Macroscopic Mixing Investigation in a Compressible Accelerated Nozzle Flow using Toluene Tracer LIF

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ABSTRACT

The macroscopic mixing behavior behind central injectors in a compressible accelerated nozzle flow has been investigated experimentally using schlieren imaging and planar laser induced fluorescence (LIF) of injected toluene. For this purpose, a new modular flow channel exhibiting a rectangular cross section, optical access from all four sides, and the opportunity to exchange the nozzle and wall geometries for future experiments has been designed and tested successfully. Our experiments were conducted using a convergent-divergent nozzle designed for an exit Mach number of 1.7 and two different central injectors. The injectors were both mounted upstream of the nozzle while their trailing edges and, thus, their exit holes reached into the subsonic and supersonic region of the nozzle flow, respectively. Analysis of the LIF images revealed self-similarity of the lateral intensity profiles for both wake flows some distance downstream of the injector trailing edges. The observed growth rate of the shear layers agreed with the $\frac{1}{2}$-power law described in literature for compressible and incompressible flows.

1 Introduction

Because of the large practical importance of mixing processes in supersonic propulsion and nanoparticle synthesis, extensive research on wake flows behind central injectors at different flow conditions has been conducted. Recently, shock-wave flow reactors have been developed to generate non-agglomerating nanoparticles from the gas phase [9, 26]. The continuous reactors initiate nucleation and particle growth by gasdynamically heating a precursor-seeded flow as it passes a stationary shock that forms after accelerating the gas flow through an over-expanding convergent-divergent nozzle. Sufficient mixing of the cold reactants in the transonic flow region of the nozzle is crucial to achieve narrow particle-size distributions [26]. Studies showed that central injectors are efficient for mixing of fluids in both, subsonic and supersonic flows [2, 8, 24]. However, in most of these works, either incompressible or supersonic wake flows were investigated and only little knowledge of transonic mixing behavior is available. Previous studies in our labs have shown high potential of transonic mixing including self-similarity [28]. During
these investigations, the geometry of the flow channel was pre-determined, such as changing wall inclination and pressure conditions. For the experiments presented here, a new flow channel has been built at the University of Stuttgart to fundamentally investigate the mixing behavior in a compressible accelerated nozzle flow under controlled flow conditions. Laser-induced fluorescence (LIF) has been used to qualitatively image the downstream mixing behavior of a tracer-laden flow injected into a parallel co-flow. The blunt-body wakes of two different central injectors have been investigated: a “subsonic” injector and a “supersonic” injector, meaning that the point of injection is located prior and after the critical cross section of the nozzle, respectively. As shown in Fig. 1, both injectors only vary in the extension of the trailing edge. Four exit holes (2.5 mm diameter, 4.8 mm distance from center to center) deliver the center flow in line with the main flow. The experiments were carried out in a high-enthalpy flow facility at the Institute of Aerospace Thermodynamics (ITLR) at the University of Stuttgart which supplies dry air to the test section at a total temperature of $T_0 = 380$ K and a total pressure of $p_0 = 2.5$ bar in case of the “subsonic” injector and $p_0 = 3.1$ bar in case of the “supersonic” injector, thus maintaining the same injected mass flow in both cases. The Reynolds numbers according to equation 1 at the point of tracer injection were $Re_{\text{subsonic}} = 0.79 \times 10^5$ and $Re_{\text{supersonic}} = 1.38 \times 10^5$, respectively.

$$Re = \frac{u_{\text{ITE}} h_{\text{ITE}}}{v_{\text{ITE}}}$$

(1)

Here, $h_{\text{ITE}}$ is the height of the injector trailing edge, while the main stream velocity $u_{\text{in}}$ and the kinematic viscosity $v_{\text{in}}$ are determined from the wall static pressure measured at the position of the injector trailing edge (cf. Fig. 1) assuming adiabatic flow condition.

![Fig. 1 Geometry of the central injector.](image-url)
2 Theory

2.1 Wake flow

Wake flows behave systematically different in subsonic and supersonic co-flows. In case of subsonic wake flows, a shear layer forms downstream of the wake generator and periodic groups of eddies (“entrainment eddies”) arise at the edge of the shear layer as shown in Fig. 2a [25]. This applies to the “subsonic” injector investigated, whose trailing edge is positioned in the subsonic region of the nozzle. In case of supersonic wake flows, expansion fans leave from the trailing edge of the wake generator, forming an expansion and a recompression zone including an area of recirculation as outlined in Fig. 2b [17]. Large-scale structures similar to those of subsonic wake flows have been observed downstream of the recompression zone [6, 18]. This wake-flow behavior applies to the “supersonic” injector investigated, whose trailing edge is positioned in the supersonic region of the nozzle.

Planar turbulent bluff-body wakes have been extensively studied in incompressible and supersonic co-flows. Wake flows are commonly characterized by the local centerline deficit velocity, \( u_\infty(x) \) and the local half width, \( \delta(x) \) as sketched in Fig. 2, where \( u(x, \pm \delta(x)) = u_\infty - \frac{1}{2} u_\infty(x) \). Assuming uniform turbulent viscosity, an analytical solution of the self-similar boundary-layer approximation for incompressible, 2D, turbulent wake flow with zero pressure gradient exists (equation 2).
\[ f_{\text{analytical}}(\eta) = \exp \left( -\ln \left( \frac{S}{2\vartheta_T} \right) \eta^2 \right) \quad (2) \]

Here, \( S \) is a spreading parameter, \( \vartheta_T \) is the non-dimensional turbulent viscosity and \( \eta = z/\delta, (x) \) is a non-dimensional cross-stream variable. It has also been shown that \( u_\infty(x) \) varies with \( x^+ \) and \( \delta_\infty(x) \) varies with \( x^+ \) (half-power law) [19].

Asymptotic self-preserving wakes have been observed in various experiments. In the case of 2D, turbulent, incompressible, subsonic wake flow, the most notable experiments were conducted by Wygnanski et al. [30]. They observed that the normalized shape of the velocity, \( f(\eta) \), stays invariant for different wake generators sufficiently far downstream. However, the shape determined experimentally slightly differed from the analytical solution and the authors proposed a corrected function (equation 3) with two experimentally determined coefficients \( A \) and \( B \).

\[ f_{\text{Wygnanski}}(\eta) = \exp(A\eta^2 + B\eta^4) \quad (3) \]

Even in the event of compressible wake flows, large, highly turbulent scales are formed and self-similarity is reached some distance downstream of the wake generator [22]. Besides, compressibility effects reduce the spreading rate and the turbulence level, especially in supersonic flows, if Mach waves are present. Self-similar supersonic [12, 18] and transonic [28] wake flows following the \( \frac{1}{2} \)-power law have been experimentally observed in the recent past.

### 2.2 Laser induced fluorescence

Laser-induced fluorescence (LIF) imaging allows to visualize mixing processes in flows when tracer species are added to one component of the mixing fluids. Due to their large absorption cross-sections in the ultraviolet and high fluorescence quantum yield, aromatic hydrocarbons, such as toluene, are often used as fluorescent tracers and their spectrally selective detection of its fluorescence is then used to, e.g., measure the local concentration or temperature of the respective flow [21].

Equation 4 gives the fluorescence signal \( S_\nu \) for an excitation at a convenient laser wavelength \( \lambda \) depending on the absorption cross-section \( \sigma \) of the respective tracer species, its number density \( n_\text{tracer} \), the laser fluence \( I_\text{laser} \), the detection efficiency \( \theta \), and the fluorescence quantum yield \( \phi \).

\[ S_{\text{LIF}} \sim I_{\text{laser}} \delta n_{\text{tracer}} \sigma(\lambda, T) \phi(\lambda, T, p_{O_2}) \quad (4) \]
As the absorption cross-section and the quantum yield both depend on temperature, toluene has shown to be a suitable tracer not only for imaging the tracer concentration but also for thermometry which has been applied to engine diagnostics [3, 13, 23] and studies investigating supersonic flows [7, 14]. As with most aromatic hydrocarbons, the fluorescence quantum yield depends sensitively on oxygen concentration or partial pressure, $p_{\mathrm{O}_2}$, as it acts as a strong quencher for the fluorescence signal [10] which has been exploited to monitor oxygen concentrations of mixing flows or fuel/air ratios [5, 11, 20]. This approach allows to measure the mixing level on a molecular scale (micromixing) as quenching only occurs if tracer molecules collide with oxygen molecules [1].

The O-concentration imaging approach of Mohri et al. [15] makes use of the red shift of the toluene fluorescence spectrum induced by collisions of excited toluene with oxygen. Two cameras equipped with different filters capture sections of the fluorescence spectra so that the ratio of both images can be related to the O concentration using a calibration curve recorded under known conditions. However, increasing temperature also causes a red shift of the fluorescence spectrum [4] making a temperature correction crucial.

This work uses qualitative toluene LIF to study the macroscopic mixing behavior. The peak of the toluene absorption spectrum is close to 250 nm and thus toluene can be effectively excited with 248-nm light from a krypton fluoride (KrF) excimer laser. The fluorescence is red shifted with a peak at approximately 280 nm and can thus be well separated from elastically-scattered laser light [21].

3 Experiment

3.1 Flow facility

The experiments were carried out in a high-enthalpy flow facility shown in Fig. 3. Ambient air is compressed up to 10 bar (with a maximum flow rate of 1.45 kg/s) and dried to a residual moisture of 0.1%. Three electrical heaters can be operated to heat the air flow up to a maximum of 1500 K. The total pressure and the total temperature were measured at the exit of the last electrical heater with uncertainties of ±5 kPa and ±2%, respectively.

3.2 Transonic flow channel

The flow channel consists of two optically-accessible modules of rectangular cross sections as shown in Fig. 4. The first module includes an interchangeable convergent–divergent nozzle,
currently designed for $Ma_{max} = 1.7$ and a holding plate for different central injectors. The second module is currently assembled as a planar flow channel. In later experiments, the top and bottom walls can be replaced by walls with a certain opening angle to investigate wakes under variable pressure gradients. The specific feature of this flow channel is the optical access from all four sides allowing for the utilization of LIF imaging. There are three downstream locations (W1, W2, and W3) that are accessible through pairs of opposing observation windows and pairs of laser light-sheet slot windows.

![Fig. 3 Schematics of the high-enthalpy flow facility at ITLR.](image)

The channel itself is equipped with 40 pressure taps located along the top wall of the flow channel providing pressure profiles in flow direction. Those are used together with schlieren measurements for the verification of the flow conditions, especially in the supersonic region. All optical equipment (including laser and detector) described in the following sections is mounted on an optical table that allows translation in all three directions. Thus, the whole channel can be scanned along the direction of the flow. The coordinate system is such that the $x$-axis follows the streamwise direction, the $y$-axis defines the spanwise direction, and the $z$-axis represents the direction normal to the main flow as seen through the windows (cf. Fig. 4). The origin of the coordinate system is located in the nozzle throat.
3.3 Toluene LIF

The flow injected into the main flow of the channel consists of nitrogen seeded with gaseous toluene. A vaporizer system (Brooks DLI) provides the necessary flow rates of 0.42 g/s nitrogen and 0.14 g/s toluene leading to an injection velocity of 9.8 m/s for the “subsonic” and 53.7 m/s for the “supersonic” injector. Note that apart from the subsonic and supersonic co-flow, the injector flow itself stays subsonic at both injection positions.

![Diagram of LIF arrangement](image)

**Fig. 4** Arrangement for LIF-imaging measurements. The optical components are mounted on an optical table that can be transversed parallel to the flow direction.

Toluene is excited by a KrF excimer laser (Lambda Physik LPX 100) at 248 nm (20 ns pulses). With a single cylindrical lens ($f = 400$ mm) a light sheet (25 mm wide, 0.33 mm thick) is formed from the laser beam with a rectangular beam profile resulting in a fluence of approximately 150 mJ/cm². The light sheet is directed through the flow channel as shown in Fig. 4. A beam splitter and an
energy monitor are installed to record the laser beam energy to correct for pulse-to-pulse energy variations during post-processing. An intensified CCD camera (LaVision Nanostar, 200 ns gate) equipped with a 280±14 nm bandpass filter captures the fluorescence signal under a 90° viewing angle with respect to the light sheet. Prior to the experiments, 200 background images have been recorded at each position without flow in order to compensate for laser light scattering. Flat-field images were taken similarly but after the experiments by filling the flow channel with a homogeneous tracer mixture (cf. section 4.2.2).

For the present work, we selected 18 positions for taking LIF images where the first two positions account for window W1 while 8 positions were required for W2 and W3 each.

3.4 Schlieren imaging

For the schlieren setup, a point light source (white LED and aperture), two achromatic lenses ($f = 1000$ mm), an adjustable aperture as schlieren edge, and a Canon EOS 600 DSLR camera equipped with a zoom lens are mounted in line on an optical rail. Schlieren images are taken at five different positions, one for window W1 and two for W2 and W3 each.

4 Results

4.1 Static wall-pressure measurements

Figure 5 shows the wall static pressure distribution (normalized by the total pressure for each injector), the flow channel wall contour, and the position of the injectors and side windows. Since the main flow accelerates from subsonic ($Ma = 0.25$) to supersonic speed ($Ma = 1.65$ in case of the “subsonic” injector and $Ma = 1.95$ in case of the “supersonic” injector) passing the convergent-divergent nozzle, the pressure decreases and reaches a minimum just downstream of the nozzle. From this point on, the wall static pressure increases slightly due to the planar channel walls in combination with boundary-layer thickening. The largest difference between both pressure distributions occurs at the beginning of window W2 where the wake flow field of the “supersonic” injector undergoes a sudden pressure jump. This can be traced back to the shock system arising from the trailing edge, which is reflected at the wall (Fig. 6). Altogether, the wall static pressure distribution confirms reasonable and stable flow conditions.
4.2 Schlieren imaging

The schlieren images of the wake flows behind the two injectors are plotted in Fig. 6. As discussed in section 2.1, the trailing edges of the two injector types are located in the subsonic region, 42.1 mm upstream of the nozzle throat and in the supersonic region, 10 mm downstream of the nozzle throat, respectively. Thus, two different kinds of wake flows are observed.

Since the frequency of the eddies formed at the trailing edge of the “subsonic” injector is too high to be captured by the schlieren camera (1 ms exposure time), the coherent structures can only be identified in the LIF images (Fig. 8). However, the schlieren images (Fig. 6a) clearly show the onset of the wake flow, the expansion waves forming just downstream of the Laval nozzle throat, and a weak shock wave system in the adjacent planar channel module.

**Fig. 5** Wall static pressure distribution in flow direction (x-coordinate). The channel contour, injector position, and the window positions are sketched in the lower section of the graph.

**Fig. 6** Stitched schlieren images of (a) the wake flow of the “subsonic” injector and (b) the wake flow of the “supersonic” injector.
In case of the “supersonic” injector, the “throat” of the recompression zone can be clearly identified in the schlieren images (Fig. 6b). The schlieren images further show the expansion fans at the trailing edge and the two shock waves forming at the end of the recompression zone. Those shock waves are reflected at the channel walls and continue to exist down to the last window.

4.2.1 Instantaneous LIF images

Instantaneous background and flat-field corrected LIF images are displayed in Figs. 8 and 9 showing the tracer distribution in the wake flow. Similar to previous work at lower Mach numbers [16, 27, 28], the images show an apparent alternating eddy separation forming from the subsonic injector trailing edge increasing in size while traveling downstream the channel (Fig. 8). The vortices are larger leading to a wider mixing zone compared to the injection into the supersonic flow region where the toluene-seeded flow is primarily concentrated in the center of the duct flow (Fig. 9).

4.2.2 Time-averaged LIF images

In order to evaluate the mixing behavior regarding the shape and growth rate of the shear layer, LIF intensity profiles were extracted from averaged and flat-field corrected LIF images (200 laser pulses) shown in Figs. 10 and 11.

The flat-field images were taken directly after the experiments by closing the outlet of the test section and injecting a small amount of the tracer/nitrogen mixture from the evaporator system. After waiting for several minutes, the tracer was distributed homogeneously within the flow channel. During this time, the whole flow facility including the test section cools down from the test temperature towards room temperature causing thermal contraction. This misalignment causes a shift of the entry point of the light sheet for the flat-field images which causes the vertical artifacts in Figs. 10 and 11. While shifts in x-direction can be compensated for, this is not possible for shifts normal to the flow direction without further effort. For future work, we will modify the facility to provide a homogeneously tracer-laden flow at testing conditions. However, the resulting stripes have little influence on the intensity profiles perpendicular to the flow as normalizing by their peak value cancels the influence of light inhomogeneity.

Figure 7 displays averaged intensity profiles along positions normal to the main flow direction for both injectors extracted from the averaged images at various downstream positions. The profiles
are normalized by their maximum intensity while their abscissa depicts the z-coordinate normalized by the half width \( \delta_{1/2} \) of the respective profile. The latter is obtained by a Gaussian fit.

Assuming that the LIF intensity is proportional to the tracer concentration, both cases reveal self-similarity regarding their downstream mixing behavior. The coefficients resulting from a best fit of equation (3) listed in Tab. 1 are close to those of Wygnansi et al. [30] determined for the turbulent, incompressible wake flow behind different wake generators.

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( B )</th>
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<tr>
<td>Subsonic injection</td>
<td>-0.685</td>
<td>-0.018</td>
</tr>
<tr>
<td>Supersonic injection</td>
<td>-0.595</td>
<td>-0.064</td>
</tr>
<tr>
<td>Wygnansi et al. [30]</td>
<td>-0.637</td>
<td>-0.056</td>
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**Tab. 1** Fit coefficients for the function (equation 3) proposed by Wygnansi et al. [30].
Fig. 8 Instantaneous flat-field corrected LIF images of the wake flow for the “subsonic” injector.

Fig. 9 Instantaneous flat-field corrected LIF images of the wake flow for the “supersonic” injector.
Fig. 10 Averaged (200 shots) and flat-field-corrected LIF images for the “subsonic” injector.

Fig. 11 Averaged (200 shots) and flat-field-corrected LIF images for the “supersonic” injector.
Figure 12a shows the half width of the extracted LIF intensity profiles ascertained from a Gaussian fit. It implies that in case of the “subsonic” injector the toluene carrying structures reach the wall boundary layer as the half width of the profile comes close to the wall of the duct located at $z = 17.7$ mm. In case of the “supersonic” injector an interaction between the shear and boundary layer is very unlikely since the injected flow stays closer to the centerline. Figure 12b shows the squared half width revealing a linear relation between $\delta_{1/2}^2$ and the distance from the injector trailing edge. Therefore, the growth rates of the present cases agree with the $1/2$-power law described in section 2.1.

![Figure 12a](image1.png)  
![Figure 12b](image2.png)  

**Fig. 12** Variation of the intensity profile half width $\delta_{1/2}$ (a) and the squared half width $\delta_{1/2}^2$ (b) along the downstream coordinate $x$. Red lines (b) represent a linear function corresponding to the $1/2$-power law fitted to the data from $x = 100–400$ mm.

Furthermore, the virtual origin of the two injectors investigated is determined. The virtual origin $x_0$ equals the $x$-coordinate of an equivalent point singularity that removes momentum from the flow at the same rate as the actual wake source [17]. For this paper, the virtual origin is extrapolated by fitting a linear function to the squared half width obtained in the non-accelerated region observed through window W2 and W3 (starting at $x = 100$ mm). From this, the growth rate $\Delta$ defined as shown in equation 1 [17, 28] is calculated and compared to previous work.

$$\Delta = \sqrt{\delta_{1/2}(x)^2/(x-x_0)} \quad (5)$$
Table 2 shows the corresponding values compared to the results Wohler [29] obtained at ITLR with the same injectors mounted in a nozzle designed for $Ma_{\text{nozzle}} = 1.3$. An increase in Mach number is known to yield to a more stable flow due to compressibility effects [22]. Thus, a decreased growth rate and a virtual origin further upstream compared to Wohler’s experiments are expected. In addition, the flow channel walls of Wohler exhibited an opening angle of $0.23^\circ$ downstream of the nozzle providing approximately constant pressure conditions, while the new flow channel is currently equipped with planar walls causing a weak positive pressure gradient (Fig. 5). This is known to have an opposite, destabilizing effect on the wake flow [12]. The values calculated for the “supersonic” injector (referred to as “Inj5” in Wohler’s work) reveal that the stabilizing effect predominates. However, the data obtained for the “subsonic” injection does not fit this theory at all because the discrepancy between the values is too high. Most likely, effects of interaction between the shear and boundary layer could have distorted Wohler’s data because the flow channel was only 20.4–21 mm high in the channel section of interest while the half width was already 5.5–6.7 mm. Hence, the much smaller growth rate could be attributed to the side walls having a damping effect on the wake flow. In case of the “supersonic” injector, an interaction between the shear and boundary layer is more unlikely since the half width is much smaller in the channel section considered (3–4.1 mm). For the new flow channel designed for this work, a larger cross section with a height of 34.4 mm has been chosen to avoid such effects.

<table>
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<tr>
<th>$Ma_{\text{nozzle}}$ = 1.7</th>
<th>$x_0$ / mm</th>
<th>$\Delta$ / mm$^{1/2}$</th>
<th>$Ma_{\text{nozzle}}$ = 1.3</th>
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<tr>
<td>(this work)</td>
<td>12.238</td>
<td>0.637</td>
<td>(Wohler [29])</td>
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<tr>
<td>$x_0$ / mm</td>
<td>−211.155</td>
<td>0.314</td>
<td>$\Delta$ / mm$^{1/2}$</td>
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<tr>
<td>$\Delta$ / mm$^{1/2}$</td>
<td>−62.733</td>
<td>0.207</td>
<td>0.252</td>
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</tbody>
</table>

Tab. 2 Growth rate and virtual origin of the wake flow.

5 Conclusions

The mixing behavior in a transonic channel flow has been studied using qualitative tracer LIF imaging for injection of a gaseous tracer-laden flow into an accelerated gas flow of the subsonic and supersonic flow region of a Laval nozzle. Following previous work, a new modular flow channel has been designed and tested allowing measurements under controlled pressure conditions and at higher Mach number than possible before. Results of evaluating the divergence
of LIF intensity profiles from recorded LIF images downstream of the nozzle throat show a similar mixing behavior for both injectors investigated compared to previous work [16, 27, 28] conducted at a lower Mach number. The self-similarity of the concentration profiles downstream of the tracer injections could be ascertained in both cases, and the growth rate of the shear layers followed the $\frac{1}{2}$-power law.

6 Outlook

While the macroscopic mixing behavior was studied in this research, follow up work will focus on the mixing on molecular level utilizing the method of toluene LIF proposed by Mohri et al. [15] using a single-color excitation and two-color detection scheme (cf. section 2.2). Therefore, calibration curves at flow conditions will be obtained in a miniature flow channel in our labs for various temperatures. In addition, the numerical simulation of the channel flow is under progress.

7 Acknowledgement

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8 References