Skin friction fields from TSP surface patterns on an underwater cylinder in crossflow

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ABSTRACT

The relationship between the Temperature Sensitive Paints (TSP) surface pattern and the corresponding relative skin-friction field is investigated for an underwater cylinder in crossflow at subcritical regime. The temperature map evolution can be decomposed in the contribution of a time-averaged, a phase-averaged and a random component. The first two contributions are discussed here. The asymptotic form of the energy equation at wall, solved by following the variational approach, provides relative skin friction fields from the TSP surface temperature maps. Results about the spatial distribution of the time averaged relative skin-friction profiles are compared with literature data. The comparison shows an excellent agreement on the whole laminar boundary layer up to the laminar separation line. Downstream, TSP-related results capture the secondary reattachment/separation events, which are lost in the literature data. Phase-averaged skin friction sequences capture the evolution of the laminar separation, of the turbulent reattachment and of the final separation, providing a description of the Laminar Separation Bubble (LSB) over the whole shedding period. Evolution with Re of the time averaged position of the LSB is in agreement with literature data.

1. Introduction

The problem of measuring the skin friction is one of the topics of the experimental fluid mechanics, because of its implications in the study of the turbulent boundary layers and separated flows. As a matter of fact, the estimation of the friction velocity is fundamental in the scaling of the turbulent boundary layers (Schlichting et al (2000)) and the knowledge of the skin friction topological map provides a rational theory and an experimental explanation of the 3D flow separation (Lighthill (1963), Tobak and Peake (1981), Surana et al (2006)). The method of Temperature-Sensitive Paint, hereafter TSP, is based on temperature-sensitive coatings to obtain qualitative and quantitative evaluation of the temperature field on a model surface. TSP coatings, when excited by narrow-band, incident light, emit a temperature-dependent longer wavelength light (Stokes shifted) that can be detected by photosensitive devices (i.e.
CCD/CMOS cameras). Recently, a method for a global skin friction diagnostic using TSP has
been introduced by Liu and Woodiga (2011) and Liu (2013), showing its feasibility in providing
highly resolved relative skin friction maps. The capability of the method in identifying loci of
flow separation with very high resolution has been shown, among the others, in Capone et al
(2015) for a cylinder in cross-flow, where the succession of separation/reattachment/separation
takes place along lines very close together. Here we want to report the feasibility of the technique
in providing quantitative estimation of the relative skin friction for the same set of data of
Capone et al (2015). To this aim, results from TSP are rearranged via triple-decomposition on the
basis of the shedding periodicity. A time-average, a phase-average and a random component are
extracted from the original data and analyzed on the basis of Liu (2013). Here only the time and
phase averaged data and related extraction procedure are reported, while the random
components will be discussed in a future paper. The comparison of the time averaged components
and literature data shows an excellent agreement in the region of the laminar boundary layer up to the laminar separation line. Downstream, the sequence of secondary
reattachment/separation is still captured by the TSP related profiles, while the available
literature data completely misses it. Phase-averaged skin friction sequences capture the evolution
of the laminar separation, of the turbulent reattachment and of the final separation, providing a
description of the Laminar Separation Bubble (LSB) over the whole shedding period. Evolution
with Re of the time averaged position of the LSB is in agreement with literature data.

2. Global skin friction diagnostics

2.1. Basic equations

By using the Taylor expansions of the velocity and temperature fields near a wall in a flow, the
asymptotic form of the energy equation at the wall can be written as:

\[ F + \tau_i \frac{\partial \tau}{\partial X_i} = 0 \]  \hspace{1cm} (1)

where \( T_w \) is the wall temperature, and \( \tau_i = \mu(\partial u_i/\partial X_j)_w \) are the skin-friction components. The
source term \( F \) in Eq. 1 is defined as:

\[ F = -\frac{\mu}{k_f} \frac{\partial u_i}{\partial \tau} + \epsilon \]  \hspace{1cm} (2a)

\[ \epsilon = \frac{\mu a}{k_f} \frac{\partial^2 q_w}{\partial X_i \partial X_j} - \mu a \left( \frac{\partial^2 T}{\partial X^2} \right)_w + \frac{4\mu}{\rho c_p} \tau_i \left( \frac{\partial^2 u_i}{\partial X^2} \right) \]  \hspace{1cm} (2b)
and \( \alpha = k_f / \rho c_p \) is the diffusivity of fluid, \( \rho \), \( c_p \), \( \mu \) and \( k \) are density, specific heat, dynamic viscosity and thermal conductivity of fluid, respectively. The heat flux \( q_w = -k_f(\partial T / \partial X_i)_w \) at the wall is positive when the heat enters into fluid from the wall. Eq. 1 gives a relation between skin friction vector, heat flux and temperature at the wall. It represents a balance between the skin friction vector projected on the normal vector \( \nabla T \) to an isotherm line \( T_w = \text{const} \) and the term \( F \) containing the time rate of heat flux. Alternatively, \( \tau_i \partial T_w / \partial X_i \) can be formally interpreted as the flux of a scalar \( T_w \) transported by a skin friction field \( \tau_i \) while \( F \) is considered as a source term. In a sense, it is a differential form of the Reynolds analogy. The second and third terms in \( F \) are related to the thermal diffusion and the viscous dissipation term, respectively. We introduce the following decomposition:

\[ T_w = \langle T_w \rangle_t + T_w' \quad (3a) \]
\[ \tau_i = \langle \tau_i \rangle_t + \tau_i' \quad (3b) \]
\[ q_w = \langle q_w \rangle_t + q_w' \quad (3c) \]
\[ \epsilon = \langle \epsilon \rangle_t + \epsilon' \quad (3d) \]

where \( \langle \cdot \rangle \) is a time-average operator (or ensemble-average operator), and the prime denotes the fluctuation (or perturbation). Substituting Eqs. 3a-3d into Eq. 1 and neglecting the higher order small terms, we have the equation for the averaged quantities, i.e.,

\[ G_0 + \frac{\langle \tau_i \rangle_t}{\tau_{\text{ref}}} \frac{\partial \langle T_w \rangle_t}{\partial X_i} = 0, \quad (4) \]

where

\[ G_0 = \frac{\langle \epsilon \rangle_t}{\tau_{\text{ref}}} + \frac{1}{\tau_{\text{ref}}} \langle \tau_i \rangle_t \frac{\partial \langle T_w \rangle_t}{\partial X_i} \quad (5) \]

and \( \tau_{\text{ref}} \) is a reference value of skin friction.

### 2.2 Phase averaged equations

The strong periodicity which characterize the investigated flow suggests to expand the decomposition in Eqs. 3 to take into account the phase average of the turbulent signal. Following Reynolds (1972), the triple decomposition can be defined as:
\[
f(x, t) = \bar{f}(x) + \hat{f}(x, t) + f'(x, t) = \langle f \rangle_p + f'(x, t) \quad (6)
\]

where \(\bar{f}(x)\) is the time average, \(\langle f \rangle_p\) is the phase average based on a reference signal, \(\hat{f}(x, t)\) is the statistic contribution of the wave and \(f'(x, t)\) is the turbulent component, already defined in Eqs. 3. It is to notice that in absence of a wave contribution, \(f(x, t) = 0\) and Eq. 6 reduces to Eqs. 3.

The procedure described in Section 2.1 can be applied to the estimation of the phase average. By applying the decomposition in Eq. 6 and substituting the results in Eq. 1, the resulting phase averaged equations can be written as follows:

\[
\begin{align*}
\mathcal{G}_0 + \frac{\langle \tau_i \rangle_p}{\tau_{ref}} \frac{\partial \langle U \rangle_p}{\partial x_i} &= 0 \\
\mathcal{G}_0 &= \frac{\langle \tau_i' \rangle_p}{\tau_{ref}} \frac{\partial \langle U'' \rangle_p}{\partial x_i} + k_p \frac{1}{k_f Re_{ref} u_{ref}} \frac{\partial \langle T_w \rangle_p}{\partial t}
\end{align*}
\]

(7) (8)

2.3. Variational approach

In image-based temperature measurements, Eq. 4 should be projected onto the image plane of a camera. As shown in Fig. 1, a special object-space coordinate system \((\bar{X}_1, \bar{X}_2, \bar{X}_3)\) is considered, in which the plane \((\bar{X}_1, \bar{X}_2)\) is parallel to the image plane \((x_1, x_2)\). Therefore, a relation between \((\bar{X}_1, \bar{X}_2)\) and \((x_1, x_2)\) is \(\frac{\partial}{\partial \bar{x}_i} = \lambda \frac{\partial}{\partial x_i}\) where \(\lambda\) is a scaling
constant. An image on a camera is the scaled projection of the surface onto the plane \((\bar{X}_1, \bar{X}_2)\). If the transformation between \((X_1, \bar{X}_2)\) and \((X_1, X_2)\) is given by \(\bar{X}_i = F_i(X_1, X_2)\) then \(\frac{\partial}{\partial X_i} = h_{ij} \frac{\partial}{\partial \bar{X}_j} = \lambda h_{ij} \frac{\partial}{\partial \bar{X}_j}(i,j = 1,2)\), where \(h_{ij} = \frac{\partial F_j}{\partial x_i}\) depend on the geometric properties of the surface. Therefore, Eq. 4 can be expressed in the image coordinates, i.e.,

\[
G + \dot{\tau}_i \frac{\partial (T_w)}{\partial x_i} = 0, \tag{9}
\]

where \(\dot{\tau}_i = \lambda h_{ij} \tau_j (i,j = 1,2)\) is the projected skin-friction vector on a surface onto the image plane. Eq. 9 has a similar form to the Horn-Schunck optical flow equation except that the optical flow is replaced by the projected skin friction vector \(\dot{\tau}\) and the time derivative of the image intensity by the source term \(G\). In principle, if \(G\) and \((T_w)\) are known, Eq. 9 can be solved as an inverse problem for the skin-fiction vector \(\dot{\tau}\). By minimizing the following functional with the Horn-Schunck regularization term on an image domain \(\Omega\):

\[
J(\dot{\tau}) = \int_{\Omega} (G + \dot{\tau} \cdot \nabla(T_w))^2 dx_1 dx_2 + \alpha \int_{\Omega} (|\nabla \dot{\tau}_1|^2 + |\nabla \dot{\tau}_2|^2) dx_1 dx_2,
\]

the Euler-Lagrange equations for \(\dot{\tau} = (\dot{\tau}_1, \dot{\tau}_2)\) are obtained, i.e.

\[
[G + \dot{\tau} \cdot \nabla(T_w)] \nabla(T_w) - \alpha \nabla^2 \dot{\tau} = 0 \tag{10}
\]

where \(\nabla = \frac{\partial}{\partial x_i}\) \(\nabla^2 = \frac{\partial^2}{\partial x_i \partial x_l}\) and \(\alpha\) is a Lagrange multiplier. Given \(G\), from an average surface temperature image, Eq. 10 can be solved numerically with the Neumann condition \(\frac{\partial \dot{\tau}}{\partial n} = 0\) imposed on the domain boundary.

3. Methods and set-up

The facility consists of a closed-loop water tunnel having a 1:5.96 contraction nozzle and a square test section of side \(B = 600\) mm. Free-stream turbulence intensity and flow uniformity at the channel centerline are respectively 1.5\% and 0.4 \%. The hollow cylinder model is made from aluminum, diameter \(D = 36\) mm and 13.5 mm thickness.
Cylinder length $L = 600$ mm is equal to the test section width and consequently the aspect ratio is $L/D = 16.7$ whereas the blockage factor is $D/B = 0.06$.

The experiments include five flow speeds, with free stream velocity $U$, ranging from 2 m/s to 5 m/s. Reynolds numbers, based on cylinder diameter and water kinematic viscosity at 25°C, range from 69,000 to 238,000. Details on the TSP setup can be found in Capone et al (2015). Images are processed according to the procedure set out in Fey et al (2013) and Capone et al (2015). With an appropriate calibration curve, mean temperature on the cylinder surface are extracted from TSP images (see Tropea et al (2007), Liu (2004) for further details).

4. Results

The application of the procedure described in Section 2.1 to the time averaged temperature map provides a skin friction field whose topology allows to clarify the flow behavior on the surface and close to the separation lines. As expected from literature data, at the investigated Re the flow undergoes to a laminar separation (at $\phi = \phi_{ls}$), followed by a turbulent reattachment (at $\phi =$...
and by a final separation (at $\phi = \phi_{rs}$). It is interesting to notice that the wall-normal extension of the recirculating bubble, which develops between the laminar separation and the successive reattachment, is really small. It is almost hidden to flow velocimetry not specifically devoted to the investigation of the near wall phenomena. In studying the flow around a cylinder in crossflow, these difficulties become prohibitive when the critical regime approaches, because of the strongest velocity gradients that appear close to the cylinder’s surface.

An example of the TSP results at $Re = 150,000$ is reported in Figs. 4 left) and right) (the flow comes from the left). The temperature map shows a remarkable vertical lighter line that, due to its higher temperature, is the signal of a drop in the flow heat removal efficiency because of the laminar separation occurrence. Moving downstream (to the right), another flow feature can be observed, but its assignment to some flow occurrence is less evident. By looking at the skin friction streamlines in Fig. 4 right), two characteristic behaviors are evident, which can be classified as loci of convergence and divergence of skin friction streamlines. Their spatial evolution underlines the occurrence of separation and reattachment lines respectively (Wu et al (2000)).

A more quantitative result can be gained by taking the spatial average along Z of the field in Fig. 4 right), i.e. the skin friction distribution along the cylinder circumference. Fig. 5 reports a comparison with experimental data at $Re = 6000$ (Olson et al (2015)) and $Re = 100,000$ (Achenbach (1968)), and with LES data at $Re = 10,000$ (Rizzetta and Visbal (2009)). As a further reference, the Schlichtings series solution for laminar boundary layer is plotted. All data series have been normalized by their maximum. Here it can be observed that the analytic solution departs from

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**Fig. 4:** Time averaged temperature map (left) and a detail of the associated skin friction streamlines (right).
the other series just after the maximum of skin friction has been reached, while all the other profiles collapse in a large part of the laminar region.

Close to the laminar separation line ($\tau = 0$), only the data from Achenbach (1968) agrees with the present results up to $\phi \approx 80^\circ$, where the laminar separation occurs. Downstream of this position, the TSP-related relative skin friction profile captures the fine details of the secondary reattachment-separation process (crossings of the $\tau = 0$ line), thus confirming the high spatial resolution of the technique and its feasibility in skin friction diagnostic.

The procedure described in Section 2.2, applied to the phase averaged temperature components, provides the evolution of the laminar separation, of the turbulent reattachment and final separation $\phi_{ls}$, $\phi_{tr}$ and $\phi_{ts}$ versus the shedding phase $\theta$. Results are shown in Fig. 6 (with their 95% of confidence interval) at $Re$ 75,000 and 150,000. There is an accordance within the literature data about $\phi_{ls}$, $\phi_{tr}$ and $\phi_{ts}$ mean values (see Capone et al 2015) but it is not known to the authors a description of their evolution during the shedding period.

![Fig. 5: Comparison between literature time averaged relative skin friction distributions and present data (continuous line). Symbols are: (o) - Rizzetta and Visbal 2009, (Δ) - Olson et al 2015, (□) - Achenbach 1968, (*) - Schlichting et al 2000](image-url)
The data in Fig. 6 describes the laminar separation bubble evolution along the whole shedding period. Higher Re shows a shorter laminar separation bubble extension (given by $R(\theta_{tr} - \theta_{ts})$), while the distance between the turbulent reattachment and the final separation seems to be less prone to Re variations. The amplitude of the oscillation of $\theta_{ts}$ decreases when Re increases (from $\pm 2^\circ$ to $\pm 1.5^\circ$), while both $\phi_{tr}$ and $\theta_{ts}$ oscillation amplitudes are almost constant ($\pm 1.1^\circ$ and $\pm 0.55^\circ$ respectively). A slight phase delay appears between $\phi_{ts}$ and the couple $\phi_{tr}$, $\theta_{ts}$.

Those data confirms the feasibility of the described method in providing, by means of time/space highly resolved TSP maps, a deeper description of the skin friction exerted by a fluid on a body surface. As the meaningful consequence, the capture of the flow topology on a body surface provides a new, exciting point of view useful in the understanding of the fluid-body interactions.

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References


