitizer provides a method to exclude low amplitude signal bursts and background noise by setting the trigger level near the top of the desired burst, e.g., center trigger. The pre-trigger capability has the effect of shrinking the size of the measurement volume when monodisperse particles are used. While this is ideal for laser velocimetry applications, it is completely opposite of PDI requirements. Thus a compromise was chosen for the simulation. Since the desired signal strength was from 1.0 to 2.5 volts, the trigger was set in the center trigger configuration, but the level was set to 0.4 volts. This allows some range for the smaller particles, yet still having some background noise rejection capabilities.

4.4 Signal Processing

One of the advantages of Monte Carlo simulations is that it allows the development and testing of various signal processing techniques. Since the answers are known, i.e., the particle velocity and size, a direct path exists to determine the exact measurement errors particle by particle. Unfortunately, this very capability showed that the signal processing technique as used in the analytic simulation had a large measurement uncertainty here because, even at the ideal signal burst amplitudes, there was sufficient noise at the zero crossing locations that sliding the three zero crossing maps to minimize time differences between apertures A:C and twice A:B did not yield definitive results.

Mie scattering characteristics

In order to keep the signal burst levels within the ideal range of 1.0 to 2.5 volts, the Mie scattering characteristics had to be investigated. Given the scattering angle of 70 degrees, the Mie scattering characteristics for: pedestal beam A, pedestal beam B, fringe amplitude and phase, were calculated for particles from 1 to 50 microns. Since the two pedestal amplitudes would be virtually identical and would
be unaffected by visibility, a plot of pedestal beam A amplitudes is presented in Figure 9.a. and used as a guide to select the input laser power to the PDI simulation, also shown in Figure 9.b., as a function of particle size. Since the large dynamic range found for the input laser power needed to obtain the ideal signal levels, the simulation was run with a single particle size from 1 to 35 microns every micron in one micron increments. Five thousand particles traveling at 100 m/s with a trajectory orthogonal to the fringe plane were launched from a vertical plane at the 1/e³ laser intensity location from the measurement volume. The vertical and horizontal launch point was located within the plane using uniform random number generators to select a location within the 1/e³ by 1/e³ extents of the plane, as defined by the 1/e³ intensity locations of the measurement volume length and height, respectively. The use of a 100 micron slit reduced the number of particles passing through the viewed volume. Further reduction occurred because of low signal levels from small particles passing through the upper and lower regions of the measurement volume. Approximately ten percent of the input particles yielded measurable signal bursts.

**Zero crossing and alignment signal processing**

Although expected, the initial calibration sweep in particle size clearly showed issues, not only with signal processing but the effects of changing signal visibility. Since the analytic results indicated that the inclusion of a Bragg cell would yield signal visibility nulls at various particle sizes depending on the particle velocity, Figures 4 and 5, the investigation first concentrated on this issue. The calculated visibility, along with the measured results are shown in Figure 10.a. There appears to be a strong null between 18 and 19 microns. A null was also reported by Saffman et. al. near 18 microns. The fringe spacing was then reduced by changing the focal distance of the transmitter from 450 mm to 300 mm, to only find that the overall visibility changed, but not the location of the null, Figure 10.b. Next the particle velocity was changed which produced the same results, Figure 10.c. Finally the Bragg frequency shift was...
changed, Figure 10.d, again with the same results. The analytic simulation was over emphasizing the visibility nulls obtained when a Bragg cell was present, probably caused by the many assumptions involved in the simulation. However, the analytic simulation did point out that nulls in the visibility were possible, and that they would have an effect on the measurements.

![Graph showing signal visibility](image)

**Fig. 10** Signal visibility null found at 18.5 microns and checked for changes based on results from the analytic simulation.

The results from the full calibration yielded very interesting results, shown in Figures 11.a and 11.b. The delay shift in time between apertures A and B, and A and C was linear, but not the same slope as theory predicted from 1 to 12 microns. Then from 13 microns to 24 microns, the A:B results matched theory albeit with increasing standard deviations as the particle size was increased. The A:C results started from the theoretical value at 13 microns, but quickly deviated from theory with very large standard deviations. Taking the mean and median values from each particle ensemble of time shifts and converting to particle size, the results are reasonable, but deviate above 18 microns to unacceptable levels, Figures 11.c and 11.d.
In order to obtain greater insight into the behavior of the data, the signal visibility and temporal shift data was plotted in a scatter plot format, Figure 12. The top two figures show the behavior of signal burst visibility as a function of peak signal voltage. The trigger level at 0.4 volts is clearly seen showing a large spread in visibility, most likely from small particles where the trigger level was at the peak of the

signal burst to slightly larger particles where the threshold was located below the peak. It is also clear that the visibility levels did not match theory except at the smaller particles. The time shift for the 12 micron particle (final particle on the first slope, Figures 11.c. and 11.d.), shows a very stable result albeit below the theoretical levels because of the different slope from theory. At 13 microns the time shifts matched theory, but scatter was quite high, especially for A:C apertures and signals below 1.0 volt.

This same presentation technique was used for 18.0 micron particles, a point just before the visibility null, and 18.5 microns at the bottom of the null, Figure 12. The scatter in visibility and time shift for the A:C apertures have increased in both cases, matching the characteristics of increased standard deviations found
in the calibration plots for larger particles. However, the scatter found from the 18.5 micron particles is much smaller, most likely because the low visibility also reduced the dynamic range of the signals to a point just over threshold.

**Fig. 12** Scatter plots of signal visibility and time shift in the neighborhood of the break in calibration near 12 microns.

**Envelope binomial curve fit signal processing**

The large scatter in the data led to the conclusion that the zero crossing alignment technique was most likely the cause. Any noise in the signal would impose uncertainties on the zero crossing location which would lead to errors in aligning the A:B and A:C mappings, even to adding an extra cycle. Thus a technique was developed that made use of the filtered signal envelope to provide a more accurate measurement of the time shifts among the signal bursts obtained from the three apertures. Since the envelope was Gaussian, a binomial curve fit of each cycle peak location would provide a more accurate location of the center of the burst. Further, also fitting the profile of the cycle minimums would provide a second, and probably better estimate, since they contain less noise, of the burst center. An example of the binomial curve fit of the high-pass filtered signal burst cycle peaks and minimums is shown in Figure 14.
Fig. 13 Scatter plots of signal visibility and time shift in the neighborhood of the visibility null at 18.5 microns.

Fig. 14 Binomial curve fit of a filtered signal burst envelope of cycle peaks and minimums.
A series of comparisons between the two processing techniques are presented in Figures 15 to 18. First the histograms of temporal shifts are presented, followed by scatter plots of the same data. The particle sizes chosen were 12 microns (Figure 15), 13 microns (Figure 16), 18 microns (Figure 17), and 18.5 microns (Figure 18) to match the particle sizes encompassing the major points of concern in the calibrations: the break in the linear calibration, and the visibility null. At 12 microns the curve fit technique had a little more scatter than the zero crossing method but, for the remaining particle sizes the curve fit technique maintained its scatter distribution whereas the zero crossing method greatly expanded the scatter in the data.
Fig. 15 Data presentations of 12 microns obtained from zero crossing (left) and curve fit signal processing.
Fig. 16 Data presentations of 13 microns obtained from zero crossing (left) and curve fit signal processing.
Fig. 17 Data presentations of 18 microns obtained from zero crossing (left) and curve fit signal processing.
Fig. 18 Data presentations of 18.5 microns obtained from zero crossing (left) and curve fit signal processing.
Histogram approach to signal processing

However, even the reduced scatter in the time shifts would still affect particle size measurement accuracy. Thus additional signal processing ideas were investigated. Since it is always better to let the data speak for itself than place artificial constraints on the data, e.g., curve fits, it was time to go back to the basics. Since the captured signal bursts are made up of 1.0 nanosecond samples of the waveforms generated by the PMT as the particle passes through the fringe pattern, why not treat the signal burst as a histogram? Furthermore, split the filtered version of the burst to yield two histograms, one from the positive half and the other from the negative half of the signal, then average the two results.

In order to minimize uncertainties at the leading and trailing portions of the signal burst, the leading portion was eliminated by starting the histogram where the burst signal first crosses a threshold whose amplitude was 10 percent of the peak signal burst amplitude. The trailing portion was eliminated by determining the location of the first threshold crossing starting from the end of the burst. Again, key particle sizes were processed and the results plotted in both histogram and scatter formats. The results from particle sizes 12 and 13 microns are presented in Figure 19, with data for particle sizes 18 and 18.3 microns (18.4 to 18.6 microns yielded no data) are presented in Figure 20. While visibility remained the same, the temporal shift scatter was reduced. Using the histogram approach to signal processing, new calibrations were obtained, Figure 21. The average measurements are presented on the left side, and the median measurements presented on the right side. Since the median measurements are not as affected by large distributions as simple averages would be, the median data better represents the system characteristics for calibration purposes. However, although the calibration plots have improved, the anomaly between 12 and 13 microns has now shifted to between 11 and 12 microns.

A detailed investigation was conducted to determine exactly where the anomaly was located. The histogram results presented in Figure 22. show a significant increase in spatial shift between 11.6 and 11.7 microns. Calibration results and scatter results follow in Figure 22. At this time, the cause of this change, or the source of the anomaly, is unknown.
Fig. 19 Data presentations of 12 (left) and 13 micron particles with histogram processing.
Fig. 20 Data presentations of 18 (left) and 18.3 micron particles with histogram processing.
Fig. 21 Visibility and calibration results using histogram processing.

4.5 Particle Size Measurements

With the histogram based signal processing technique yielding repeatable, low noise results, the technique could be used to investigate distributions of particle sizes in the range from 1 to 24 microns even though there is a break in the calibrations. As a reminder, the program will calculate the size of each particle that produced a signal burst with a voltage greater than the 0.4 volt threshold. That measured particle size could then be compared directly to the particle's actual size and thus the measurement accuracy for each and every particle would be determined. This not only allowed the determination of the particle size distribution, but the distribution of measurement errors for the entire ensemble based on the statistics obtained from each particle. This would result in determining the overall measurement accuracy of the PDI technique as used in the classic configuration.
Fig. 22 Histogram, visibility and time shift results near 11.6 and 11.7 microns using histogram processing.
Referring back to an original issue regarding the Mie scattering dynamic range, two test cases were developed and processed. First a uniform distribution from 1 micron to 10 microns, with a size spacing of 0.1 microns, was processed with 10,000 particles generated. For single particle testing the 1 micron particle required 4 watts of laser power, the 4 micron particle required 1 watt, and the 10 micron particle
required 0.2 watts. If a laser power of 0.5 watts was used in this test, one would expect that the particles that triggered the analog-to-digital converter, ADC, (i.e., the selected particles) would be larger with a greater probability of being measured. This is because the larger particles with their higher signal strength would effectively increase the measurement volume, whereas the smaller particles would be confined to near the center of the measurement volume. The second test used a uniform distribution from 10 microns to 20 microns, again with a size spacing of 0.1 microns, and also was processed with 10,000 particles generated. In this case, the required laser power was virtually constant at 0.2 watts.

**Uniform distribution: 1 – 10 microns**

The histograms presented in Figure 23 show the input particle size distribution from 1 to 10 microns being approximately uniform, with the selected particle size distribution clearly showing the effect of the Mie scattering dynamic range. Particle sizes from 1.0 to 2.8 are completely missing, and the remaining distribution is heavily skewed toward the 10 micron particle size. The temporal shift between apertures A:B is trending to be uniform, but the A:C temporal shift is not. Figure 23 also contains the scatter results for visibility and time shifts as a function of peak signal voltages along with particle size. Notice that several signal bursts have exceeded saturation (PMT saturation of 2.2 volts, but when the signal voltage was increased by the ADC amplifier, saturation becomes 3.5 volts). Also when plotted against particle size, there is a large amount of scatter in signal visibility, but a very linear calibration with low noise.

The resulting measured particle size histograms are shown in Figure 24, along with histograms of measurement error based on the results from the errors originating from each signal burst processed. In Figure 24 the histogram of selected particles is presented in greater detail than the previous on-line results shown in Figure 23, by plotting the output from the program log file in 0.1 micron steps. The median, mean, and particle-by-particle measurement errors are also shown in the figure for the results from apertures A:B and A:C along with the average of these two results in micron steps. The measurement errors are small until 8 microns is reached when the errors begin to increase primarily due to aperture C. The particle size is also plotted every 0.1 microns to display the uncertainties directly from measurements obtained from A:B and A:C time shifts.
Fig. 23 Histograms of the input and selected particle size, and scatter maps of visibility and time shift as a function of peak signal voltages and particle size.
Fig. 24 Average and detailed presentation of particle size measurements from a 1 to 10 micron uniform size distribution.
Uniform distribution: 10 – 20 microns

The histograms presented in Figure 25 show a completely different response with the selected particle size histogram being very close to a uniform distribution. However, there are two visible notches near 18 microns in the selected particle distribution. The notches were caused by the null points in signal visibility at the 17.7 and 18.5 micron particle sizes. There are no indications of the missing particle data in the signal shift plots. Apparently the randomness of the measurement errors along with the coarse resolution of the on-line plotting routine was sufficient to fill in this missing information. As with Figure 23, the scatter maps of signal visibility and time shifts are shown as a function of signal voltage, and as a function of particle size. The signal visibility scatter map clearly shows the missing data at 17.7 and 18.5 microns along with several other locations of lower visibility. Also the break in the calibration plots at 11.6 microns is clearly found along with missing data at the two visibility null points. There is an anomaly from 14.4 to 14.6 microns caused by several bad measurements from aperture C.

As in the manner of Figure 24, the particle size measurements are presented in Figure 26 in both averaged micron resolution along with the detailed 0.1 micron plot of means and standard deviations. The effect of the null points is clearly seen.
Fig. 23 Histograms of the input and selected particle size, and scatter maps of visibility and time shift as a function of peak signal voltages and particle size.
Fig. 24 Average and detailed presentation of particle size measurements from a 10 to 20 micron uniform size distribution.
5. Concluding Remarks

An investigation into the limits and measurement accuracy of Phase Doppler Interferometry using analytic and Monte Carlo simulations was presented. The technique appears to be fairly robust in its linear relationship of temporal shift of scattered light arrival among three (or two) detectors and particle size. However, the user of this technology should be aware of the issues regarding the huge dynamic range of scattering efficiency as a function of particle size and its effect on the uniform probability of measuring all particle sizes equally. A second area of concern is the loss of signal visibility at various particle sizes, again affecting the uniform probability of measuring all particle sizes equally.

The analytic simulation alerted the reader of the issues with signal visibility, but showed dependencies of these issues on the presence of the imposed optical frequency shift from a Bragg cell and its interaction with particle velocity that turned out to be unfounded. It also brought into question whether the “classic” configuration was optimal. The Monte Carlo simulation brought the ability to divide the instrument into smaller functions, thus reducing the assumptions necessary in any simulation. Further, its ability to adjust randomness to better fit the processes in nature provided greater insight into the physical processes involved in the PDI technology. Also it allowed the determination of the exact measurement error for each measured particle because the size, velocity and trajectory for each particle was known and could be compared with the measured results in the simulation. Finally it provided signal bursts with the same characteristics as found in an actual PDI instrument that could and were used to develop and evaluate signal processing techniques leading to a novel, but more accurate technique than the others tested including the standard cross correlation approach.

Since this investigation is a work in progress, the Monte Carlo simulator will be used to investigate several issues that surfaced in the investigation so far, including the break in the calibration, using particles larger than 25 microns, and if the measurement characteristics change when allowing variations in particle velocities and trajectories as well as large increases in the particle flow rate. Additionally other configurations, including the in-plane configuration, will be investigated to determine the optimum configuration for the investigation of sprays.
6. References


