An Experimental Study of Wall-Injected Flows in a Rectangular Chamber

A. Perrotta¹*, G.P. Romano¹, B. Favini¹
1: Dept. of Mechanical and Aerospace Engineering, Sapienza University of Rome, Italy
* Correspondent author: andrea.perrotta@uniroma1.it

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ABSTRACT

We performed an experimental investigation of the flow inside a rectangular chamber with two of the six sides injecting air. The flow, which enters perpendicularly to the injecting wall, turns suddenly its direction being pushed out to the exit of the chamber. This kind of flows is a bi-dimensional approximation of what happens inside a solid rocket motor, where the lateral grain burns expelling exhaust gas, which goes out through the nozzle and pushes the launcher. Under the incompressible and inviscid hypothesis, two analytical solutions were proposed. The first one, known as Hart-McClure solution, is irrotational and with non-perpendicular injection velocity. The other one, due to Taylor and Culick, has non zero vorticity and constant, imposed injection velocity. We propose a brief derivation of those solutions and a comparison between the analytical solutions and experimental data. Mean velocity data are obtained by averaging 500 instantaneous PIV flow field reconstructions, which are captured at a frequency of 5 Hz. The characteristic Reynolds number inside the chamber is based on the half height of the chamber (H/2) itself and the orthogonal injection velocity (y-component), which is though really hard to measure because of the high, with respect to the orthogonal one, parallel component (z-component). Anyhow, we estimate a value around 20 ÷ 40, whereas the Reynolds number based on the exit velocity is around (1 ÷ 2)×10³.

1. Introduction

The fluid flow inside a rocket motor is, at least in principle, very simple. In fact, gas is emitted from the inside lateral wall of a nearly cylindrical tube, then it flows along this tube and it is accelerated and expelled by a nozzle. The global thrust of a launcher depends on the pressure and velocity of the exhaust gas. Any fluctuation of such quantities causes a direct fluctuation of the thrust. Actually, the internal fluid exerts lateral force anytime there is an unbalance of pressure on the sides. Thus, fluctuations are both dangerous for the trajectory and for the thrust. The main cause of instability identified during the last two decades is the resonance behavior of the pressure field inside a cavity. A simple but clear model, which explains this resonant behavior, is the model by Rossiter J (1964). It is based on the acoustic waves and travelling vortices coupling. Three vorticity generation processes have been identified: the presence of corners, the presence of obstacles and the intrinsic instability of the mean flow. The former two
are well understood. The third one, instead, depends on the mean flow stability properties that make things more complicated. First of all because we should find the mean flow, which depends on the geometry of the domain, then it is necessary to understand what process activates its instability.

The mean flow corresponds to the lowest order term of an expansion with respect to the Mach number of the Navier-Stokes equations. In order to obtain a simple analytical expression, viscosity has been neglected in the model and two simple geometrical configurations have been studied: cylindrical and planar. The first solution of this field was deduced by the Hiemenz flow and it is known as Hart-McClure (H-C) solution because was adopted as base model by Hart R.W, McClure F.T (1959) and McClure et al (1963). It is irrotational and has oblique injection velocity. A rotational solution was proposed by Culick F (1966) (T-C) in the axisymmetric case and by Varapaev V, Yagodkin V (1969) in the planar case. Both were already found by Taylor G (1956) as limit cases of the injection from the wall of a cone and a wedge, respectively. More complex solutions exist for the cylindrical configuration, see Majdalani J, Fist A (2014), but they do not have a planar counterpart.

Linear stability analysis was carried out by Casalis G, Avalon G, Pineau JP (1998) and Griffond J, Casalis G, Pineau JP (2000) superposing a normal perturbation to the mean flow field. Even though a normal perturbation is not consistent with a non-parallel flow, it fits very well experimental results obtained by Avalon G, Casalis G, Griffond J (1998) and Griffond J, Casalis G, Pineau JP (2000). These results are obtained with the VECLA facility at the ONERA-Palaiseau laboratory, which is a channel with one injecting wall, by mean of hot wire measurements. A feature of this linear stability analysis is that for a given unstable mode and position, there is a continuous range of unstable frequencies. A discrete spectrum is, instead, yielded by a technique that does not make any assumption on the longitudinal shape of the eigenfunctions as stated by Férraille T, Casalis G (2005), Casalis G et al (2006) and Chedevergne F, Casalis G, Férraille T (2006). Anyhow, only temporally damped modes are obtained but the shape of the eigenmodes shows a quasi-exponential growth in space, which is supposed to be the cause of hydrodynamic instability. According to Avalon G, Casalis G, Griffond J (1998), hydrodynamic instability is activated when amplitude of perturbation and acoustic frequencies of the chamber matches, in some sense. Also PLIF visualizations of the hydrodynamic instabilities have been conducted by Avalon G et al (2001) with the VECLA.

As said before, measurements with intrusive techniques (hot wire) and visualizations have been conducted in a half planar channel. We propose in this work a reconstruction of the velocity field inside a planar chamber, with two injecting walls, by mean of Particle Image Velocimetry
(PIV), which is a non-intrusive technique. The aim of the paper is to determine the mean flow in steady condition trying to retrieve auto-oscillations.

2. Analytical solutions

In order to find an analytical solution of the mean flow, as already stated in the introduction, we consider the incompressible and inviscid Navier-Stokes equations

\[
\nabla \cdot \mathbf{U} = 0 \quad (1)
\]

\[
(U \cdot \nabla)U = -\nabla P \quad (2)
\]

The first solution, which is due to Hart-McClure, makes the irrotational assumption \( \nabla \times \mathbf{U} = 0 \). Let us assume a planar, bi-dimensional configuration as shown in Fig. 1 so that we have the existence of a stream function \( \psi \). Then, the two component of the velocity field are \( U_z = \partial \psi / \partial y \) and \( U_y = -\partial \psi / \partial z \). The zero vorticity hypothesis becomes

\[
\Omega = \nabla^2 \psi = 0 . \quad (3)
\]

![Fig. 1 Reference frame of the planar configuration.](image)

Let be the stream function linear in the \( z \)-direction, \( \psi(y, z) = zF(y) \). Thus, equation (3) becomes an ordinal differential equation in \( F \)

\[
zF''(y) = 0 \Rightarrow F(y) = Ay + B. \quad (4)
\]

Now, imposing the symmetry condition on the centerline \( (y = 0) \) and the boundary condition at the wall
\[ U_y(y = 0) = 0 \]  \hspace{1cm} (5)
\[ U_y(y = \pm 1) = \mp 1, \]  \hspace{1cm} (6)

where we have re-scaled variables with respect to the half height and the wall injection velocity, yields \( B = 0 \) and \( A = 1 \). Then \( U_z = z \) and \( U_y = -y \).

Varapaev and Yagodkin did not imposed the vorticity to zero but solved the vorticity equation, which is actually scalar, with the constant injection from the wall, symmetry at the centerline and no-slip at the wall

\[
\begin{align*}
\nabla \times (U \times \Omega) &= z (F F''' - F' F'') = 0 \\
U_y(1, z) &= -F(1) = 1 \\
U_y(0, z) &= -F(0) = 0 \\
U_z(1, z) &= F'(1) = 0 \\
\end{align*}
\]  \hspace{1cm} (7)

that admits the non-unique solution

\[ F(y) = (-1)^n \sin \left( \frac{\pi}{2} + n\pi \right) y, \]  \hspace{1cm} (8)

but the one that gives a radial velocity which vanishes only on the axis is the case \( n = 0 \), the other solutions are unphysical. Therefore \( U_z = \pi z / 2 \cos(\pi y / 2) \) and \( U_y = -\sin(\pi y / 2) \). Note that \( \nabla \times U \neq 0 \), in fact

\[ \Omega = \nabla^2 \psi = -z \pi^2 / 4 \sin(\pi y / 2). \]  \hspace{1cm} (9)

3. Experimental set-up

In order to reproduce the flow configuration described in Fig. 1, a rectangular chamber, shown in Fig. 2, is considered. It is made of Plexiglas to allow optical access and has two lateral section walls made of porous Polythene that makes the flow injected by six, three per side, inlets uniform. A compressor feeds the chamber with air mixed with droplets of oil as tracer, 1\( \pm \)5 \( \mu \)m big, generated with a Laskin nozzle. A cross-correlation camera acquires a couple of images each 0.2 seconds. Each picture is illuminated by a laser sheet, which is generated by a Nd:Yag Laser and synchronized with the camera by a pulse generator. The laser sheet comes from above and it is an orthogonal plane to the camera that frames approximately a 2 cm by 2.3 cm region covered by roughly 910\( \times \)1040 pixels. A detailed scheme of the experimental set-up is shown in Fig. 2.
order to cover the whole length of the chamber, which equals 25 cm, we analyzed thirteen section of it. For PIV processing, we used the free software PIVlab developed by Thielicke W, Stamhuis E.J (2014). For each section a sequence of 500 couples is analyzed with an interrogation area of 64 pixels for the Pass 1, followed by Pass 2 with an area of 32 pixels and Pass 3 with 16 pixels (see Fig. 3); each Pass has an overlapping of the 50%.

Fig. 2 Scheme of the experimental set-up (left) and front view of the test chamber (right).

Fig. 3 An example of acquired picture with the illustration of typical interrogation areas.

We performed two measurements: one with an exit pressure from the compressor of 1 bar, the other one with 2.5 bars. In order to ensure that particles travel at most for 20 pixels within Δt, i.e. the time step of the PIV analysis, we used for the first test (1 bar) a time step Δt = 200 μs and for
the second one (2.5 bars), since \( U_z \) grows linearly with \( z \), a \( \Delta t = 100 \mu s \) for the 11\textsuperscript{th}, 12\textsuperscript{th} and 13\textsuperscript{th} section and a \( \Delta t = 200 \) for the others.

4. Results: mean velocity field

We show in the following the mean field obtained as average of a sequence of 500 instantaneous velocity fields. We used two different injection velocities, which are driven by the pressure at which we inject air. However, due to the high parallel component \( (U_z) \), it is very difficult to directly estimate the injection velocity. Therefore we use as test-indicator the pressure \( P \) at which the compressor is set during that test and we derive the injection velocity and, as a consequence the Reynolds number, by averaging over the chamber length

\[
U_{\text{inj}} = \frac{1}{2L} \int_{-L}^{L} U_z(y)dy \Rightarrow \text{Re} = \frac{hU_{\text{inj}}}{\mu} \quad (10)
\]

where \( h = H/2 \), \( \mu \) is the kinematic viscosity and \( L \) is the length of the channel. Each component of the velocity has been scaled with respect to \( U_{\text{inj}} \) so that we can compare them with the analytical solution. We show in Fig. 4 the \( z \)-component of the Taylor-Culick (T-C) solution and in Fig. 5 the vorticity, which are the only non-trivial plots of the analytical solutions because the Hart-McClure solution is linear both in \( z \) and \( y \) and has zero vorticity.

In the first case, \( P = 1 \text{ bar} \), we obtained a Reynolds number \( \text{Re} \sim 18 \). While in the second one, \( P = 2.5 \text{ bar} \), we had \( \text{Re} \sim 40 \). In Fig. 6 we show the shape of \( U_z \), while in Fig. 7 there is \( U_y \). In order to appreciate the differences between the analytical solutions and the experimental data, we report in Fig. 8 three experimental profiles that correspond to three different sections of the channel. Another interesting plot is vorticity, see Fig. 9. Since we expect oscillations within the chamber it is useful to observe the standard deviation from the mean value, see Fig. 10 and Fig. 11. The same sequence of images is given in the following for the second test.

Fig. 4 \( z \)-component of the Taylor-Culick solution. Red corresponds to 36 and blue is 0.
Fig. 5 Vorticity of the Taylor-Culick solution. Red corresponds to 57.2 and blue is −57.2.

Fig. 6 $z$-component ($U_z$) of the experimental velocity field for the 1 bar test. Red corresponds to 65.9 and blue is 0.

Fig. 7 $y$-component ($U_y$) of the experimental velocity field for the 1 bar test. Red corresponds to 5 and blue is −5.
Fig. 8 Profiles at the right end of three sections, against the two analytical solutions: Taylor-Culick (T-C) and Hart-McClure (H-C); for the 1 bar test.

Fig. 9 Vorticity plot of the experimental data for the 1 bar test.
Red corresponds to $10^4$ and blue is $-10^4$.

Fig. 10 $\sigma^2$ of $U_z$ for the 1 bar test. Red corresponds to $2\times10^2$ and blue is 0.
Fig. 11 $\sigma^2$ of $U_y$ for the 1 bar test. Red corresponds to $9 \times 10^3$ and blue is 0.

Fig. 12 $z$-component ($U_z$) of the experimental velocity field for the 2.5 bar test. Red corresponds to 53.4 and blue is 0.

Fig. 13 $y$-component ($U_y$) of the experimental velocity field for the 2.5 bar test. Red corresponds to 5 and blue is $-5$. 
**Fig. 14** Profiles at the right end of three sections, against the two analytical solutions: Taylor-Culick (T-C) and Hart-McClure (H-C); for the 2.5 bar test.

**Fig. 15** Vorticity plot of the experimental data for the 2.5 bar test. Red corresponds to $10^4$ and blue is $-10^4$.

**Fig. 16** $\sigma^2$ of $U_z$ for the 2.5 bar test. Red corresponds to $2 \times 10^4$ and blue is 1.
One can note that a higher injection pressure obviously yielded a higher Reynolds number, we switched from 18 to 40, but the centerline parallel velocity is higher in the first case than when the Reynolds number is bigger, as shown in Fig. 7 and Fig. 14. Furthermore, vorticity, which is higher where the flow has to turn its direction from orthogonal to parallel, is much more confined to the wall in the second test than in the first one as if the change of direction is more abruptly in the second test, which is reasonable. The structure of $U_z$ is retained, even though the flow rate increases as shown with the equation (10). Also the standard deviations have the same structure, but the values are much higher in the second test, which is reasonable if we are expecting some sort of instabilities. Anyhow, we cannot gauge oscillations under the sampling rate, which is 5 Hz, imposed by technical limitations of the laser. The radial velocity $U_y$ behavior is a mess within the first section where we could expect recirculation phenomena, more evident in Fig. 6 than elsewhere, but there is also an anomalous tendency at the centerline to be positive instead of zero.

5. Conclusion

Preliminary Particle Image Velocimetry (PIV) measurements of the wall-injected flow inside a rectangular chamber are conducted. The goal was to have an experimental reconstruction of the global velocity field with a non-intrusive technique. With the set-up presented here, we have been able to reach only low Reynolds numbers. In our opinion this is the reason why we are so far from the analytical solution, anyhow our results shown a high variability in time of the $U_z$ as far as $z$ increases, which is a clue of the instability arising from the wall not in boundary layer sense but as intrinsic instability of the flow itself. While $U_y$ is more subjected to variations at the head of the chamber, probably due to recirculation. We will investigate in the future higher
Reynolds numbers, different porosities of the wall and the effect of external acoustic source in order to estimate how and if acoustic waves can trigger hydrodynamic instabilities.

References

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