Volumetric measurements of vortex packet recovery downstream of a perturbation

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ABSTRACT

Volumetric measurements were acquired in the logarithmic region of a turbulent boundary layer with \( \text{Re}_t = 2500 \) using a 3-D PTV technique. Flow perturbed by spanwise cylinder arrays with \( H = 0.2\delta \) and \( H = \delta \) was compared against unperturbed flow at multiple streamwise locations. Measurement volumes were \( 0.7\delta \times 0.9\delta \times 0.12\delta \) in the streamwise, spanwise and wall normal direction respectively. Cross correlations were obtained between velocity vectors at separate wall-normal locations within the volumes. In unperturbed flow, low momentum regions frequently extended across the wall-normal volume dimension. Cross correlations indicated forward inclined structures and a persistent spanwise scale across the wall normal direction associated with alternating low and high speed momentum regions. Immediately downstream of each array in the perturbed flow, the cross correlations were altered significantly due to the presence of wake structures. Further downstream, the correlation shapes recovered significantly. For \( H = 0.2\delta \), the correlations essentially recovered to the unperturbed shape and values by \( 2.4\delta \) downstream. By contrast, the correlations for the \( H = \delta \) case remained altered a distance of \( 4.7\delta \) downstream. Compared to the unperturbed case, these correlations exhibited smaller peaks, less skew in the streamwise direction, and weaker negative lobes in the spanwise direction implying clear differences in both the wall-normal and spanwise flow organization.

1. Introduction

Coherent structures within turbulent boundary layers (TBL) have been observed in many experiments including those of Head & Bandyopadhyay (1981), Adrian et al. (2000), and Dennis & Nickels (2011). In all of these experiments, hairpin-like vortices were ubiquitous throughout the logarithmic region. In a number of experimental studies, groups of hairpin structures have been observed to align in the streamwise direction whilst travelling at roughly the same speed, forming what are known as hairpin packets (e.g. Adrian et al., 2000). Ganapathisubramani et al. (2003) showed that hairpin packets contribute significantly to the Reynolds stresses in the log region, whilst occupying a relatively small area. Many techniques ranging from hot-wire...
anemometry to various forms of imaging velocimetry have been devoted to the study of these packet-like structures.

Recent developments in the volumetric measurement arena have enabled glimpses of hairpin packets in 3-D space. These include tomographic PIV measurements by Elsinga et al (2007, 2012), Schröder et al. (2008, 2011), Gao et al. (2013), and Jodai and Elsinga (2016). In addition, Dennis and Nickels (2011) used high-repetition rate stereo PIV and Taylor’s hypothesis to reconstruct volumetric velocity fields over a large streamwise domain.

In the present study, a turbulent boundary layer was perturbed by a spanwise array of cylinders extending outward from the bounding surface in order to investigate the distortion and eventual recovery or re-generation of hairpin packets. The work follows that of Ortiz Duenas et al. (2007), and Zheng and Longmire (2014), who observed that the incoming packet organization seemed to reappear quickly downstream of a narrowly spaced array extending through the log region. Zheng and Longmire hypothesized that the unperturbed packet organization above the array propagated downwards toward the wall so that the original organization was re-established.

To investigate this hypothesis, we considered arrays with different heights: \( H = 0.2\delta \), extending through the log region, and \( H = \delta \), extending to the edge of the boundary layer. The array spacing considered was 0.2\( \delta \), which is significantly narrower than the natural spanwise packet spacing \( \approx 0.6\delta \). In our previous work (Tan & Longmire, 2015), data were obtained in streamwise-spanwise planes at three measurement heights downstream of the aforementioned arrays using flying PIV. The flying PIV system was traversed at the local mean velocity in order to track flow structures as they convected downstream. In the flying PIV data downstream of the \( H = \delta \) array, packets similar to those in unperturbed flow were observed first closer to the wall and later further from the wall in a ‘bottom up’ development. By contrast, downstream of the \( H = 0.2\delta \) array, the recovery occurred last closer to the wall. In this paper, we consider data from volumetric measurements to understand how, in unperturbed flow, narrow low momentum regions associated with packets closer to the wall relate to the wider and shorter regions further from the wall. In addition, we seek to understand more about the three-dimensional organization of packet-like structures developing downstream of the perturbations.

2. Methodology

Experiment Set-up

Measurements were conducted in the University of Minnesota - Department of Aerospace Engineering - Turbulent Shear Flows Laboratory open water channel with a freestream velocity
Figure 1. Schematic of the experimental setup (flow is into the page) showing the cameras viewing the boundary layer through a transparent box placed at the water surface, and the lasers illuminating the measurement volume. Not to scale.

of 0.508 m/s and a water depth of 390 mm. The boundary layer is tripped at the beginning of the test section and develops along the bottom surface over a distance of 6.1m. The x, y & z directions refer to streamwise, spanwise and wall normal directions respectively. At the array location (defined as x/δ = 0), the boundary layer thickness, δ, was 125 mm, and Re· was 2500.

A 3D PTV system was used to measure volumetric velocity fields in unperturbed flow and flow perturbed by an array of cylinders. Two array heights (H = 0.2δ & H = δ) were considered. Cylinder diameter was 6.35mm. The measurement location for the unperturbed flow spanned 0.15 < x/δ < 0.85. Data was acquired at three measurement locations downstream of the respective perturbations. For H = 0.2δ, the measurement locations covered 0.15 < x/δ < 0.85, 2 < x/δ < 2.7 and 3.3 < x/δ < 4. For H = δ, the locations spanned 0.15 < x/δ < 0.85, 2.2 < x/δ < 2.9 and 4.3 < x/δ < 5.0.

Four TSI Powerview 8MP cameras fitted with Scheimpflug mounts and 60mm Nikon lenses were mounted above the water channel in a rectangular configuration with approximately 250 mm between cameras aimed downward to view the boundary layer through a transparent viewing box placed on the water surface. The viewing box was located at the top water surface and was located far enough away from the measurement volume that it did not interfere with the boundary layer measurement location. The cameras and viewing box were mounted to a carriage that was able to move in the streamwise direction, allowing for data to be taken at multiple locations downstream of the cylinder array without the need for adjusting the cameras or recalibrating the system. The measurement volume of 88 mm × 115 mm × 16 mm (0.7δ × 0.9δ × 0.12δ) in the x,y,z directions respectively was illuminated with two dual-head Nd:YAG lasers.
operating at 5 Hz, with nominally 380 mJ and 200 mJ per pulse, respectively and positioned on either side of the channel. The flow was seeded with 13 micron silver-coated hollow glass spheres. A schematic showing an end view of the experimental setup can be seen in Figure 1.

Camera Calibration

The four camera system was calibrated using a single-plane, back-lit calibration target with a rectangular grid of dots spaced evenly at 5 mm. The calibration plate was attached to a vertical traverse by a series of stainless steel rods. The plate had a thickness of approximately 6 mm, and was placed so that the calibration began with the plate nearly contacting the floor of the water channel. The plate was traversed in increments of 1 mm from the bottom-most location throughout the depth of the measurement volume. Target images were captured by all four cameras at each location. A least-squares fit was performed to determine a set of de-warping polynomials to map real-world calibration point locations onto pixel locations on the four cameras.

Due to the inherent thickness of the calibration plate, calibration images were not captured all the way to the bottom surface of the channel. Therefore, a new technique was implemented, whereby additional calibration planes were created by extrapolating trajectories of the calibration points determined from the target-based calibration. The predicted sub-pixel locations of calibration points on the sensor of each camera were extrapolated and added to the calibration.

This initial calibration was used as the basis for every data collection zone, as the cameras and laser sheets were moved to different streamwise measurement locations, in order to account for slight variations in the camera positions relative to each other and the measurement volume. The corrections were performed after the data collection using an auto-calibration reconstruction (ACR) technique based on the particle images, and described in detail in Boomsma et al. (2016).

Data Capture and Processing

The cameras and laser pulses were timed such that pairs of images of illuminated particles within the measurement volume were captured from each of the four cameras simultaneously with a time separation, Δt, of 750 µs. The seeding density was approximately 0.12 ppp. The positions of individual particles were identified on the images through thresholding, and a 2D Gaussian fit was applied to find the sub-pixel center of each particle which was then mapped into real-world space through a polynomial mapping function created from the previously performed calibration, resulting in a particle cloud at two times, t and t+Δt. The particle displacements were tracked between the two time instances using a robust point
matching technique described in Stellmacher and Obermayer (2000) to arrive at a volumetric velocity field. The particle vector volumes contained 50,000 to 55,000 randomly-spaced vectors per field. The volumes of randomly-spaced particle vectors were interpolated using Gaussian-weighting, with a 0.7 ratio between the Gaussian radius and the node volume half-size, onto a rectangular grid with node dimensions of 4 mm × 7 mm × 3 mm in the streamwise, spanwise, and wall normal directions, respectively, with a node volume overlap of 75%.

**Uncertainty**

The uncertainty in the particle position measurement is a function of a number of experimental parameters including the number of cameras, the angle between cameras, the accuracy of the 2D Gaussian fit (which depends on the magnification, particle size, and pixel size), and the least squares mapping error. A root sum of squares uncertainty analysis was performed for the parameters in the current study resulting in an uncertainty of 4.5 microns in the streamwise and spanwise directions, and 14 microns in the wall-normal direction. The uncertainty in the velocity measurements depends on particle displacement uncertainty as well as the time between laser pulses. For the current study the velocity uncertainty was less than 1% of $U_\infty$ in the streamwise and spanwise directions and less than 3% in the wall-normal direction.

**3. Results**

**Unperturbed Flow**

The mean and rms velocity were computed by averaging individual particle vectors from 100 volumetric fields within bins of depth $\Delta z^+ = 30$, or ~2.5 mm. The mean velocity plotted in Figure 2a agrees well with the law of the wall with $\kappa = 0.41$ and $B = 4.95$. The rms velocity in Figure 2b agrees well with previous LDV measurements taken under the same conditions by Gao (2011).

In the unperturbed flow, hairpin packet signatures frequently extended throughout the depth of the measurement volume ($155 < z^+ < 465$). This observation was consistent with previous measurements by Gao et al. (2011) who performed tomographic PIV on volumes spanning $z^+ = 100-300$ and 300-500. Figure 3 shows an instantaneous volume in which a coherent low momentum region (LMR) at $y/\delta \sim 0.15$ extends across the entire depth as well as the streamwise domain. In previous planar PIV measurements at three heights, $z^+ = 125, 300, 500$ (Tan & Longmire, 2015), we saw a distinct difference between packets at $z^+ = 125$ and those at the other two heights. Packet signatures closer to the wall were frequently narrow and long. With increasing height, packets became wider and shorter. This trend was also observed in the volumetric data. For example, a narrow and long LMR can be seen in Figure 3a at $z^+ = 155$ ($y/\delta =$
The signature of the same packet was observed at roughly the same spanwise location at both \( z^+ = 300 \) and 465. The packet looked wider at \( z^+ = 300 \) and shorter at \( z^+ = 465 \).

Autocorrelations of streamwise-spanwise slices at \( z^+ = 155, 300 \) & 465 are shown in Figure 4. The trends are consistent with previous data and with our observations that low momentum regions become wider and shorter with increasing height. In Figures 4d and 4e, which show streamwise and spanwise cuts with \( \Delta y = 0 \) and \( \Delta x = 0 \) respectively, two points are notable. First, the streamwise correlation at \( \Delta y = 0 \) is distinctly lower near the tails (\( |\Delta x| > 0.5 \)) at \( z^+ = 465 \) than at 155 or 300 where the values are similar. This result implies that fewer low and high momentum regions are present. Second, the spanwise cut with \( \Delta x = 0 \) shows that the central positive lobe grows wider with increasing height indicative of growing spanwise length scales.

Cross correlations of streamwise velocity from planes at different depths are shown in Figure 5. Values from a reference plane at \( z_{ref}^+ = 155 \) are correlated with values at depths of \( z_c^+ = 200 \) (Fig. 5a), 300 (Fig. 5b) and 465 (Fig. 5c) respectively. In the plots, positive values of \( \Delta x \) correspond with a positive offset in the streamwise location in the upper \( z \) plane compared with the streamwise location at \( z_{ref}^+ \). As would be expected, the cross correlation peak values decrease with greater separation between the reference plane and the correlated plane. This trend is most obvious in the streamwise cuts with \( \Delta y = 0 \) shown in Fig. 5d. Furthermore, the correlation peaks shift toward positive \( \Delta x \) as the wall-normal separation increases, and values for positive \( \Delta x \) drop off more slowly than those for negative \( \Delta x \). This trend, which is consistent with previous streamwise-wall normal correlations (e.g. Dennis and Nickels, 2011), may be attributed to the presence of forward leaning structures (e.g. hairpin packets). In Fig. 5e, which shows the
Figure 3. Unperturbed flow. a) Instantaneous volume from 3-D PTV. Colors show the fractional deviation from the local mean velocity for the plane at $z^* = 155$. Iso-surfaces show regions with $U < 0.95U_m$. Streamwise-spanwise slices at b) $z^* = 155$, c) $z^* = 300$ and d) $z^* = 465$.

spanwise cut with $\Delta x = 0$, the central positive region became wider with increasing wall-normal separation. Interestingly, the negative lobes were very similar in all three correlations, which might suggest that many of the neighboring low and high speed structures have similar spanwise spacing across the depth of the measurement domain.

Flow perturbed by array with $H = \delta$

Instantaneous volumetric fields some distance downstream of the taller array indicated that LMRs at $z^* = 155$ were frequently disconnected from LMRs observed at $z^* = 465$, in contrast to the structure observed in unperturbed flow where LMRs frequently extended through the
Figure 4. Unperturbed flow. Autocorrelation contours of the streamwise velocity at a) $z^+ = 155$, b) $z^+ = 300$ and c) $z^+ = 465$. Slices of the autocorrelations taken at d) $\Delta y/\delta = 0$ and e) $\Delta x/\delta = 0$. 100 independent fields correlated.

volume in the $z$ direction. Figure 6a shows a representative example of such a scenario at $x/\delta = 4.6$, where the isosurfaces of the LMRs at $y/\delta = -0.2$ and $0.1$ are limited in their extension across the depth. Beginning at $z^+ = 155$ and toggling through the streamwise-spanwise slices for increasing $z$, the LMRs observed at the aforementioned spanwise locations weaken with increasing height until they are hardly visible at $z^+ = 465$ (Figures. 6b-d).

In order to quantify this effect statistically, cross correlations similar to those in Figure 5 were computed at three streamwise locations downstream of the perturbation. The resulting contour plots (not shown) indicate reduced streamwise coherence compared to the unperturbed flow. This effect can be seen clearly in the line plots with $\Delta y = 0$ in Figures 7a-c. Immediately downstream of the cylinders (red curves), the correlations exhibit periodic oscillations in the streamwise direction. This behavior was also visible in autocorrelations of the same data at $z^+ = 155, 300$ and $465$. These periodic oscillations can be attributed to very regular spanwise oscillations in wake position downstream of the cylinders. The regularity of the oscillations in
all cross correlations also suggests that the wakes are very two-dimensional across the thickness of the measurement volume.

The peak values of all cross correlations are smaller than those in unperturbed flow. At $x/\delta = 2.6$ and 4.6, the reduction in the peak correlation value was minimal for $z_c^+ = 200$, but significant for $z_c^+ = 300$ and $z_c^+ = 465$. In addition, the correlation values were suppressed everywhere along the $\Delta x$ direction compared to the unperturbed flow. At the two larger wall-normal separations corresponding with $z_c^+ = 300$ and $z_c^+ = 465$, the shift in the correlation toward positive $\Delta x$ values is suppressed significantly compared with the results in unperturbed flow, possibly suggesting fewer LMRs with forward inclination across the measurement volume.

When comparing the results at $x/\delta = 2.6$ and 4.6, the correlations corresponding with $z_c^+ = 200$ (Fig. 7a) and $z_c^+ = 300$ (Fig. 7b) seem to indicate some recovery towards the unperturbed condition, but the correlations at $z_c^+ = 465$ (Fig. 7c) do not. These trends are consistent with our
Figure 6. a) Instantaneous volume showing streamwise velocity starting at \(x/\delta \sim 4.3\) behind \(H = \delta\) array. Iso-surfaces of regions with 0.95 local mean velocity are shown, with the streamwise spanwise contours of streamwise velocity at \(z^* = 155\). Z-planes at b) \(z^* = 155\), c) \(z^* = 300\) and d) \(z^* = 465\) of streamwise velocity.

Observations in planar PIV fields at different heights where the packet organization seems to recover initially closer to the wall (Tan & Longmire, 2015).

The spanwise cross correlation curves with \(\Delta x = 0\) are plotted in Fig. 7d-f. Immediately behind the array, all cross correlations have strong periodicity associated with alternating wakes behind cylinders and accelerated zones between them. The magnitudes of the peaks and valleys adjacent to the central positive lobe are similar for all cross correlations, also suggesting that the wakes are fairly two-dimensional across the thickness of the volume. Also at this streamwise location, all cross correlations indicate strong suppression of the central positive lobe compared...
Figure 7. Cross correlation slices for 100 fields at Δy/δ = 0 for flow behind H = δ array, z_{ref} = 155 correlated with a) z_c = 200, b) z_c = 300, & c) z_c = 465. Slices at Δx/δ = 0 for the aforementioned cross correlations shown in d, e & f.

to the unperturbed flow. This can be explained by the shift toward smaller spanwise scales induced by the narrowly-spaced array. Further downstream, the width of the central positive lobe increases, but remains slightly suppressed compared to the unperturbed width in all three correlations. At x/δ = 2.6 and 4.6, the adjoining negative lobes are clearly damped compared with the results in unperturbed flow indicating persistent alterations of the spanwise organization.

Flow perturbed by array with H = 0.2δ

The flying PIV measurements from Tan & Longmire (2015) indicate that packet signatures similar to those in unperturbed flow re-appeared first in planes away from the wall, i.e. at the cylinder tip height. Looking at instantaneous volumes directly behind the H = 0.2δ (H^+ = 500) array, we observed that wakes generally weakened with increasing z^+, in particular as the cylinder tip height was approached. An example of this is seen in Figure 8. In Figure 8a, wakes were observed directly behind each cylinder, but the local momentum deficits are weaker and less correlated to the cylinder positions in Figure 8c. Previous stereo PIV and V3V measurements by Ortiz-Duenas et al. (2007) suggested that downwash induced immediately downstream of the cylinder tips induced faster moving fluid towards the wall. Furthermore, as a
result of the downwash, wakes in the averaged velocity fields split and merged into slow moving regions aligned with spanwise locations midway between cylinders. Thus, on average, the zones aligned with the cylinders became high momentum regions (Zheng & Longmire, 2014). This behavior can be seen in the averaged streamwise velocity plots shown in Figure 9. Note that the inversion from low to high velocity occurs sooner near the cylinder tips at \( z^+ = 465 \) (at \( x/\delta \sim 0.4 \) in Fig.9c), then later at \( x/\delta \sim 0.6 \) and 0.8 at \( z^+ = 300 \) and 155 respectively (Figs. 9b and 9c). In the subsequent measurement locations centered on \( x/\delta = 2.4 \) and \( x/\delta = 3.7 \), LMRs were frequently observed to extend across the depth of the measurement volume with their size and shape resembling LMRs in the unperturbed fields. At similar streamwise locations and \( z^+ = 300 \), Zheng and Longmire (2014) found that the dominant spanwise modes were fairly similar to
those in unperturbed flow. Additionally, their plots of mean streamwise velocity did not reveal any obvious spanwise variations.

Cross correlations were computed between the same reference plane of $z_{ref}^+ = 155$ and $z_c^+ = 155, 300 \& 465$. The resulting line plots for $\Delta y = 0$ are shown in Figs. 10a-c. Immediately behind the array, all three cross correlations ($z_c^+ = 200, 300 \& 465$) are strongly suppressed compared to the values in unperturbed flow. The periodicity observed in Figure 7 ($H = \delta$) for all three of the correlations is much weaker for $H = 0.2\delta$ indicating either less or less regular spanwise displacement of cylinder wakes. The periodicity in Figure 10c (red curve), which correlates $z_{ref}^+ = 155$ with $z_c^+ = 465$, is barely noticeable, suggesting further that the two-dimensionality associated with wakes behind infinite cylinders is disrupted by the proximity of the cylinder tips. The correlations exhibit increasing streamwise asymmetry with increasing separation between correlated planes. Although all three correlations peak at $\Delta x = 0$, the values drop off faster for positive $\Delta x$ and even become negative for $\Delta x/\delta > 0.4$ for both $z_c^+ = 300$ and 465. The negative values can be attributed to the streamwise inversion in streamwise velocity deviation shown in Figure 9. Further downstream, at $x/\delta = 2.4$ and $x/\delta = 3.7$, the correlation curves overlap for the perturbed and unperturbed flow. This result is consistent with the instantaneous observations.
where LMR shapes downstream of this array appeared similar to those in unperturbed flow and with the data of Zheng & Longmire (2014).

Spanwise variations in the cross correlations with $\Delta x = 0$ are plotted in Figure 10d-f. Immediately behind the array, the correlations show strong periodicity very similar to the results in Figure 7. The magnitudes of the positive and negative lobes are similar to the $H = \delta$ result at $z^* = 200$, but decrease with greater separation in $z$. Furthermore, the central positive lobe for $z^* = 465$ was wider for $H = 0.2\delta$ than for $H = \delta$ as might be expected for weaker velocity deficits and weaker initial perturbations at the cylinder tip height. Further downstream, the correlations have largely recovered to the unperturbed values. We do note that the negative lobes are slightly weaker for the perturbed flow, perhaps owing to a persistent alteration of the spanwise structure organization. In comparison, the suppression of the negative lobes was much stronger for the flow downstream of the $H = \delta$ array.

Conclusions

A 3-D PTV method was used to investigate how long, narrow low momentum regions associated with packet signatures closer to the wall relate to the wider shorter structures observed further away from the wall and how structures were correlated across the depth of the measurement volume in both unperturbed flow and flow perturbed by cylinder arrays with heights of $0.2\delta$ and $\delta$. The measurement volume was positioned immediately beneath the tip height of the shorter cylinders and spanned a significant portion of the logarithmic region. Cross correlations from the volumetric data were used to extract trends related to flow recovery behind the two arrays.

Slow moving regions associated with packets frequently extended across the measurement volume from $z^* = 155$ to $z^* = 465$, consistent with the results of Gao et al. (2011), Dennis & Nickels (2011) and Elsinga et al., (2007). Trends observed in instantaneous fields and cross correlations were consistent with those in the literature. Packets were longer and narrower closer to the wall and grew wider and shorter with increasing wall normal distance. Cross correlations between slices at different wall normal locations suggest the presence of forward leaning structures. Moreover, the similar strength in negative lobes for all cross correlations alludes to the dominance of a similar spanwise spacing between faster and slower moving structures across the volume depth.

Both cylinder arrays elicited wake structures associated with smaller spanwise length scales than those present in the unperturbed flow. Immediately downstream of each array, correlation peaks were reduced and the correlation shapes were altered significantly. Judging from the cross correlations, the streamwise development of the flow was quite different for the
two arrays. The shorter $H = 0.2\delta$ cylinders yielded correlations that became highly asymmetric as the wall-normal offset was increased, consistent with current and previous mean velocity measurements showing wake ‘splitting and merging’ related to strong wall-normal motions induced by cylinder tip effects. According to the cross correlations and instantaneous velocity fields, the flow had recovered nearly to the unperturbed structure by $x/\delta = 2.4$. On the other hand, the taller $H = \delta$ array generated wakes that seemed more two-dimensional and consistent over the depth of the measurement volume. In this case, cross correlations did not recover at the furthest downstream measurement location ($x/\delta = 4.6$). At this streamwise location, low momentum regions often did not extend across the depth from $z' = 155$ to $z' = 465$, and the related cross correlations were more symmetric about $\Delta x = 0$. In addition, the primary negative lobes in the $\Delta y$ direction were suppressed suggesting a persistent disruption of the spanwise scales.

The cross correlations presented are consistent with previous results on packet recovery by Tan & Longmire (2015) and Zheng and Longmire (2014), although they leave various questions unanswered. In current and future work, we will employ conditional cross-correlation methods to separately analyze the relationships between low and high momentum regions across the measurement depth for both perturbed and unperturbed flow.

References


