The turbulent/non-turbulent interface in the near-field region of a round jet

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Keywords: Jets, PLIF/PIV applications, Entrainment, Turbulence

ABSTRACT

We experimentally investigate the turbulent/non-turbulent interface (TNTI) in the near-field of a round jet at $Re = 24,000$. Simultaneous stitched-particle image velocimetry (PIV) and planar laser-induced fluorescence (PLIF) are implemented to perform a multi-scale analysis on the TNTI. We use isocontours of a passive dye in water ($Sc \gg 1$) to identify the TNTI, and we show that there is a jump in spanwise vorticity across it. A spatial-filtering method is implemented to characterise the scale-similarity of the TNTI geometry. This is achieved by filtering the TNTI with box-averaging filters that span over two decades in length and then measuring the mean TNTI length across short streamwise spans. The TNTI length exhibits a power-law scaling as a function of filter size across a decade of scale, which is indicative of a fractal geometry. However, the fractal dimension, $D_f$, of the TNTI evolves as a function of streamwise distance. The fractal dimension reaches a plateau beyond $x/D \approx 4$, which is the end of the potential core. More importantly, in this region we show that $D_f = 0.32 – 0.33$, which agrees with reported values of $D_f$ measured in fully-developed turbulent flows, such as the far-field of jets and turbulent boundary layers. The onset of this fractal behaviour also coincides with evidence of flow homogeneity based on the radial auto-correlation functions of axial and radial velocity fluctuations. We suggest that the flow about the TNTI beyond the potential core exhibits a hierarchy of scales that is comparable to fully-developed turbulence. This is because the TNTI is contorted by a hierarchy of turbulent eddies (i.e. turbulent cascade), and the fractal dimension of the interface scaling is a “footprint” of these eddies.

1. Introduction

Turbulent entrainment is a process that describes the transport of irrotational fluid across the boundary between the turbulent and non-turbulent regions of a flow; we refer to this boundary as the turbulent/non-turbulent interface (TNTI). The TNTI plays an important role in the understanding of entrainment problems because it is along the TNTI that vorticity diffuses to the non-turbulent fluid (Corrsin & Kistler 1955). Although viscous diffusion, and molecular diffusion for scalars, is a very slow process, it is expedited by acting over the very large surface area of the TNTI. The prevalence of natural and industrial entrainment scenarios makes the TNTI an important flow feature to study. For example, the flame front in combustion problems is analogous to the TNTI except in a reacting flow. Modifying the surface area of the flame front can alter the consumption rate of the premixed gas and influence the efficiency and thermoacoustic characteristics of the flow (Tamadonfar & Gülder 2015). In this regard, understanding the
characteristics of the TNTI in a simple, non-reacting flow, such as an axisymmetric jet, offers insight that can be applied to industrial flow scenarios that are significantly more complex.

There have been many studies that have characterised the TNTI in fully-developed, non-reacting turbulent flows (see da Silva et al 2014 and references therein), which include turbulent boundary layers, mixing layers, and the far-field of turbulent jets. However, it is the near-field region that is of particular relevance to combustion problems where external factors (e.g. external geometry) disrupt the flow before it can become fully-developed. In this paper we examine the TNTI in the near-field of a turbulent, round jet at high Reynolds number, with particular emphasis on the properties of the TNTI geometry. We focus on the geometry because recent work on a stratified mixing layer flow by Krug et al (2015) reports larger overall entrainment rates for flow conditions that yield a larger TNTI surface area. Thus, the geometry of the TNTI (i.e. surface area) may influence the global entrainment rate and it is of interest to investigate how these two features are coupled.

![Image](image-url)

**Fig. 1** Instantaneous scalar concentration field (normalised by exit concentration) in the near-field of a turbulent jet at \( Re = 24,000 \). The TNTI is denoted by the blue line.

The corrugated TNTI (see **Fig. 1**) is shaped and contorted by the adjacent turbulent eddies, and these eddies are characterised by a large span of length, time, and velocity scales (i.e. energy cascade). The effect of this hierarchy of turbulent scales on the TNTI may be quantified by analysing the TNTI from a multi-scale fractal perspective. Mandelbrot (1982) describes fractal self-similarity as “[invariance] under certain transformations of scale.” Thus, we expect the contortions and patterns that exist along the interface in the inertial range to also exist at smaller scales. The result of this self-similarity is the non-trivial scaling of surface area (or length in 2D) as a function of the measurement resolution. Moreover, this multi-scale analysis of the TNTI offers insight into the connection between the local, instantaneous entrainment and the global entrainment (Mathew & Basu 2002).
Box counting techniques are often used to quantify the multi-scale nature of turbulent surfaces (Sreenivasan et al 1989). Simply, this technique counts the number of boxes, \( N \), that are needed to occupy a surface and then evaluates the scaling of the box count with the box size. Sreenivasan et al (1989) showed that there is an intermediate range of scales (inertial range) across which the box count scales as a power-law with the box size. This scaling relationship was reported to be \( N \sim \Delta^{-D_f^\eta} \), where \( D_f^\eta = 7/3 \) is the fractal dimension (Sreenivasan et al 1989). For a 2D contour that is measured in a single plane the equivalent scaling is \( \langle L_s \rangle \sim \Delta^{-1/3} \) (see Mandelbrot 1982). Care must be taken when interpreting early fractal scaling results because these studies were performed at low Reynolds numbers that are restricted to a narrow scale-separation over which the power-law scaling is measured (Catrakis 2000). With the advent of improved experimental techniques, such as high-resolution cameras, the topic of fractal scaling of the TNTI has been revisited. Recent work on this subject in a high-Reynolds number turbulent boundary layer has reported that the fractal dimension of the TNTI falls in the range \( D_f = 0.3 - 0.4 \), where \( \langle L_s \rangle \sim \Delta^{-D_f} \) (de Silva et al 2013). This fractal dimension range is measured using the classic box counting technique as well as a spatial-filtering technique. Unlike the turbulent boundary layer, the near-field region of a turbulent jet is not a fully-developed flow, and also does not exhibit the self-similarity that characterises the far-field region of a jet. Hence, we seek to characterise the evolution of the TNTI geometry downstream of the nozzle-exit to determine the streamwise distance at which the expected fractal scaling \( (D_f = 0.3 - 0.4) \) is recovered.

In this paper we examine the evolution of the fractal scaling of the TNTI in the near-field region of a turbulent, round jet. We interpret these results in the context of flow homogeneity with the use of auto-correlation functions. This analysis is applied to data that was experimentally measured in the radial-streamwise plane along the jet centreline. We first summarise the experimental apparatus and characterise the near-field region of the jet. We then provide a description of the simultaneous particle image velocimetry (PIV) and planar laser-induced fluorescence (PLIF) experiments that are used to identify the TNTI. Finally, we present the results from the fractal scaling analysis and discuss the implications thereof.

2. Experimental methods

2.1 Apparatus

A turbulent jet was produced in an 8 m long water tank with a 0.9 m x 0.5 m cross-section and an open top (see Fig. 2). The walls and floor of the tank are made of Perspex, which provides optical access to the jet flow. The turbulent jet was supplied by a series of pumps to produce a nozzle-exit Reynolds number of \( Re = 24000 \), where \( Re = U_e D / \nu \), \( U_e \) is the nozzle-exit velocity, \( D \) is the diameter of the jet, and \( \nu \) is the kinematic viscosity. The exit diameter of the nozzle is \( D = 40 \) mm; such a large diameter is necessary to capture the small-scale turbulent structures in the near-field. The confinement effects of the water tank facility are expected to be negligible in the near-field.
region because the cross-sectional area of the jet flow in the near-field \((x/D < 7)\) is much smaller than the cross-sectional area of the tank (see eq. B10 in Hussein et al 1994). The nozzle assembly contains a series of meshes and honeycomb flow-straighteners that homogenise the flow before entering the contraction. The contraction is defined by a 5th-order polynomial that reduces the boundary layer thickness at the nozzle exit. A constant flow rate of the jet was maintained by a series of valves and was validated with measurements of the pressure drop across an orifice plate. A container of dyed fluid is used to supply the jet flow for the scalar measurements that are discussed in §2.3.

**Fig. 2** Schematic of the turbulent jet apparatus: (a) high-speed laser, (b) laser-sheet optics, (c) water tank, (d) nozzle assembly, (e) stitched-PIV high-speed cameras, (f) PLIF high-speed camera, (g) dyed-fluid reservoir. Tank section is cropped for clarity.

### 2.2 Flow characterisation

Two-dimensional PIV measurements were taken in the near-field region of the jet to characterise the boundary conditions of the flow. A more comprehensive description of the PIV set-up is given in section §2.3 for the simultaneous PIV/PLIF measurements. The experimental arrangement is similar to the PIV set-up described here and will only be summarised here. Briefly, a Nd-YLF laser, in combination with sheet-forming optics, was used to generate a thin light sheet in a vertical plane passing through the jet centreline. Neutrally-buoyant particles were added to the jet flow and the ambient fluid. Two high-speed cameras in a side-by-side stitched configuration were used to capture the Mie scattering from the particles in the image plane. The paired particle images are separated in time by 1 ms; the resultant velocity fields are separated by 20 ms. The particle images were processed using DaVis (LaVision GmbH) with a final interrogation window size of \(32 \times 32\) px, which equates to a spatial resolution of 4.7 mm. The scalar concentration data presented in this section is described in detail in §2.3. A total of 5457 vector fields are used for the
statistical velocity quantities presented in this section. A summary of the flow characteristics is presented in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle-exit diameter</td>
<td>40 mm</td>
</tr>
<tr>
<td>Kinematic viscosity (water)</td>
<td>$1.0 \times 10^{-6}$ m²s⁻¹</td>
</tr>
<tr>
<td>Molecular diffusivity (rhodamine 6G)</td>
<td>$1.2 \times 10^{-10}$ m²s⁻¹</td>
</tr>
<tr>
<td>Nozzle-exit velocity</td>
<td>0.60 ms⁻¹</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>24 000</td>
</tr>
<tr>
<td>Schmidt number</td>
<td>8000</td>
</tr>
</tbody>
</table>

Table 1 Fluid properties and flow characteristics of the jet.

The contraction profile of the nozzle is designed to generate a top-hat velocity profile at the nozzle-exit. This is illustrated in Fig. 3(a), in which the axial velocity only varies by ±3% in the region $r/D = 0 \pm 0.4$. The expected velocity step-profile between the ambient region, $|r| > 0.5D$, and the jet flow is smoothed due to the finite spatial resolution of the PIV measurements. The PIV spatial resolution is shown by the grey bar in Fig. 3(a). The scalar concentration profile of the passive dye (blue line) in Fig. 3(a) shows a top-hat profile that exhibits a step-change at $r/D = ±0.5$.

The centreline velocity and scalar concentration profiles in Fig. 3(b) illustrate the length of the potential core of the jet. We define the potential core length as the streamwise distance from the nozzle exit to the point at which the mean centreline axial velocity drops to 95% of the exit velocity; the scalar concentration potential core is similarly defined. The potential core length is measured to be 4.6D and 3.4D for the velocity and scalar fields, respectively. The velocity potential core length agrees with comparable studies of the near-field in jets by Ashforth-Frost & Jambunathan (1996), who measure a potential core length of 4.5D for a top-hat profile jet at a similar Reynolds number. The scalar concentration potential core is shorter than that reported by Mi et al. (2001), who measure a length of approximately 5D. This difference may be attributed to the lower Reynolds number in the work of Mi et al. (2001). In figure Fig. 3(b) we also plot the integral velocity, $u_m = M/Q$, where $M = 2 \int_0^\infty \pi^2 r dr$ and $Q = 2 \int_0^\infty \pi r dr$; we only evaluate these integrals to the radial extents of the FOV. The decay of $u_m$ demonstrates that the jet transfers momentum to the ambient fluid even in the potential core region.
A point-source jet in an infinite environment will exhibit a constant momentum flux as a function of streamwise distance (Pope 2000); this characteristic is often used as a test for the validity of jet data (Hussein et al 1994). We plot the momentum flux, normalised by the source momentum flux $M_0$, in Fig. 4 (blue +’s). Although there is a slight increase in the momentum flux between $0 \leq x/D \leq 1$, the profile remains relatively constant (within 4%) in the region $1 \leq x/D \leq 7$. The source momentum flux is likely to be affected by the finite spatial resolution that cannot recover the sharp-edged top-hat velocity profile in the very near-field region, $x/D \leq 1$. Nonetheless, the constancy of the momentum flux profile further downstream indicates that the jet follows the classic momentum scaling for a free jet.

From a global (time-averaged) perspective, the entrainment rate is represented by the streamwise derivative of the mass-flux,

$$\frac{d\Phi}{dx} = \frac{d}{dx} \left( 2\pi \rho \int_0^\infty \bar{u} r \, dr \right)$$

(Crow & Champagne 1971). The basis of the entrainment hypothesis (Morton, Taylor, & Turner 1956) is that the entrainment rate in a free jet is constant, which means that the radial inflow of ambient fluid is proportional to some velocity scale in the jet. We do not expect the entrainment rate, $d\Phi/dx$, to be constant in the near-field of a turbulent jet because the flow evolves from the potential core region to a transitional region, and finally the self-similar region. In Fig. 4 we present the global mass-flux as a function of streamwise distance (red +’s). The global entrainment rate, $d\Phi/dx$, is determined by fitting a line to the mass-flux data in the region $1 \leq x/D \leq 6$; this is measured to be 5.07 kg/ms. Moreover, the gradient of the mass-flux is relatively constant beyond $x/D = 1$. This indicates that the entrainment achieves some degree of self-similarity very near to the nozzle exit. This is in agreement with Ricou & Spalding (1961) and Liepman & Gharib (1992), who report a linearly increasing mass-flux rate in the near-field. There is some evidence of the decay in the mass-flux rate beyond $x/D = 6$ in Fig. 4 that is also observed by Liepman & Gharib (1992), except they report a decay in $d\Phi/dx$ beyond $x/D = 5$. It is interesting that the potential
core extends to $x/D \approx 4$ (Fig. 3) and yet the entrainment rate achieves a self-similar entrainment state upstream of this point. However, we caution that further measurements are required to better characterise the decay in the mass-flux rate beyond the potential core region.

![Global (integral) mass-flux (red +'s, left axis), and the momentum-flux (blue ×'s, right axis) as a function of streamwise distance.](image)

**Fig. 4** Global (integral) mass-flux (red +'s, left axis), and the momentum-flux (blue ×'s, right axis) as a function of streamwise distance.

We further characterise the flow evolution in the near-field by considering mean and rms radial profiles. These profiles for the velocity and scalar concentration fields are presented in **Fig. 5**. The flow is shown to evolve from a step profile for the axial velocity and scalar concentration at $x/D = 1$ to the smoother, near-Gaussian profiles towards $x/D = 6$. The mean scalar concentration profiles are shown to be wider than the velocity profiles by $x/D = 6$ (see profiles at $r/D = 1.6$). Westerweel et al (2009) use a superlayer analysis to explain the larger spreading rate of the scalar field compared to the axial velocity. They reproduce the Eulerian time-averaged velocity and scalar concentration profiles with a convolution of a step profile with a Gaussian distribution of the superlayer. The scalar concentration jump across the TNTI is larger than the velocity jump, which yields a wider profile of the scalar field compared to the axial velocity field.
Fig. 5 Radial profiles of (a) axial velocity, (b) radial velocity, and (c) scalar concentration. Mean profiles are shown in top row and the respective rms-profiles are shown in the bottom row.

2.3 Simultaneous PIV/PLIF measurements

Examination of the TNTI in the near-field of the jet requires identification of the TNTI and measurement of the velocity fields around it. This is achieved with simultaneous PLIF and PIV measurements in which the PLIF measures the scalar concentration field of a passive dye that identifies the TNTI and the PIV measures the surrounding velocity field. We perform planar measurements in the streamwise-radial plane along the centerline of the jet from the nozzle exit up to approximately 7D downstream. A single high-speed 527 nm Nd-YLF laser (Darwin-Duo 527-60-M, Quantronix) was used to (i) illuminate the particles added to the flow for the PIV measurement and (ii) excite the fluorescent dye for the PLIF measurement. Two PIV cameras were positioned in a side-by-side stitched-configuration such that a longer streamwise extent of the flow was captured (see Fig. 1). A single PLIF camera was used to capture the field-of-view (FOV) covered by the stitched PIV cameras. Additional experimental parameters are detailed in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIV field of view</td>
<td>270 mm×140 mm</td>
</tr>
<tr>
<td>PIV measurement region</td>
<td>(-1.8 \leq r/D \leq 1.8; 0 \leq x/D \leq 6.8)</td>
</tr>
<tr>
<td>PIV spatial resolution</td>
<td>(\Delta x) 3.20 mm</td>
</tr>
<tr>
<td>PIV vector spacing</td>
<td>0.80 mm</td>
</tr>
<tr>
<td>PIV/PLIF camera resolution</td>
<td>1024×1024 px</td>
</tr>
<tr>
<td>PIV camera lenses</td>
<td>180 mm Sigma macro lens (f/4.0)</td>
</tr>
<tr>
<td>Particle image separation time</td>
<td>(\delta t) 1.0 ms</td>
</tr>
<tr>
<td>Vector field separation time</td>
<td>(Dt) 1.0 ms</td>
</tr>
<tr>
<td>PLIF field of view</td>
<td>270 mm×270 mm</td>
</tr>
</tbody>
</table>
Neutrally-buoyant particles (1.05 kg/l) of mean diameter 20 µm (Microbeads AS) were added to the reservoir that supplied the jet fluid and also to the ambient region surrounding the nozzle. The Mie scattering of the particles was captured by the PIV high-speed cameras (SA1.1, Photron Ltd.). We used notch filters (F10-527.0-4-2.00, CVI Melles Griot) to isolate the Mie scattering of the PIV tracer particles from the fluorescence of the passive dye. These filters were positioned in between the CMOS sensor of the PIV cameras and the camera lens. We used rhodamine 6G (Sigma-Aldrich Co. LLC) as the fluorescent dye, which exhibits maximum light absorptivity at 525 nm and maximum light emissivity at 555 nm (Crimaldi 2008). An optical edge filter was mounted in between the CMOS sensor of the PLIF camera and the camera lens, which isolated the fluorescence of the passive dye from the Mie scattering of the particles. An example of the effect of the optical filters for the PIV and the PLIF cameras is presented in Fig. 6. Here, we show an image (a) from a colour digital camera that captures the Mie scattering from the particles and the fluorescence from the passive dye. The PIV notch filter is positioned in front of the digital camera in Fig. 6 (b), which shows that only a narrow band of wavelengths around the laser wavelength (527 nm) passes through the filter. The PLIF edge filter is applied in Fig. 6(c), which shows that the green light from the laser is filtered out and the orange fluorescence from the passive dye remains.

![Fig. 6](image-url) Application of optical filters to a colour digital camera. (a) The true-colour image showing the nozzle (dashed white line), seeding particles, and fluorescent dye. (b) PIV notch filter positioned in front of the camera to block out the fluorescence of the rhodamine 6G. (c) PLIF edge filter positioned in front of the camera to block out the Mie scattering from the particles.

The PIV particle fields and PLIF scalar fields were captured at rate of 1 kHz and two time-sequences of 5456 images each were recorded and saved. The PIV particle images were imported into DaVis (LaVision GmbH) and processed with multiple-passes of decreasing window sizes; the first passes were performed with 64×64 px interrogation windows, then 32×32 px windows, and finally 24×24 px windows with 75% overlap. Window-shift and window-deformation features were implemented to improve the accuracy of the PIV cross-correlation algorithm (Adrian &

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<table>
<thead>
<tr>
<th>PLIF measurement region</th>
<th>$-3.4 \leq r/D \leq 3.4; \ 0 \leq x/D \leq 6.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLIF pixel spacing $\Delta x$</td>
<td>0.26 mm</td>
</tr>
<tr>
<td>PLIF camera lens</td>
<td>85 mm Nikon lens (f/3.5)</td>
</tr>
<tr>
<td>Total number of fields</td>
<td>10 912</td>
</tr>
<tr>
<td>Number of down-sampled fields $N$</td>
<td>500</td>
</tr>
</tbody>
</table>

**Table 2** Experimental parameters of the simultaneous PIV/PLIF measurement.
Westerweel 2014). The resultant vector fields exhibit a spatial resolution of 3.20 mm (0.08D) based on the size of the final interrogation windows, and vector spacing of 0.80 mm (0.02D).

The raw passive dye images captured by the PLIF camera require extensive post-processing to yield a scalar concentration field in which the pixel brightness is linearly-proportional to the local scalar concentration. The raw dye images are affected by (i) imperfections cast by the Perspex window, (ii) the non-uniform laser intensity distribution, and (iii) the absorption of light energy along the beam path, as described by the Beer-Lambert law (Crimaldi 2008). We compensate for each of these imaging effects in a process that is illustrated in Fig. 7 for an instantaneous scalar field. The shadows cast by imperfections in the Perspex window are exhibited in Fig. 7(a). These are removed in Fig. 7(b) by evaluating the time-averaged scalar concentration field, for which the turbulent structures are smoothed-out and the shadows become more discernible. The effect of the laser-sheet intensity profile is then compensated in Fig. 7(c). This is achieved by imaging a clear box containing a homogeneous solution of rhodamine 6G. Imaging such a box should yield a uniform scalar concentration field, which isolates the variations of the laser-sheet intensity from the scalar concentration field. Finally, we invoke the Beer-Lambert law that describes the attenuation of light energy along a beam path. The attenuation of light energy may be described by the following expression:

\[ I(r) = I(r_0) \exp \left( -\epsilon_\alpha \int_{r_0}^{r_1} \phi(r)dr \right), \]

where \( \epsilon_\alpha \) is the absorption coefficient (\( \epsilon_\alpha = 1.1 \times 10^5 \text{ cm}^{-1}\text{M}^{-1} \) for rhodamine 6G; Crimaldi 2008). This expression shows that the light intensity, \( I \), along beam path \( r \) decays as an exponential function. We integrate this expression along columns in the PLIF image rather than along the beam path. This is valid because the laser-sheet is slowly expanding and therefore the beam paths are close to being vertical. The resultant scalar concentration field is illustrated in Fig. 7(d).

2.4 Identifying the turbulent/non-turbulent interface

The TNTI represents a region of finite thickness over which the vorticity field transitions from non-turbulent levels (i.e. zero) to a magnitude that is representative of the turbulent region (da Silva et al 2014). This region may be identified by applying a threshold of vorticity and identifying the corresponding isosurface. However, this is not feasible for planar measurements, such as that described here, which only capture one component of vorticity. To overcome this limitation researchers have used surrogate measures to identify the TNTI, such as the turbulence kinetic energy (de Silva 2013), axial velocity (Anand et al 2009, Khashehchi et al 2013), or the scalar concentration field of a passive scalar (Westerweel et al 2002). In this study we use the scalar concentration field to identify the TNTI because passive scalars (\( Sc \gg 1 \)) have been shown to agree well with the local vorticity field (Westerweel et al 2009). Moreover, PLIF measurements offer
greater spatial resolution and dynamic range compared to PIV, which is necessary for the fractal scaling analysis presented in §3.

We implement an empirical approach to identify the scalar concentration threshold that corresponds to the TNTI; Prasad & Sreenivasan (1989) and Westerweel et al (2002) have used similar approaches. We consider the conditional-averages of (i) scalar concentration, (ii) spanwise vorticity, (iii) turbulence kinetic energy, and (iv) axial velocity as a function of the scalar concentration threshold. The conditional-average is determined by extracting all the points from the fields that exceed the given scalar threshold and calculating the ensemble average, $\langle \chi | _{\phi > \phi_t} \rangle$, where $\chi$ is one of the aforementioned variables and $\phi_t$ is the scalar threshold value. This process is repeated for a wide range of scalar thresholds and is presented in the top row of Fig. 8.

![Fig. 7 Normalisation procedure applied to the scalar images.](image-url)
The scalar threshold that identifies the TNTI is given by the inflection point in the conditionally-averaged profiles (Prasad & Sreenivasan 1989). We consider the gradient of the conditionally-averaged profiles to show the inflection point more clearly. In each of the gradient profiles in the bottom row of Fig. 6 we observe a plateau in the gradient at $\phi_t/\phi_e = 0.07$, which represents the inflection point in the conditionally-averaged profiles. That all 4 conditionally-averaged variables exhibit a gradient change at the same scalar threshold gives us confidence that a threshold of $\phi_t/\phi_e = 0.07$ does indeed coincide with the transition from the non-turbulent to the turbulent regions of the jet. Hence, we use this scalar concentration threshold to identify the TNTI in the turbulent jet flow. This threshold is applied to each instantaneous scalar concentration field using the “contour” function in Matlab. The longest continuous contour lines in the $r > 0$ and $r < 0$ regions are extracted; these lines represent the TNTI for the top-half and bottom-half of the jet in the measurement plane.

The effectiveness of the scalar concentration criterion as an identifier of the TNTI is illustrated in Fig. 9. In this figure we plot the conditionally-averaged profiles of (a) scalar concentration and (b) spanwise-vorticity magnitude along coordinates that are locally-normal to the TNTI. These profiles are generated as follows: at each point along the TNTI we determine the local-normal unit vector, $\mathbf{n} = (\nabla \phi/|\nabla \phi|)_n$, where subscript I represents points along the TNTI. The local-normal vector is used to generate a local coordinate, $x_n$, that extends $-0.5 \leq x_n/D \leq 1$, where negative distances represent the non-turbulent region. The spacing of the new coordinate is selected to match the vector spacing of the PIV measurements. At each point along the $x_n$ coordinate we interpolate the local velocity and scalar concentration values, as well as the spanwise vorticity values that are measured from the instantaneous Eulerian field. In some instances the interface-normal coordinate crosses the TNTI again. In these cases, we exclude all
points beyond any secondary transitions from turbulent to non-turbulent regions (or vice versa). The instantaneous conditional profiles for all fields are saved and then averaged to generate the profiles in Fig. 9.

Fig. 9 (a) Conditionally-averaged profile of scalar concentration along coordinates that are locally-normal to the TNTI. The profiles are measured across ±0.5x/D from the streamwise positions indicated in the legend. (b) Conditionally-averaged profile of the spanwise-vorticity magnitude in logarithmic scaling. Inset plot shows spanwise-vorticity profile for x/D = 6.5 ± 0.5 with linear scaling.

The scalar concentration profiles in Fig. 9(a) show that there is a steep increase in the scalar concentration across the TNTI, x_n/D = 0. Interestingly, the magnitude of the scalar jump across the TNTI that is observed in the streamwise region x/D ≥ 3.5 (Δϕ ≈ 0.2ϕ_e) is comparable to the scalar jump that exists across the TNTI in the far-field of turbulent jets (Mistry et al 2016). This suggests that the TNTI from x/D ≥ 3.5 exhibits some semblance to that in a fully-developed turbulent field.

The spanwise-vorticity profiles in Fig. 9(b), presented in logarithmic scaling, demonstrates that the TNTI as identified by a Sc ≫ 1 passive scalar does indeed isolate the turbulent from the non-turbulent region of a flow. This is because the jump in vorticity from non-turbulent levels, O(10^{-2}), to the turbulent levels, O(10^1), coincides with the isocontours of scalar concentration, ϕ_t/ϕ_e = 0.07. The jump in vorticity is more clearly shown for the x/D = 6.5 ± 0.5 case in the linearly-scaled inset plot in Fig. 9(b). Thus, the scalar concentration threshold that we identify in Fig. 8 yields isocontours that fall within the finite region of the TNTI, as defined by the region over which the vorticity transitions to the turbulent levels.
Fig. 10 (a) Instantaneous scalar concentration field in the near-field of a turbulent jet (background contours), superimposed with the TNTI (blue line), and local velocity vectors (purple arrows). (b) As above, but with the spanwise-vorticity field as the background contours.

The culmination of the above-described experimental set-up is illustrated in Fig. 10(a): the planar measurement of the scalar concentration field in combination with the local velocity vectors. The scalar concentration field identifies the TNTI, which encapsulates the corresponding spanwise vorticity field that is illustrated in Fig. 10(b). The time-resolved measurements yield 10 912 velocity and scalar fields that are spaced 1 ms apart. We down-sample this data to create a data-set of 500 fields for the calculation of the statistical quantities presented in this paper. Although not presented here, the time-resolved data is used to calculate the local entrainment velocity, $v_n$, along the TNTI using an interface-tracking technique.

3. The scale-dependence of the TNTI

In this section, we implement a multi-scale methodology to characterise the streamwise evolution of the TNTI surface area (or length in 2D). This is achieved by evaluating the scale-dependence of the TNTI length using a spatial-filtering technique. Measurements of the TNTI in fully-developed turbulent flows have indicated that this interface exhibits a power-law scaling of length as a function of measurement resolution, which is characteristic of fractal scaling.
(Sreenivasan et al 1989). The exponent of the power-law scaling represents the fractal dimension, $D_f$, of the surface. We use the fractal dimension to determine the streamwise distance at which the expected TNTI fractal scaling ($D_f = 0.3 - 0.4$, de Silva et al 2013) is recovered.

We spatially-filter the scalar concentration fields with a box-averaging filter that span a range of filter sizes, $6.6\times10^{-3} \leq \Delta/D \leq 3.6$. For each filter size we identify the TNTI from the filtered instantaneous fields using a constant threshold value, $\phi_c/\phi_e = 0.07$ that is determined in §2.4. We then calculate the mean interface length, $\langle L_s \rangle$, across short streamwise extents of $x/D \pm 1$ that span the measurement region. This allows us to examine the evolution of the interface length scaling with streamwise distance, which is presented in Fig. 11(a). For very large filter sizes, $\Delta/D \approx 1$, the interface is very smooth and essentially one-dimensional. Hence, the mean interface length, normalised by the streamwise extent of the interface, collapses onto $\langle L_s \rangle/L_x \approx 1$. This is true for all streamwise positions considered. For small filter sizes, $\Delta/D \leq 3\times10^{-2}$, the scaling of $\langle L_s \rangle/L_x$ levels off, which denotes the end of the fractal scaling. This indicates that the filter sizes are approaching the smallest length-scales in the flow, or may be an artefact of the spatial-filtering process (de Silva et al 2013). At the smallest filter size (i.e. unfiltered), $\Delta/D = 6.6\times10^{-3}$, the interface length ratio of reaches a maximum of $\langle L_s \rangle/L_x \approx 3.3$ for $x/D \geq 5$. In 3D we estimate that the surface area of the TNTI in this region would be approximately 11 times larger than its projected area. This is comparable to the surface area estimation of Chauhan et al (2014), who reported that the TNTI of a turbulent boundary layer is expected to be 9 times larger than the unit area on which it resides.

![Fig. 11](image-url) **Fig. 11** (a) Mean interface length as a function of filter size, $\Delta$, and for streamwise distances over spans of $x/D \pm 1$. (b) The fractal dimension of the interface length scaling. The horizontal grey bar represents covers the region $D_f = 0.32 - 0.33$. The power-law scaling is measured in the region $0.03 \leq \Delta/D \leq 0.31$, as indicated by the vertical green ticks in (a).

The fractal dimension, $D_f$, of the TNTI is determined by evaluating the exponent of the power-law scaling across the mid-range of filter sizes in Fig. 11(a). We apply a least-squares fit to the region $0.03 \leq \Delta/D \leq 0.31$, which represents a decade in filter size lengths (see green tick marks in Fig. 11a). The resultant fractal dimensions are plotted in Fig. 11(b) for the range of streamwise distances considered. Note that the streamwise distances in this plot represent the centre-points about which $x/D \pm 1$ segments are extracted for measurement of the fractal dimension. Near the
nozzle-exit ($x/D = 1$) the fractal dimension is $D_f = 0.15$, which increases to $D_f = 0.32 – 0.33$ in the region $x/D > 4$. The larger fractal dimension observed in this downstream region may be attributed to the streamwise vortex structures that evolve from azimuthal instabilities of the initial roll-up structure (Liepmann & Gharib 1992). The fractal dimension in the region $x/D > 4$ agrees well with studies of the TNTI in fully-developed turbulent flows for which $D_f = 0.3 – 0.4$ (Zubair et al 2009; de Silva et al 2013). Similarly, the fractal dimension measured here agrees well with the theoretical analysis of Sreenivasan et al (1989), which estimates that $D_f = 1/3$ based on the Reynolds number similarity hypothesis. This suggests that the turbulence that is modulating the TNTI surface geometry beyond $x/D \approx 4$ (i.e. beyond the potential core region) exhibits a hierarchy of scales that is comparable to that in a fully-developed flow, such as the far-field of a round jet. This is because turbulent eddies across a range of sizes contort the TNTI across a range of scales to increase the surface area, and the fractal dimension of the TNTI is a “footprint” of these eddies. Thus, we suggest that the turbulence that shapes the TNTI from $x/D \approx 4$ is reminiscent of the homogeneous turbulence that exists in the far-field region of the jet.

Evidence of a homogenous flow-field is supported by the velocity correlation functions along the radial axis. The correlation functions presented in Fig. 12 are calculated using (a) the axial velocity fluctuations (lateral correlation function) and (b) the radial velocity fluctuations (longitudinal correlation function) along the radial axis from the jet centreline ($r = 0$). In the region immediately adjacent to the nozzle, $x/D < 1$, the correlation functions exhibit a steep decay. This is because there are only weak fluctuations in the potential core region of the jet. The peak in the correlation function at $r/D = \pm 0.5$ for the $x/D = 1$ profiles is associated with the axisymmetric vortex roll-up that is evident in Fig. 1. This large-scale structure breaks down into small turbulent eddies, which results in the typical correlation function profiles shown for $x/D \geq 4$. In fact, the lateral correlation functions for $x/D \geq 4$ exhibit good collapse, which indicates that the flow has achieved homogeneity (Davidson 2004). This collapse is similarly evident in the longitudinal correlation functions in Fig. 12(b), although the profiles only become similar in the region $x/D \geq 5$. Evidence of homogeneity is further supported with the use of the integral length-scale based on the correlation functions, $L_{u'u'} = \int_0^\infty R_{u'u'}dr$, which is presented in Fig. 12(c). The integral length-scales for axial velocity fluctuations begin to plateau for $x/D \geq 4$; for radial fluctuations the integral length-scale begins to level off from $x/D \approx 5$. Thus, analysis of the correlation functions suggests that the flow has achieved a good degree of homogeneity by $x/D \approx 4$, which approximately coincides with the onset of the fully-developed turbulence fractal dimension, $D_f = 0.32 – 0.33$. 
In this section we demonstrate that the onset of flow homogeneity beyond the potential core of a round jet is observed in the scaling of the TNTI surface area and also with the radial auto-correlation functions. The mixing performance (i.e. entrainment rate) of the jet may be improved with the use of grids (Sponfeldner et al 2015), adding swirl (Freitag et al 2006), or acoustically forcing the jet (Crow & Champagne 1971), to name a few examples. We anticipate that the result of these passive and active flow control techniques would be to increase the turbulence intensity, which in turn yields a more contorted interface. Hence, these flow control techniques may yield a larger surface area, which allows for greater diffusion of vorticity and scalar concentration to the ambient fluid. With respect to the multi-scale geometry of the TNTI, we anticipate that the effect of increasing the surface area via flow control would be a reduction in streamwise distance at which the fractal dimension reaches the fully-developed values ($D_f = 0.3 – 0.4$, de Silva et al 2013).

5. Summary & conclusions
We performed simultaneous stitched-PIV/PLIF measurements in the near-field region of a round jet at high Reynolds number. We use isocontours of a passive scalar ($Sc \gg 1$) to identify the turbulent/non-turbulent interface. The TNTI is identified using an empirical approach in which we evaluate conditionally-averaged variables as a function of scalar threshold. The TNTI that we identify exhibits a jump in spanwise vorticity across it, which indicates that isocontours of a $Sc \gg 1$ scalar can indeed demarcate the boundary of the vorticity field. We analyse the TNTI using a multi-scale methodology that evaluates the scale-dependence of the TNTI length. This is achieved by spatially-filtering the TNTI and measuring the mean length of the interface; the resultant scaling of length with filter size gives the fractal dimension of the TNTI. The fractal dimension, $D_f$, increases from the nozzle-exit and levels-off near the end of the potential core. The plateau of the TNTI fractal dimension coincides with onset of homogeneity based on auto-correlation functions of velocity fluctuations. Interestingly, the value of $D_f$ beyond the end of the potential core agrees well with the fractal dimension measured by other authors in fully-developed
turbulence. We may interpret $D_f$ as a “footprint” of the eddies that contort the TNTI. From this perspective, we suggest that there is an extensive hierarchy of turbulent eddies acting on the TNTI beyond the potential core that is reminiscent of the turbulence cascade observed in fully-developed turbulent flows.

6. References


