Quantification of the loss-of-correlation due to PIV image noise

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ABSTRACT

The effect of image noise on the uncertainty of velocity fields measured with particle image velocimetry (PIV) is still an unsolved problem. Image noise reduces the correlation signal and thus affects the estimation of the particle image displacement. However, a systematic quantification of the effect of the noise level on the loss-of-correlation is missing. In this work, a new method is proposed to estimate the loss-of-correlation due to image noise $F_o$ from the auto-correlation function of PIV images. Furthermore, a new definition of the signal-to-noise ratio $SNR$ for PIV images is suggested, which results in a bijective relation between $F_o$ and $SNR$. Based on the newly defined $SNR$ it becomes possible to estimate the signal level and the noise level itself by adding additional noise with known intensity to the images. The presented method is very general because the estimation of $F_o$ and $SNR$ works independently of various parameters, including the particle image intensity, the particle image density and the particle image size. The findings lead to an extension of the fundamental PIV equation $N = N_1 F_1 F_o F_\Delta$ and enable PIV users to optimize their measurement setup with respect to the image noise and not only based on the loss-of-correlation due to in-plane motion, out-of-plane motion and displacement gradients. Furthermore, the new definition of $SNR$ allows for a characterization and comparison of PIV images. Finally, the quantification of the uncertainty contribution due to image noise is possible.

1. Introduction

Thanks to the enormous improvements regarding the quality of the equipment and the evaluation techniques during the last decades, nowadays PIV provides reliable velocity field distributions in transparent fluids even when the in-plane and out-of-plane loss-of-pairs is significant and strong gradients are present (Stanislasm et al., 2003, 2005, 2008). The valid detection probability of a shift vector depends on the height of the correlation peak with respect to the correlation noise. Keane and Adrian (1992) showed that the formation of a well detectable correlation peak is sufficiently likely if the number of particle images within the interrogation window is $N_1 > 7$. It is well known, that loss-of-correlation due to in-plane motion $F_1$, out-of-plane motion $F_o$ or displacement gradients $F_\Delta$ lead to a decreased probability for the detection of valid displacement vectors (Adrian and Westerweel, 2010, Raffel et al, 2007). To keep these effects into account, the optical magnification, the seeding concentration, the light-sheet
thickness and the size of the interrogation windows should be selected such that the effective number of particle images is \( N = N_1 F_1 F_0 F_A > 7 \). Following the one-quarter rule proposed by Kean and Adrian (Keane and Adrian, 1990) ensures a significantly high value of \( F_i \) for the first evaluation pass on average. For following evaluation passes, the in-plane loss-of-pair is compensated by multi-pass interrogation techniques using window shifting (Willert, 1996), such that \( F_i = 1 \) (at least for the final pass) for state-of-the-art multi-pass PIV evaluation methods. In-plane gradients can be compensated by combining multi-pass interrogation techniques with window deformation approaches (Scarano, 2001). In the case of dominant out-of-plane motion, the time separation between the double images \( \Delta t \) must be selected carefully to find a compromise between a large dynamic velocity range and a small amount of loss-of-pairs (Adrian, 1997, Scharnowski and Kähler, 2016).

After optimizing the probability in detecting a valid vector the problem of quantifying the uncertainty of the results emerged. Promising approaches for the uncertainty quantification are published by Charonko and Vlachos, 2013, Christensen and Scarano, 2015, Kähler et al, 2012a,b, Neal et al, 2015, Scharnowski and Kähler, 2016, Sciacchitano et al, 2013, 2015, Timmins et al, 2012, Wiencke, 2015, Wilson and Smith, and 2013, Xue et al, 2015. It was shown that the uncertainty of PIV measurements depends on many parameters in a complex manner, including particle image size, particle image density, turbulent fluctuations, noise level, velocity gradients and many more. Wiencke (2015) showed that knowledge about the individual parameters is not necessarily required in order to estimate the shift vectors uncertainty if one analyzes the shape of the correlation function. However, knowledge about the effect of the different parameters on the uncertainty certainly is desirable to identify the most important error sources and to optimize PIV experiments efficiently. To do so, it is necessary to determine not only the uncertainty with respect to the parameters, which can be done with the help of synthetic PIV images, but also to determine the value of these parameters from the PIV images.

One important parameter, which is difficult to determine from experimental PIV images, is the image noise level or the signal-to-noise ratio. Image noise reduces the correlation function's contrast of cross-correlated PIV images and therefore raises the uncertainty for the shift vector estimation. Figure 1 illustrates, based on synthetic images, that the noise level \( \sigma_n \), which is the standard deviation of random noise, affects the shift vector uncertainty \( \Delta X_{rms} \) quite strongly. While the particle image diameter \( D \) was kept constant, different numbers of particle images per pixel \( N_{pp} \) were tested. The lower the particle image density is the higher the shift vector uncertainty becomes. Interestingly the effect of \( N_{pp} \) on \( \Delta X_{rms} \) becomes larger with increasing noise levels. This illustrates that knowledge about all individual parameters is required to optimize PIV measurements.
Unfortunately, no method exists to determine the image noise level reliably from PIV images. To overcome this problem a new method to estimate the loss-of-correlation due to image noise \( F_\sigma \) from the height of the auto-correlation function is proposed in this work. It will be shown that a new definition of the signal-to-noise ratio \( SNR \) allows for an analytical solution for the loss-of-correlation due to image noise with respect to \( SNR \). Synthetic images are used to analyze the effect of different parameters on the correlation function. The generation of the synthetic images is briefly discussed in the following section. Section 3 describes how the loss-of-correlation is determined from PIV images and Sec. 4 discusses the new \( SNR \) definition. Section 5 shows how the noise level and the signal level are extracted from the \( SNR \) by applying additional noise with known standard deviation.

Fig. 1 Shift vector uncertainty \( \Delta X_{ms} \) computed from synthetic PIV images as a function of the image noise level \( \sigma_n / I_0 \) for different particle image densities \( N_{ppp} \). Particle image diameter and interrogation window size were set to \( D = 3.0 \) px and 32×32 px, respectively.

2. Synthetic PIV images

The generation and analysis of synthetic PIV images is a well-established method to investigate the effect of different parameters on the shift vector uncertainty, as discussed in Kähler et al (2012a), Stanislas et al (2003, 2005, 2008) and other works. One of the main advantages of synthetic PIV image analysis is that all parameters can be precisely controlled and varied independently over a range not achievable in experiments. On the other hand, all important parameters must be considered in order to generate realistic data.

All synthetic images were generated using MATLAB functions, as discussed in detail in Scharnowski and Kähler (2016). The controlled image parameters include: the maximum particle
image intensity $I_0$, the particle image diameter $D$ (width at which the intensity drops to $I_0/e^2$), number of particle images per pixel $N_{ppp}$, noise level standard deviation $\sigma_n$ and others. The discrete pixels’ gray values were computed from the integral over the pixels' areas, corresponding to a sensor fill-factor of 1. Between the first synthetic PIV image $A$ and the second one $B$ a small particle image displacement of $\pm 1$px, with a constant gradient in $y$-direction, was simulated to capture all possible sub-pixel displacements in the analysis. This is important to achieve representative results. To account for the discrete nature of digital images, the intensity distribution was converted to 16 bit unsigned integer numbers. The image parameters were varied to study the effect of the noise level on the loss-of-correlation and the shift vector uncertainty. Figure 2 shows small sections (100×100 px) of example images with different noise levels. The full size of the synthetic images was 1024×1024 px.

![Fig. 2 Examples of synthetic PIV images with constant particle image diameter $D=3$ px, constant particle image density $N_{ppp}=0.1$ but increasing noise level from left to right.](image)

3. Loss-of-correlation

The function of the cross-correlation coefficient $C(\xi,\psi)$ of a PIV image pair is computed from the averaged product of the intensity variations normalized by the product of the standard deviations within the interrogation windows of the two images $A$ and $B$:

$$C(\xi,\psi) = \frac{\sum_{x=1}^{X} \sum_{y=1}^{Y} I'_A(x,y) I'_B(x,y)}{X \cdot Y \cdot \sigma_A \cdot \sigma_B} \quad (1)$$

Where $(x,y)$ are coordinates of the image plane within the interrogation window, $(\xi,\psi)$ are the displacement coordinates and $\sigma_A$ and $\sigma_B$ are the standard deviations within the interrogation windows of the images $A$ and $B$, respectively. The intensity variations $I'$ are the difference between the image intensity $I$ and the mean intensity $\langle I \rangle$. 
If random noise is added to the image intensity the shape of the correlation function, computed from Eq. (1), remains unchanged on average. However, the denominator in Eq. (1) increases. The standard deviation of a noisy image $\sigma_{A,n}$ is:

$$
\sigma_{A,n} = \sqrt{\sigma_A^2 + \sigma_n^2}
$$

(2)

where $\sigma_n$ is the standard deviation of the noise level. Thus, image noise results in a decreased correlation signal. The loss-of-correlation due to image noise can be defined as the ratio of the averaged correlation function with and without image noise:

$$
F_\sigma = \frac{\langle C_n \rangle}{\langle C \rangle} = \frac{\sigma_A \cdot \sigma_B}{\sqrt{\sigma_A^2 + \sigma_n^2} \cdot \sqrt{\sigma_B^2 + \sigma_n^2}}
$$

(3)

It is important to note that the loss-of-correlation due to image noise $F_\sigma$ not only depends on the noise level but also on the standard deviation of the PIV images. Where $\sigma_A$ is a function of the maximum particle image intensity $I_0$, the particle image diameter $D$ and the particle image density. Based on the analysis of synthetic images it can be shown that the standard deviation of a noise free PIV image is:

$$
\sigma_A = \frac{I_0}{2} \sqrt{\frac{N_{ppp} \left( \frac{\pi}{4} D^2 - 1 \right)}{}}
$$

(4)

for $D \leq 2$ and $0.01 \leq N_{ppp} \leq 0.1$ as shown in Fig. 3.

While the loss-of-correlation due to image noise $F_\sigma$ is superimposed by the loss-of-correlation due to in-plane motion $F_I$, out-of-plane motion $F_O$ and velocity gradients $F_\Delta$ in the case of the cross correlation (see Adrian (1988), Soria and Willert (2012) or Scharnowski et al (2012)), the auto-correlation function depends on the noise alone. Figure 4 illustrates the normalized auto-correlation function $R$ for three different noise levels. The center value of the

![Fig. 3 Image standard deviation as a function of the particle image diameter for different particle image densities.](image)
normalized auto-correlation function always equals one due to the self-correlation of the noise.

By using the intensity of the surrounding pixels to approximate the discrete auto-correlation function by a Gaussian fit function, the estimation of the loss-of-correlation from the peak height of the fit function $R_{\text{max}}$ becomes possible. It is evident from the figure that $R_{\text{max}}$ decreases with increasing noise level.

According to equations (3) and (4) the loss-of-correlation due to image noise varies with $D$ and $N_{\text{ppp}}$ for a constant noise level. This is shown in Fig. 5, where the maximum intensity of the particle images $I_0$ and the image noise standard deviation $\sigma_n$ were kept constant, while $D$ and $N_{\text{ppp}}$ were changed over a broad range. It becomes clear from the figure and from Eq. (4), that the ratio $I_0/\sigma_n$, which is often used as signal-to-noise ratio for PIV images, is not suited for the estimation of the loss-of-correlation due to image noise. This is obvious for the following two reasons: First, not only the center of the particle images contributes to the correlation function but also the lower intensity values located around the maximum. Thus, it is evident that larger particle images lead to higher signal and to less loss-of-correlation. Second, more particle images increase the standard deviation of the image intensity, according to Eq. (4), which results in a
reduced effect of the noise level on the loss-of-correlation, according to Eq. (3). Thus, higher particle image densities also represent higher signals and lead to less loss-of-correlation.

4. Signal-to-noise ratio

The maximum of the Gaussian fit function $R_{\text{max}}$ of the auto-correlation function is equal to the loss-of-correlation of the correlation function. Thus, $F_\sigma$ in Eq. (4) can be computed from image $A$, assuming that $A$ and $B$ have the same maximum intensity and noise level:

$$F_\sigma = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_n^2}$$  \hspace{1cm} (5)

Based on this equation, it is proposed to use the ratio of the standard deviations of the image intensity and the noise as signal-to-noise ratio:

$$SNR = \frac{\sigma_A}{\sigma_n}$$  \hspace{1cm} (6)

which results in:

$$F_\sigma = \left(1 + \frac{1}{SNR^2}\right)^{-1}$$  \hspace{1cm} (7)

Following the definition of Eq. (6), the $SNR$ becomes 1 if $\sigma_A$ and $\sigma_n$ are equal. In this case the loss-of-correlation due to image noise is 50%. For smaller noise levels $F_\sigma$ becomes larger and reaches 80% for $SNR = 2$ or 96% for $SNR \approx 5$, respectively.

The proposed definition of $SNR$, shown in Eq. (6), is universal, because it results in a unique relation between $F_\sigma$ and $SNR$, which includes the effects of the particle image size and the particle image density, according to Eq. (4). Figure 6 illustrates the estimated $F_\sigma$ from synthetic data.

![Fig. 6 Loss-of-correlation due to image noise as a function of the signal-to-noise ratio as defined in Eq. (6). Each symbol contains a variation of $D$ and $N_{\text{ppp}}$.](image-url)
PIV images with varying image intensity and varying noise level. For each symbol type in the figure, the particle image diameter was varied between $2 \leq D \leq 10$ px and the particle image density varied between $0.01 \leq N_{ppp} \leq 0.1$, like in Fig. 3. The results clearly show that all estimated values collapse nicely with the theoretical curve given by Eq. (7). It is evident from Fig. 6 that the often used definition, $I_0 / \sigma_n = SNR$, does not result in a constant loss-of-correlation. The ratio $I_0 / \sigma_n$ only represents the same $SNR$ as $\sigma_A / \sigma_n$ if $N_{ppp} = 4/(D^2 \cdot \pi / 4 - 1)$, according to Eq. (4). For $D = 3$ px, for instance, the number of particle images per pixel must be $N_{ppp} \approx 0.65$ to fulfill the condition $I_0 = \sigma_A$, which is a rather high particle image density.

On the one hand, the effect of particle image density on the loss-of-correlation is very small for low noise levels $SNR > 10$ (red symbols in Fig. 6). In this region, improving $SNR$ has only little effect on $F_\sigma$. On the other hand, at higher noise levels, particle image density and particle image diameter strongly affect $F_\sigma$, in agreement with the results of Fig. 4. The steepest slope of the function $F_\sigma(SNR)$ is at $SNR = 1$, where $\sigma_A = \sigma_n$ and $F_\sigma = 0.5$. In this region, improving $SNR$ by increasing the particle image intensity or increasing the particle image density is most efficient and results in the best gain for $F_\sigma$.

5. Estimation of $\sigma_A$ and $\sigma_n$

The signal-to-noise ratio is estimated from the height of the auto-correlation function. The inverse function of Eq. (7) is:

$$SNR = \frac{\sigma_A}{\sigma_n} = \left( \frac{1}{F_\sigma} - 1 \right)^{-0.5}$$  \hspace{1cm} (8)

If $F_\sigma$ is measured, the ratio $\sigma_A / \sigma_n$ can be computed. The absolute values of $\sigma_A$ and $\sigma_n$ however, remain unknown. In order to extract these quantities, additional noise with the known standard deviation $\sigma_{add}$ can be added to the PIV image. Let $F_\sigma,\text{add}$ be the resulting height of the normalized auto-correlation function of the PIV images with additional noise. Hence, Eq. (8) becomes:

$$\frac{\sigma_A^2}{\sigma_n^2 + \sigma_{add}^2} = \left( \frac{1}{F_\sigma,\text{add}} - 1 \right)^{-1}$$  \hspace{1cm} (9)

Where $F_\sigma,\text{add}$ is again a measurable quantity. Combining equations (8) and (9) results in:
\[
\sigma_n = \left[ \frac{(1 - F_\sigma) \sigma_{\text{add}}^2 F_{\sigma, \text{add}}}{F_\sigma - F_{\sigma, \text{add}}} \right]^{0.5}
\]  

(10)

and Eq. (8) gives:

\[
\sigma_A = \sigma_n \left( \frac{1}{F_\sigma} - 1 \right)^{0.5}
\]

(11)

Thus, the signal \( \sigma_A \) and the noise level \( \sigma_n \) can be extracted from PIV images, theoretically. Therefore, \( F_\sigma \) and \( F_{\sigma, \text{add}} \) must be estimated with high confidence. Figure 7 illustrates how the window size affects the estimation of \( F_\sigma \). As before, synthetic PIV images with \( D = 3 \text{ px} \), \( I_0 = 1024 \text{ counts} \) and \( N_{\text{ppp}} = 0.1 \) were generated. Gaussian noise was added to the images to obtain different values for the loss-of-correlation. The window size, from which the auto-correlation function was computed, varied between \( 32 \times 32 \text{ px} \) and \( 256 \times 256 \text{ px} \). The markers in the figure represent the mean value of the estimated \( F_\sigma \) averaged over 100 windows and the error bars correspond to the standard deviation. The figure shows that the smallest window size considered \( (32 \times 32 \text{ px}) \) results in a rather large scatter of the estimated \( F_\sigma \). For a window size of \( 64 \times 64 \text{ px} \) and \( 128 \times 128 \text{ px} \), the estimated \( F_\sigma \) scatters much less. However, it seems to be slightly underestimated in the range between \( F_\sigma \approx 0.2 \) and \( 0.8 \), for these window sizes. The largest windows investigated \( (256 \times 256 \text{ px}) \) allow for a reliable estimation of \( F_\sigma \). This also holds for the estimation of \( F_{\sigma, \text{add}} \). Thus, it can be concluded that several thousand particle images are required for the estimation of the loss-of-correlation due to image noise from the auto-correlation function. In this work, the auto-correlation function was computed from \( 1024 \times 1024 \text{ px} \), if not stated differently. In principle, the auto-correlation function can also be computed from averaged window correlation (Meinhart et al, 2000) or single-pixel ensemble-correlation (Kähler et al, 2006, Westerweel et al, 2004).

Fig. 7 Estimated versus simulated loss-of-correlation due to image noise for different interrogation windows.
From the denominator in Eq. (10) it is obvious that the added noise must result in a loss-of-correlation $F_{\sigma_{\text{add}}}$ that differs significantly from the original $F_{\sigma}$. In order to analyze the effect of $\sigma_n$ and $\sigma_{\text{add}}$ on the estimated noise level, synthetic PIV images with $D = 3\text{ px}$, $I_0 = 1024\text{ counts}$ and $N_{\text{ppp}} = 0.1$ were generated with different noise levels ranging from $\sigma_n = 0.01I_0$ to $1.0I_0$. Additional Gaussian noise with a standard deviation between $0.02I_0$ and $0.5I_0$ was added to the images. Figure 8 shows the estimated image noise level computed by using Eq. (10) for the different levels of additional noise. From the figure the following conclusions can be drawn: First, the standard deviation of the additional noise $\sigma_{\text{add}}$ must be at least $\approx 20\%$ of $\sigma_n$ in order to estimate $\sigma_n$ with high confidence. Second, very small image noise levels with $\sigma_n < 0.03$, corresponding to $\text{SNR} > 30$, are underestimated. Here, the slope of $F_{\sigma}(\text{SNR})$ is very small and a small bias error for the estimation of $F_{\sigma}$ could result in the observed behavior. Third, for $0.03 \leq \sigma_n / I_0 < 1.0$, corresponding to $0.4 < \text{SNR} \leq 30$, the image noise level can be estimated accurately by adding additional noise with a standard deviation of the same order as those of the original image noise.

5. Conclusions

The loss-of-correlation due to PIV image noise $F_{\sigma}$ depends on the particle image intensity, the particle image density, the particle image size and the image noise in a complex manner. The presented approach shows that the loss-of-correlation can be determined from the distribution of the auto-correlation function with high confidence if the central peak is rejected. Furthermore, it is shown that the ratio of the standard deviations of the (noise-free) PIV image and the image
noise is a useful and quite universal definition for SNR. The definition includes the effect the particle image intensity, the particle image density, the particle image size and the image noise. Due to the unique relation between SNR and $F_\sigma$, the newly defined SNR can directly be determined from the auto-correlation function, for the first time. By adding additional noise with a standard deviation in the order of the standard deviation of the original image noise, it was possible to not only extract the SNR but also to estimate the noise level $\sigma_n$ and the signal level $\sigma_A$ of PIV images.

The findings of this work are an important step towards a full characterization of PIV images. Knowledge about SNR enables PIV users to compare their measurements quantitatively and to optimize the measurement setup.

Finally, this analysis implies that the classical rule proposed by Keane and Adrian (1992), $N = N_t F_t F_o > 7$, which was extended by the loss-of-correlation due to gradients $F_\Delta$ by Westerweel (2008), must also be extended by the loss-of-correlation due to image noise. Hence, the effective number particle images is $N = N_t F_t F_o F_{\Delta F_\sigma}$. The recommendation $N > 7$ in the interrogation window is still valid, in order to keep the number of spurious vectors sufficiently low, such that they can be reliably detected.

References


