Streamline Segment Detection in a Turbulent Wavy Channel Flow via Tomographic Particle-Image Velocimetry

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ABSTRACT

The streamline segmentation method proposed by Wang is a novel way of analyzing turbulence. As a purely geometric and mathematically well-defined method, it is completely unambiguous. Previous work by Schäfer et al. resulted in a model function capable of predicting the streamline segment statistics. However, a single free parameter determining its degree of asymmetry between accelerating and decelerating segments remains. Empirical investigations found the mean local acceleration to be a key factor in determining this parameter. This study aims to better assess the limits of the empirical relation and identify the leading causes for its failure in separated flow regions.

1. Introduction

Turbulence has been in the focus of research in fluid mechanics for decades. While many approaches describe turbulence in terms of local point statistics, which has led to a wide range of turbulence models, no single model has proved to be universal. Rather, each model has case-specific strengths and weaknesses and often requires an experienced user to fine-tune model parameters for each case, thus limiting its predictive quality and proving prone to errors. While it is possible to simulate flows accurately be resolving even the smallest turbulent scales and thereby omitting any kind of modelling, this is hardly feasible for engineering scale problems.

In past efforts to improve the small scale turbulence models, a variety of structures has been considered to represent turbulent flows. While Corrsin [3] concerned himself with the question whether there were “naturally identifiable” recurring structures in turbulent flows, Hunt et al. [4] proposed to divide flows into four characteristic types of zones (“stream”, “shear”, “convergence/divergence”, and “eddies”) to simplify further modelling attempts. Another approach to decompose turbulent flows is to consider it as an ensemble of vortices. While this appears intuitive at first, the definition of vortices is ambiguous as it requires some sort of thresholding to describe the extent of each vortex [13]. Furthermore, the interaction between
multiple vortices is highly complex and non-linear, which leads to a limited promise when it comes to significant simplifications as required by turbulence models. To improve upon current ways to describe a turbulent flow’s behavior, Peters and Wang [6] proposed the dissipation element method which decomposes a scalar flow field into an ensemble of spatially non-overlapping, monotonous sub-volumes – the dissipation elements – containing the geometric information of the instantaneous flow field. Further investigations using this method were performed experimentally as well as numerically [9] [14] in a fully turbulent channel flow. Building on this approach, Wang [15] proposed a variation of this method. Instead of considering volumetric structures of an arbitrarily selected scalar, streamlines are traced throughout the instantaneous flow field and divided into sections of monotonous absolute velocities as shown in figure 1.

![Streamline segmentation concept](image)

**Fig. 1** Streamline segmentation concept.

Unlike the dissipation element method, this method follows the inherent topology of the flow rather than imposing a mathematical formulation of a scalar. In extensive works by Schäfer et al. [10] [11] [12], a model function for the normalized joint probability distribution of the velocity difference $\Delta V_{\text{abs}}$ within each segment and the corresponding segment length $l$ was derived for a multitude of synthetic turbulence cases. Its characteristic shape consisting of two asymmetrical
branches representing segments with increasing (+) and decreasing (-) absolute velocities along their trajectories has been reproduced in multiple DNS studies [15] [12] as well as experimental data [8]. Each branch’s shape can be characterized by its conditional means $<\Delta u \mid l>$ denoting the mean velocity difference $\Delta u$ along all accelerating (l) or decelerating (l) streamline segments of a given length l. In their studies, Schäfer et al. found an inherent asymmetry in segment length as well as conditional mean velocities between accelerating and decelerating segments caused by the kinematic differences between both types. However, the currently available model function cannot predict the degree of asymmetry. Rather, it is modelled as a free parameter.

Previous experiments, in which the effect of favorable and adverse pressure gradients (FPG and APG, respectively) on dissipation elements was analyzed, showed that most elements were aligned along the direction of the mean flow and that the local pressure gradient possesses a significant impact on the length distribution of the dissipation elements [7]. A comparable behavior can be found in streamline segments. The asymmetry was previously analyzed for a channel flow with a sinusoidal wall by recording high resolution tomographic snapshots of four distinct volumes corresponding to the crest (C), expansion slope (ES), trough (T), and contraction slope (CS) of the wave. A linear relation between the difference of accelerating and decelerating mean segment lengths and the local mean acceleration driven by APG and FPG was recovered (figure 2) with the notable exception of highly fluctuating or even separated flow regions. This dataset will be used as a reference in this study.

![Figure 2](image-url)  

**Fig. 2** Difference between positive and negative mean segment length depending on local flow acceleration for two wave amplitudes.
Hence, the scope of this study is to confirm the proposed relation for the asymmetry, determine its limits, and consequently analyze the underlying local differences which lead to the breakdown of the previously determined relation. For this, tomographic Particle-Image Velocimetry (PIV) measurements are performed in a fully turbulent channel flow that undergoes favorable as well as adverse pressure gradients.

2. Experimental Setup and Data Processing

All experiments were carried out in an Eiffel-type wind tunnel that provides a fully turbulent two-dimensional channel flow matching DNS results of Moser et al. [5] in the test section. The test section has an aspect ratio of 1:20 and measures 2 h x w = 100 mm x 2,000 mm (height x width) at a test section length of l = 2,500 mm, i.e., 50 h with the channel half-height h. The test section itself provides full optical access and is attached to a nine meter long inlet section (180 h) with two strips of sandpaper on its upstream end to enforce laminar-turbulent transition. The test section is equipped with exchangeable sidewall tiles allowing it to be outfitted with varying wall geometries, i.e., wavelength and amplitude of a wavy sidewall. In the current study, a sidewall with sinusoidal shape with a wavelength of 100 mm and an amplitude of 5 mm was investigated for bulk Reynolds numbers in the range of 3200 < Re_{ina} < 9600 based on the bulk streamwise velocity u_{ina} and the flat channel half height h. The trough of the wave was aligned with the previously flat surface such that the wave’s crests reached 10 mm into the channel. To obtain a suitable local resolution, the experimental setup consisted of four sCMOS cameras (PCO Edge, 2560 x 2160 px) equipped with 100mm lenses (Zeiss macro, f_{,ina} = 2) and a Nd:YAG dual cavity laser (NewWave Solo 200 XT, 200 mJ per pulse at 15 Hz). To better correlate the results with the local flow state, the setup was modified compared to previous studies [8], in which four narrow cuboid volumes along the wave were illuminated by an incoming beam from the channel’s top and recorded separately without acquiring data from the surroundings. The modified setup provides a thick light sheet of approximately 7 mm in depth which was arranged perpendicular to the wavy channel surface (figure 3). To achieve high signal-to-noise-ratios in the recordings, the laser was reflected by a chromium coated tile in the channel wall which led to favorable brightness and reduced scattered light at the wall. The camera array was placed below the light sheet capturing a large field of view spanning from the crest to the trough of the wave (figure 4).
Fig. 3 Test section and tomographic PIV setup.

Fig. 4 Field of view compared to the measurement volumes of the referenced study.
All images were recorded and processed in DaVis (LaVision GmbH, Göttingen, Germany). To reconstruct the volumetric intensity distribution, a variation of the well-established MLOS-SMART algorithm as proposed by Atkinson & Soria [2] was applied. While a large volume spanning approx. 3000 x 2500 x 500 vox was reconstructed, only the core region was used to determine the velocity fields to avoid reconstruction artifacts at the volume boundaries. The velocity field was determined by employing a multi-grid direct correlation approach starting with large interrogation volumes of 192 vox and iteratively decreasing their size to the final pass interrogation volume size of 48 vox which corresponds to a resolution of 0.8 mm³ at 75% overlap resulting in a vector spacing of 0.2 mm. Before the final volume correlation pass, the reconstructed volumes were enhanced by an MTE pass.

To verify the data quality, the resulting segment length distributions were compared to the reference dataset (figure 5). The global distribution of the acquired data exhibits a much wider profile than the reference data. However, the location of the peak segment length is nearly identical. The deviation between the distributions is due to the limited volume length in the reference dataset preventing longer segments from being detected. This reduces the probability that segments with a length of the order of the volume length are detected. When the same spatial limits are imposed onto the new data, a distribution quite similar to the reference data is recovered encouraging confidence in the current dataset since it can reproduce previously known statistics when similar regions are considered without being hampered by the same spatial limits.

Fig. 5 Exemplary comparison of streamline length distributions between current and reference data.
3. Streamline Segment Analysis

When the streamline segmentation method is applied to the current dataset, familiar segment length distributions (figure 6) are obtained. The distributions present a roughly lognormal profile which decreases in width with increasing Reynolds number. At the same time, the distribution’s peak probability moves towards smaller segment lengths. The steeper exponential decay of large elements at high Reynolds numbers is especially apparent in the logarithmic representation. Similar results can be found in investigations of a flat channel flow [1].

![Figure 6](image)

Fig. 6 Global segment length distributions.

Joint probability distributions of the segments’ length and difference in velocity within each segment are shown in figure 7. The typical asymmetry between accelerating (positive) and decelerating (negative) segments is more pronounced than previously found in DNS data of decaying and forced synthetic turbulence [10]. The positive branches exhibits a slope at approximately 80% - 85% of the negative slopes whereas the synthetic isotropic cases yield values slightly above 90%. Taking into account that the observed flow region is decelerating on average, this is expected and reflected in the overall streamline segment statistics of the reference dataset. It is worth noting, however, that especially the expansion slope and trough volume of this particular geometry do not react to the average acceleration as expected. Since the current
investigation of a much larger field of view includes the volumes that presented deviations in the reference dataset, these appear to be a highly localized phenomenon which is not captured by global statistics.

![Joint PDF distributions](image)

**Fig. 7** Joint PDF distributions for Re = 3200 (left), 6400 (center), and 9600 (right).

To recover and confirm the non-linear reaction to the background deceleration found in the reference data, a grid of 50 x 35 cells with 1 mm spacing was introduced. Each detected streamline segment was assigned to all cells whose centers were passed within a distance of 2 mm or less resulting in a 75% overlap. This is favorable for both the number of elements contributing to the local statistics and the smoothness of the resulting fields. A principal depiction of the cell locations and segments assigned to a particular cell is shown in figure 8.

![Association of streamline segments](image)

**Fig. 8** Association of streamline segments with individual cells (right) and resulting grid of cells (left).
Consequently, typical statistical quantities such as mean segment length for positive, negative, and all segments were calculated for each cell and compared to local flow conditions. A scatter plot comparable to the one presented for the reference data (figure 2) but far superior in spatial resolution is shown in figure 9. There linear relation from reference data is also plotted for comparison. First and foremost, it is worth noting that a large majority of cells yields data points that are in close vicinity to the predicted relation. However, just like in the reference dataset, a number of cells with very large decelerations present mean segment length differences vastly above the value that would correspond with the linear functional relation.

Fig. 9 Scatter plot of local acceleration and difference between positive and negative mean segment lengths in each cell for \( \text{Re} = 3200 \) (left), 6400 (center), and 9600 (right).

To further investigate the origin of those non-linearly behaving cells, the deviation from the linear relation was calculated for each cell. The resulting deviation fields are shown in figure 10. For comparison, the velocity fluctuation normalized by the local mean velocity is shown below. High values of the normalized fluctuation correspond to the separated region.
Fig. 10 Deviation from linear scaling (top) and local ratio of velocity fluctuation and mean velocity (bottom) for Re = 3200 (left), 6400 (center), and 9600 (right).

For the largest portion of the observed flow, the deviations from the linear behavior are quite low. As long as the level of fluctuations remains below 25% of the mean velocity, the linear relation is valid. High deviations are only found in the separated flow with high fluctuations. Most interestingly, however, the level of deviation is not homogeneous within the separation bubble. Rather, all three Reynolds numbers result in a very high deviation maximum at the stagnation point ahead of the separation region. Within the separation region, the deviations are still higher than elsewhere in the flow but noticeably lower than at the stagnation point. Especially the lowest Reynolds number case appears to show deviations being introduced at the stagnation point and possibly convected along the separation bubble’s shear layer. The other cases present a less clear structure behind the stagnation point so that it remains possible that the lowest Reynolds number phenomenon is due to not fully converged statistics. The general location of the non-linearly behaving streamline segment statistics is quite apparent and coincides with the separation region, which up until now was only expected based on the location of the volumes investigated in the reference dataset. Additionally, the linear behavior could be confirmed for the largest portion of the observed flow.

4. Conclusion

The shortcomings of the previously hypothesized linear relation for the difference between mean segment lengths of positive and negative segments were investigated. While large portions of the investigated flow follow the linear relation, regions with deviating behavior could be identified more accurately than in previous measurements. They coincide with the highly fluctuating separated region of the flow. Interestingly, the deviations are not distributed homogeneously within such regions. Rather, they seem to originate from the stagnation point at the beginning of
the separation bubble. This new information might serve as a first step towards the extension of the theoretical model function for streamline segment statistics to be able to predict this behavior. The fundamental mechanisms used in the current model function consider slow stretching and contraction of segments as well as rapidly occurring cutting events. Whenever a mean acceleration defines the scaling behavior of streamline segments, it can reasonably be assumed that the stretching and contraction mechanism dominates the local flow behavior as structures pass a mean flow field which largely corresponds to the well established Taylor hypothesis. Therefore, significant deviations from this scaling behavior might indicate the cutting mechanisms to play a greater role in those regions.

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References