Simple Tools to unveil characteristics of PIV coupled errors

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ABSTRACT

Digital PIV was introduced more than 20 years ago. Ever since, its use has widely spread. Currently, experimental and industrial facilities often recur to it for their measurements. In spite of its widespread use, the error of the technique is not fully characterized nor completely understood. In an attempt to increase the error knowledge in PIV and given the relevance of turbulent flows in many applications, this work focuses on the errors that spatial gradients, like the ones induced by turbulence, can produce on a PIV measurement. The objective pursued is to characterize the variation of the error magnitude with some of the measurement setup parameters (the time between laser pulses, Δt, and the laser sheet thickness). This would allow for future error-wise setup optimizations. For such purpose, a new error analysis tool is introduced (identified as PIV Simulator). It is used to study two related issues: (i) the variation of the error with the measurement parameters and (ii) the error distribution in the different length-scales of the flow (by recurring to the second-order structure function S,(r)). Due to the mixing of small scale information in the interrogation volume, this tool unveils the possible presence of errors as large as 45% of S,(r) in actual measurements where the occurrence of other errors conceal this fact, giving direct error estimations as low as 10%.

1. Introduction

Digital Particle Image Velocimetry is a measurement technique that was introduced more than 20 years ago (Willert and Gharib 1991), as a digital-image laser-based technique to obtain the instantaneous velocity flow-fields. Since its introduction, PIV has been widely used in industrial and experimental facilities. Nonetheless, the different error sources of the technique and their relevance in different measuring conditions are not yet completely determined nor well understood. Recent research has unveiled some features on this topic, but much work still needs to be done in order to understand error coupling in PIV. Moreover, the assessment of the error is especially relevant in modern measurements to provide an uncertainty interval for comparison and validation of numerical calculations.

Research in PIV errors could be divided into two complementary conceptions:
• Some authors seek tools that provide, “a posteriori” a broad estimation of the measurement uncertainty by encompassing all the error estimation at each location as a function of a few input parameters (Timmins et al. 2012, Wilson and Smith, 2013, among others).

• Other authors try to evaluate each error source separately: spatial discretization, capacity of the tracer particles to follow the flow, CCD induced errors, spatial velocity gradients, out-of-plane motion, optical distortion, etc. (Hjemfelt and Mockros 1996, Lecuona et al. 2004, Westerweel 2008, Nobach and Bodenschatz 2009, Legrand et al. 2014, among others). In this last case, also the complex coupling between the different sources requires research effort (e.g. Legrand et al. 2014).

The separate error source study allows to discriminate error sources associated with the acquisition (seeding, \( \Delta t \) delay between laser pulses, laser sheet thickness, etc.), from those associated with the image processing algorithms (interrogation window size (IW), correlation peak fitting, image interpolation, subpixel resolution algorithm, etc.). In particular, the parameters selected during the acquisition may strongly limit the dynamical range of the measurement (e.g. Lecordier et al. 2001, Nobach and Bodenschatz 2009, Nogueira et al. 2012, Nogueira et al. 2014, among others).

The acquisition of PIV images is the more expensive phase and the harder to repeat. A proper characterization of the error, allows for optimizing the acquisition parameters and thus designing an experiment where the influence of the error is reduced, increasing the value of the technique.

In this line of work, and given the relevance of turbulent flows for many applications, this paper focuses on the adequate selection of acquisition parameters for providing turbulent related magnitudes, such as the kinetic turbulent energy or the dissipation rate (Saarenrinne and Piirto, 2000, De Jong et al. 2009, Nogueira et al. 2009, among others). For specifying the length-scale interval that is correctly measured in these cases, it is essential to assess the PIV error for each particular spatial scale. Thus, the impact of the acquisition parameters on the magnitude of the error at each different scale has to be unveiled.

At present time, synthetic images are a common tool to study this kind of inter-relations. However, in this scenario, there is an excessive number of interactions between the flow variations with and the error sources integrated in the synthetic images. As a consequence, a simple dedicated tool free of some of these error coupling is proposed. The aim of this research tool, henceforth identified here as PIV Simulator, is to allow a clear view of the impact of an error source by artificially eliminating the coupling with other error sources.
In this paper, Section 2 indicates PIV Simulator working principles and locates its place within usual error assessment process. Section 3 describes the method chosen to extract error information at each length-scale. Section 4 offers the results that link PIV error to time delay, $\Delta t$, and the laser sheet thickness, $Th$. The PIV Simulator results are compared to those obtained by synthetic images as well as real images, showing the potential of the PIV Simulator for PIV error research. Section 5 draws the main conclusions of this work.

2. Error analysis tools

2.1 Location of the “PIV Simulator” in the error assessment procedures

In the literature, PIV error assessment research presents a general structure that can be split in different “steps”:

(i) Theoretical rationale: it allows considering each error source separately and estimating a gross estimate of the error order of magnitude without any coupling between them.

(ii) A dedicated experimental setup: permits checking the coherence of the theoretical reasoning on a real flow field. However, for complex situations the detailed real velocity field is unknown and the coherence check requires additional analysis tools.

(iii) Synthetic images: generating digital PIV images by directly plotting on it particle pairs, corresponding to a given flow field, allows for a PIV application in which the error can be evaluated due to the detailed knowledge of the underlying flow field. The main difficulty for error research is the coupling between error sources inherent to PIV.

It is easy to find works where some error sources are suppressed in synthetic images to uncouple them from other sources under study. Also some works extend the theoretical rationale producing models that are conceptually simple but require computer calculations to extract conclusions. Both tendencies indicate the need of additional tools in this intermediate land. In this ground, the use of the mentioned “PIV Simulator” is proposed for cases when the theoretical rationale falls short in providing the full analysis of the error characteristics and error uncoupling cannot be performed in synthetic images. In this work, it is used to better analyze the error coming from the interaction of the laser sheet thickness and the time between laser pulses with the spatial gradients produced by the turbulence of the flow, by uncoupling it from other error sources such as peak-locking and outlier production.

2.2 PIV Simulator: working principles

The PIV Simulator emulates the image correlation process in a way that is neither affected by discretization noise nor cross talk between particle images, while maintaining the PIV non-
linear behavior. Once a known test flow field is selected (either from a numerical source or artificially imposed), the PIV Simulator working principle corresponds to the following sequence of steps:

1. 3D interrogation volumes are defined by intersection between the corresponding interrogation window (IW) and the laser sheet thickness.
2. Each volume is seeded with a fixed number of particles placed randomly with uniform distribution.
3. For each volume, from the particles velocities and the time delay between laser pulses, the position of the correlation peak of each of the particles with itself (self-correlation peak) is calculated. By adding those self-correlation peaks, the analytical expression of the displacement-correlation map is obtained in a continuous domain. The self-correlation peaks are supposed to be of Gaussian shape with $\sqrt{2}D_p$ e^{-2} diameter (where $D_p$ is the e^{-2} diameter of the particles).
4. The displacement that provides the maximum correlation value is obtained for each of the correlation maps.

It is important to remark that this procedure removes the following sources of error:

- The cross-correlation between different particles (Westerweel, 1993, among others) is avoided, as Fig. 1 depicts, as well as the correlation of particles with background noise. In consequence, no “outliers” are generated. All vectors are produced from existing particles displacements.
- Particle image discretization: no images are created, therefore, the errors associated with particle image discretization (peak-locking, subpixel peak fitting, grey level interpolation) are avoided.

![Fig. 1 a) For a certain displacement, a PIV correlation would correlate each of the particle images in the initial interrogation window with all the particle images in the final interrogation window. b) In the case of the PIV simulator, only the correct particle images are correlated, avoiding cross-talk.](image-url)
3. Application to the laser sheet thickness error study

3.1 Theoretical rationale

The laser sheet thickness may introduce two very different kinds of errors when associating the measurement to the one corresponding to the meridian plane. One is the averaging effect due to contribution of particles from different depths in the plane width to the measurement in a single location. The other is an amplification of the small scale variations due to the possibility of contributions from different depths in the plane direction to adjacent measurement locations. To evaluate these errors, a tool to discriminate different length-scales is required.

3.1.1 Method of analysis for discriminating different length-scale related information

To discriminate different length-scales in the characterization of the error, three functions were considered. All of them have been employed widely in turbulence research. They are: (i) the second-order longitudinal velocity structure function, $S_{11}(r_1)$, (ii) the third-order longitudinal velocity structure function, $S_{111}(r_1)$, and (iii) the longitudinal one-dimensional spectrum, $E_{11}(\kappa_1)$.

$S_{11}(r_1)$ and $E_{11}(\kappa_1)$ are closely related and one can be calculated from the other. The greatest difference is that one is expressed as function of spatial scales while the other is expressed in the frequency domain. It has already been reported (Davidson 2004, among others) that both representations provide flawed representations of the turbulent energy. In terms of easiness for calculation, $S_{11}(r_1)$, is much more simple. Its simpler definition results also in an easier physical interpretation of its deviations due to the errors under analysis.

In respect to $S_{11}(r_1)$ and $S_{111}(r_1)$, both functions are obtained from velocity differences. In the case of $S_{11}(r_1)$ the sign of the difference does not affect the calculation (due to the square power in eq. (1)) while on the case of $S_{111}(r_1)$ the sign does affect. As a result, $S_{111}(r_1)$ is less affected by random errors on the measurement, as De Jong et al., 2009, state. While this feature could be of interest for calculations derived from the function (for example the dissipation can be obtained), it has been considered a disadvantage here, since the aim of the paper is to provide an error characterization.

Based on the previous reasoning, $S_{11}(r_1)$ has been chosen for the characterization of the error. Following Pope (2000), the definition of the second order longitudinal structure function is given by (for the first velocity component, $u_1$):

$$ S_{11}(r_1) = \left\langle (u_1(x + e_1 r_1, t) - u_1(x, t))^2 \right\rangle_{xt} $$

(1)

Where $x$ is a position within the flow field, $e_1$ is the unity vector in direction 1, and $r_1$ the distance at which the structure function is calculated. It is called longitudinal because the
component of velocity used for the calculation is aligned with the separation vector \( \mathbf{e}_1 r_1 \). The operator \( \langle \cdot \rangle_{x,t} \) indicates average on different positions and time instances. Analogously, the function could be defined for other velocity components.

### 3.1.2 Spatial versus temporal variations

Along this work, for the acquisition of the PIV images, only spatial variations are relevant as error source. This is so, because for the results presented in this work the smallest interrogation volume is of a size \( \ell \sim 4\eta \), where \( \eta \) is Kolmogorov length scale. On the other hand, the largest time delay is \( \Delta t \sim 0.25\tau_K \), where \( \tau_K \) is Kolmogorov time scale. In consequence, spatial variations will be much more important than temporal variations.

### 3.2 Implementation of the PIV Simulator

On this particular case, the PIV Simulator is based on a database containing the results of solving by Direct Numerical Simulation (DNS) a homogeneous isotropic turbulent flow (Li et al. 2008). Taylor Reynolds number, \( Re_T \), equals 433. From that data, the PIV simulator generates 2D vector maps by performing directly from the data a correlation process, without creating any images (as described in 2.2). In this DNS field, the Kolmogorov length-scale size is \( \eta = 0.00287 \) feet and its time-scale is \( \tau = 0.0446 \) seconds. The dissipation rate is \( \varepsilon = 0.0928 \text{ ft/s} \). The rms velocity fluctuation is \( u' = 0.681 \text{ ft/s} \).

#### 3.2.1 Use of the PIV Simulator for error assessment

Error predicted by the PIV Simulator in this work corresponds to the difference between the real velocity value at the center of the interrogation window and the vector provided by the PIV Simulator, \( \bar{u}_M \). Additionally, \( \bar{u}_{AVG} \) will be used to define the value corresponding to the moving average in the interrogation window. This is a paradigm often used as a first approximation in PIV (Willert & Gharib, 1991, Lavoie et al., 2007, among others), and it is useful to check deviations, \( \bar{u}_{DEV} \), between this assumption and the PIV Simulator result:

\[
\bar{u}_M = \bar{u}_{AVG} + \bar{u}_{DEV}
\]

The main effects contributing to this deviation, \( \bar{u}_{DEV} \), are:

- The convergence error: The convergence error is related to the number of samples within the interrogation volume. In general, for \( N \) samples, the convergence error standard deviation would be given by: \( \sigma_{CB} \sim \sqrt{\sigma_u^2/N} \). Where \( \sigma_u^2 \) is the variance of \( u \) inside
the volume (given by the turbulent fluctuations inside the measurement volume). \( N \) in this case is considered to be the number of self-correlation peaks of particles.

- Peak-splitting. The contribution of different particle pairs to the correlation peak may generate peak-width increase due to the spatial gradients, converging to some kind of average with increasing number of samples. This is so only if the particle size is large enough. For smaller particles, large time delays between laser pulses, \( \Delta t \), or larger local velocity gradients, the correlations peaks are split apart. The peak-splitting error occurs for correlation maps containing several correlation peaks (Westerweel 2008, among others). This is illustrated in Fig. 2 where the influence of the time delay \( \Delta t \) on the probability of occurrence of this phenomenon can be understood. Fig. 2 is plotted in function of the gradient parameter \( a \), defined by: \( a = |\Delta u|\Delta t M_0 / D_p \), where \( |\Delta u| \) is the maximum velocity difference inside the interrogation volume and \( M_0 \) the magnification. When the correlation map has peak-splitting the convergence error could be increased, since the number of particles \( N \) contributing to the highest peak will be reduced.

![Fig. 2](image)

**Fig. 2** Correlation maps for different time delays \( (a = |\Delta t|\Delta t M_0 / D_p) \), obtained from the PIV Simulator fixing the rest of the parameters: particle positions, number of particles, \( N=20 \); Particle diameter, \( D_p = 0.8 \eta \); Interrogation window size, \( IW=10.5 \eta \), laser sheet thickness, \( Th=14.9 \eta \).

The contribution of \( \bar{u}_{DEV} \), to the value of \( S_{11}(r) \) can be expressed as:

\[
S_{11}(r_1)_M = \left( (u_1 M(x + r_1) - u_1 M(x))^2 \right)
\]

\[
S_{11}(r_1)_M = \left( (u_{1AVG}(x + r_1) + u_{1DEV}(x + r_1)) - (u_{1AVG}(x) + u_{1DEV}(x)) \right)^2
\]

Regrouping the terms above and squaring:
\[ S_{11}(r_1)_M = \left( (u_{1_{AVG}}(x + r_1) - u_{1_{AVG}}(x))^2 + (u_{1_{DEV}}(x + r_1) - u_{1_{DEV}}(x))^2 \right) + 2 \left( u_{1_{AVG}}(x + r_1) - u_{1_{AVG}}(x) \right) \left( u_{1_{DEV}}(x + r_1) - u_{1_{DEV}}(x) \right) \]

For a large number of measurements, the first term would give \( S_{11}(r_1)_{AVG} \). The second term is considered to be proportional to \( u_{2_{DEV,MAX}}^2 \), the maximum deviation produced by the convergence and peak split errors mentioned above. The third term should cancel when making the average, because there will be contributions both negative and positive. The result is:

\[ S_{11}(r_1)_M = S_{11}(r_1)_{AVG} + ku_{2_{DEV,MAX}}^2 \] (3)

The first term in expression above should depend only on the interrogation volume size (averaging volume). When comparing with the interrogation volume, for small \( r \) distances, only the small eddies will contribute to \( S_r(r) \). For those eddies, \( \tilde{u}_{AVG} \) is clearly reduced in respect to the real velocity. However, when \( r \) gets large enough, the effect of large eddies start to be included in the value of \( S_r(r)_{AVG} \). For those eddies, \( \tilde{u}_{AVG} \) is not so reduced and the function should grow at a similar rate than the real one.

3.3 Synthetic images

A synthetic image generator is used as a first validation step of the PIV Simulator results. In this case, the vector fields produced include more error sources than the PIV Simulator vector fields; therefore, the relative importance of the errors under study can be assessed.

The synthetic images are generated from the same homogeneous turbulence flow database (Li et al. 2008) than the PIV Simulator vector fields. Particles are assumed to be mono-disperse in size, thus, the light intensity given by a particle is only dependent on the out-of-plane position. The laser profile is Gaussian, with its thickness defined between the \( e^{-1} \) values with respect to central intensity. Only the in-plane position of the particles is used to obtain their position in the sensor (cylindrical projection). The intensity of a particle is integrated over each square pixel with unity fill factor (Westerweel 1998) over a sensor with no background noise. Where particles overlap, intensities are added. The particles per pixel, \( ppp \), are 0.02, for all images. Magnification was such that \( \eta \approx 2.5 \text{ pixels} \) for the synthetic images of 4.2 and \( \eta \approx 7.5 \text{ pixels} \) for those images of 4.3.

All the synthetic images, as well as the real images, are processed in Davis 7.2®\textsuperscript{®}, by a multi-grid scheme which ended at 32x32 pixels\textsuperscript{2} with 50\% overlap. The image deformation was symmetric with bilinear interpolation for all the steps except for the last one, where Whittaker image deformation was employed (Scarano and Riethmuller, 2000). The last multi-grid steps (at 32x32 pixels\textsuperscript{2}) are weighted with a Gaussian function of round shape. The vectors are validated by a modified version of the median filter (available in Davis 7.2), based on Westerweel (1994)
and Nogueira et al. (1997) and global histogram filter. For the calculation of $S_i(r)$ just one on every 2 vectors is taken, to remove the overlap influence (which is a parameter that is not implemented in the PIV Simulator).

Among other considerations, differences between the PIV Simulator and the synthetic images results are due to: Out-of-plane motion, Vector validation, Image correction algoritms, Multi-grid schemes, undetected outliers, Gaussian weighted round window, Peak-locking bias, additional sources of random error, such as cross-talk between the particles or peak-locking random errors. This highlights the need of a tool like the PIV Simulator to isolate errors.

### 3.4 Experimental setup

On the measurement campaign designed to validate the rest of results, turbulence is generated by means of a perforated plate, as is used for example in Liu and Ting (2007). The turbulence is of the same characteristics of the so-called grid turbulence. The perforated plate is placed perpendicularly to a stream discharging into ambient air. The stream goes through a contraction nozzle with an exit diameter of 10cm.

The perforated plate holes are of a diameter $D=8mm$, on hexagonal disposition, with a distance between holes centers $M=12mm$, which gives for a plate solidity of 0.6. The thickness of the perforated plate is 1mm. The measurement region was placed at a distance $y/D \sim 16$ from the grid, in order to allow the turbulence to settle to a close to homogeneous state. The mixing layer of the grid-jet was excluded of the measurement zone. The $Re\_\lambda$ produced by the perforated plate at the measurement zone was estimated to be $\sim 70$.

The PIV system used to acquire the images consisted of a double cavity 380 mJ per pulse Nd-Yag laser from QUANTEL and a LaVision FlowSense CCD camera of 2k by 2k pixels covering an area of about 18 by 18 mm. The magnification, $M_s$, corresponds to 0.009 mm/pixel. The flow was seeded with a dissolution of food-grade glycols and water droplets with diameter in the order of 1 to 2 μm generated by means of submerged nozzles (Legrand et al. 2016). The laser sheet thickness, at the e$^{-1}$ points, was $\sim 1.5mm$ and the depth of field, $\delta z$, estimated by $\delta z \cong 4(1 + 1/M_0)^2 f_\#^2 \lambda$ (Adrian and Westerweel, 2011) was $\sim 0.3mm$, where $f_\#$ is the f-number of the lenses (in this case 5.6) and $\lambda$ the laser wavelength (532nm). Under these conditions, the diameter of particle images was $\sim 1.8px$. The $ppp$ was around 0.02 (the same as for the synthetic images).

The images were processed with the same parameters than the synthetic images, except that the image intensity was normalized with a natural logarithm to increase the data yield and the mean stream flow was imposed as initial velocity on the multi-grid scheme.
At each PIV image, the flow supply can be considered steady, but it had a large period variation of \(\pm 10\%\) in 6 minutes. The measurements are taken at about 1 Hz during periods of around 20 minutes averaging the effect of this nuisance.

4. Results

As commented previously, this work focuses on the influence of the time between laser pulses, \(\Delta t\), and of the laser sheet thickness, \(Th\), on the error in the presence of turbulence. Subsection 3.1.2 already indicated that these errors are mostly due to spatial gradients within the interrogation volume rather than temporal ones. As commented in Section 3.1, the error is characterized for each spatial scale available \(r\) by means of \(S_s(r)\) (in this section \(S_s(r)\) for short). It has been normalized using the local scale \(r\) and the dissipation rate \(\varepsilon\), so the final magnitude for error evaluation is \(S_s(r)/(r\varepsilon)\sqrt{3}\).

4.1 Error uncoupling and error magnitude estimation obtained with the PIV Simulator

4.1.1 Influence of the time delay \(\Delta t\)

When evaluating the PIV error related to the spatial gradients in the interrogation volume, previous laboratory findings indicated dependence with \(\Delta t\) for measurements done with the same interrogation window and laser sheet thickness. Larger \(\Delta t\) generated larger errors. If this was due to outliers (due to out of plane motion), background correlation noise or to the peak-splitting described in section 3.2.1 was not unveiled then. Here, Fig. 3 illustrates the results on \(S_{LL}(r)\) when varying the non-dimensional gradient parameter, \(a = |\Delta u|\Delta t M_0 / D_p\), (for a particle diameter \(D_p = 0.8\eta\)) in the interrogation window, by increasing \(\Delta t\).

![Fig. 3 Left: influence of \(\Delta t\) on \(S_s(r)\). Results from the PIV Simulator for a top-hat laser profile of \(Th=15\eta\) and \(IW=10.6\eta\).](image)
Fig. 3 clearly indicates the range of \( \Delta t \) than can cover the full variety of correlation peak situations, from peak agglomeration similar to a moving average to peak-splitting generating error at small scales. It is remarkable that, for \( a > 0.74 \), the smaller length-scales overpass the value corresponding to the DNS \( S_c(r) \). This indicates presence of error that amplifies small scales by mixing information from different depths in the laser sheet plane.

The convergence between the different \( S_c(r)/(re)^{\alpha} \) curves for increasing \( r \) is due to the chosen normalization. Without this normalization, the difference tends to be constant, as expression (3) suggests, by a factor defined by \( u_0^2/(re)_{MAX} \). The fact is that, at the smallest scales, the difference is the smallest and this difference varies with \( r/\eta \) a 50% (from \( 2.4v_K^2 \) to \( 3.6v_K^2 \), where \( v_K \) is Kolmogorov scale velocity).

The difference between the two PIV Simulator outermost graphs obtained by correlation \( (a=0.2 \text{ and } a=1.03) \) at \( r/\eta=10.7 \) gives a \( \Delta S_c(r)/S_c(r)=50\% \). When this difference is compared to the rms velocity fluctuation it gives \( (\Delta S_c(r))/u^\prime=15\% \). This difference should be attributable to random errors (peak-splitting, lack of convergence and the non-linear correlation process).

These tests show that the PIV simulator allows for effective uncoupling of errors indicating that peak-splitting can produce relevant errors without contributions from outliers or correlation background noise. Additionally, it can provide a magnitude of these errors for given measurement conditions (number of particles in the interrogation window, particle size, non-dimensional gradient parameter, etc.).

4.1.2 Influence of the laser sheet profile and thickness

For low values of the non-dimensional gradient parameter, \( a \), Fig. 4-left shows the variation of the low-pass filtering effect, imposed by the 3D interrogation volume, as a function of the laser sheet profile. It plots \( S_c(r)/S_c(r) \) (simulated measurement divided by real (DNS value). The Top-hat profile produces a significantly larger low pass effect. For the case of Gaussian profile, the marked difference between the simulator and the synthetic image case is commented in the next subsection. Fig. 4–right shows the effect of increasing the laser sheet thickness on \( S_c(r)/r \), which greatly increases the low-pass effect.

However, when the value of the non-dimensional gradient parameter, \( a \), is sizeable, the behavior changes. Fig. 5-left shows that, for small wavelengths, thickness increases result in larger variability of the velocity. In homogeneous turbulence, this is so because for a given \( \Delta t \cong 0.1\tau_K \), thickness variations result in variations of the non-dimensional gradient parameter, through the variation of \( |\Delta u| \). As the laser sheet thickness increases, more turbulent scales fit into the measurement volume. This can be clearly observed for scales below \( 30r/\eta \).
**Fig. 4** Value of \( S_r(r) \) measured divided by the real value. **Left:** Influence of the laser thickness profile at \( Th=15\eta \). **Right:** Influence of the laser sheet thickness on \( S_r(r) \). Results from the PIV Simulator for a Gaussian Laser profile, \( IW=10.6\eta \), when the displacement is obtained by a moving average instead of by a correlation.

**Fig. 5** Influence of the laser sheet thickness on \( S_r(r) \) with \( \Delta t \approx 0.1\tau_k \). **Left:** PIV Simulator results, with Gaussian Laser profile, \( IW \) size=10.6\( \eta \). **Right:** Comparison between the previous graph and Synthetic images results for \( IW \) size = 12.8 \( \eta \). \( u' \) is the rms velocity fluctuations expressed in pixels, for each case.
4.2 Comparison between the results from the PIV Simulator and those corresponding to synthetic images

Results from synthetic images cannot uncouple the peak-splitting effect from the cross-talk between particles in the correlation. In addition, other errors may arise corresponding to correlation background effects and the possibility of undetected outliers. Fig. 5-right, indicates that these errors can add a low pass effect. For this particular case the curves depart further from the real value, but that will not always be the case. Nevertheless, it has to be taken into account that these movements are due to averaging effects over the error, not a recovery of the real values of the flow in terms of fluctuations.

Fig. 6 Error! Reference source not found. compares between the results from the PIV Simulator and those from synthetic images when varying $\Delta t$ (i.e. non-dimensional gradient parameter, $a$, variations at constant laser sheet thickness). Comparisons are made for two different laser sheet thicknesses ($15\eta$ for Fig. 6-Left and $66\eta$ for Fig. 6-Right). The laser profile employed is Gaussian.

![Fig. 6 Comparison of PIV Simulator and synthetic images results for variations of $\Delta t$. Left: Thickness=15$\eta$. Right: Thickness=66$\eta$.](image)
The IW size=10.6η for the PIV simulator and 12.8η for synthetic image results. This difference tries to compensate for the use of the Gaussian weighted round window for the synthetic image processing, already commented in Section 3.3. The rms velocity fluctuation is $u' = 0.681$ ft/s for all cases as commented in section 3.2, but the corresponding displacement in pixels as $\Delta t$ varies has been referenced for the synthetic images to allow peak-locking or other errors envisage. The out-of-plane movement has been removed in the cases in which it is not negligible.

Despite the amplitude difference between Fig. 6-Left and Fig. 6-Right, a common feature is perceived: PIV Simulator and Synthetic images cases are closer for extreme values of $a$; and both analysis diverge for intermediate values of $a$. The divergence at intermediate values might be connected to the fact that, for the synthetic images PIV analysis, the use of an image deformation algorithm is able to reduce the effective peak-splitting. For large values of $a$, smaller scale fluctuations become relevant and this algorithm would become less effective. In addition, possible undetected outliers would also increase the measurement of fictitious local velocity variations. For all the cases the Synthetic images result contain a larger spurious low pass effect (averaging) that may come either from the image deformation algorithms or the correlation cross talk between particles.

Fig. 6 allows for the evaluation of the results for a similar non-dimensional gradient parameter, $a \sim 0.75$, when perturbations coming from $\Delta t$ are exchanged with those from the laser sheet thickness. The comparison between the green line, at Fig. 6-Left, and the red one, at Fig. 6-Right, at small scales supports the hypothesis that the mixing between velocity variations comes from the laser sheet thickness producing relevant spurious variations, while $\Delta t$ just evidences the presence of this mixing. For large scales, other error sources may reduce the low-pass effect for synthetic images as can be observed in Fig. 4-Left.

4.3 Experimental validation: coherence of the results on real images.

To evaluate the validity of this work, the comparison with the results from the experimental setup described in Section 3.4 is plotted in Fig. 7. In order to construct $S_\varepsilon(r)/(r\varepsilon)^{3/2}$ for the experimental results, the value of $S_\varepsilon(r)$ at the inertial range was employed to estimate $\varepsilon$, following De Jong et al. (2009). In the same work, this method was found as the most convenient. However, no correction factors are applied here except for the one of finite $Re_\varepsilon$. On the case of the grid experiment, just the in-plane direction perpendicular to the stream is used to calculate $S_\varepsilon(r)$. Perforated plate grid turbulence measurements are assumed to be representative of
homogeneous turbulence in that direction (at least in the inertial and dissipation range), so they are compared together with DNS results.

The experimental setup had a smaller $Re_λ$ which makes the experimental setup to have a narrower inertial range and thus reaching lower maximum $S_{LL}(r)/\langle r \epsilon \rangle^{2/3}$. Nonetheless, the maximum reached is coherent with the ones obtained by the DNS of Yeung & Zhou (1997).

For very large scales, Fig. 9-Right, shows a large discrepancy between the DNS and the synthetic image results. These are due to lack of statistical convergence. It would be unpractical to evaluate long series of large images just to fit this part of the curve.

The synthetic images are generated without out-of-plane. This is due to the differences regarding the large scale. Fig. 7 indicates that the large scales of the real experiment have lower velocities generating small out of plane displacements. The magnitude of the out of plane is given in the figure as $u'/th$ values.

![Graph showing $S_{LL}(r)/\langle r \epsilon \rangle^{2/3}$ vs $r/η$ for DNS, PIV Simulator, and Synthetic Images.](image)

**Fig. 7** $S_{LL}(r)$ obtained for a depth of field=4.5$η$ and IW=4.2$η$. **Left**: comparison of PIV Simulator results and real images. **Right**: comparison of synthetic image results and real images.

The experimental images have smaller depth of field than laser sheet thickness. So the thickness of the interrogation volume in the PIV Simulator and Synthetic Images corresponds to the depth of field. Both experimental cases have the same depth of field and IW size. Variations in $a$ are produced through variations in $Δt$ (0.12$τ$, for one case and 0.24$τ$, for the other).
Due to the differences between the DNS and the available experimental flow, the validation is limited to the order of magnitude of the errors as a function of $a$. This can be extracted by evaluating the difference, $\Delta S_u(r)$, between both available cases ($a = 0.8$ and $a = 1.6$) at $r/\eta=4.2$. It is given in Table 1, normalized with the $S_u(r)$ and $u'$ for the $a = 1.6$ case.

<table>
<thead>
<tr>
<th></th>
<th>PIV Simulator</th>
<th>Synthetic images</th>
<th>Real images</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta S_u(r/\eta=4.2)/S_u(r/\eta=4.2)$</td>
<td>13%</td>
<td>16%</td>
<td>10%</td>
</tr>
<tr>
<td>$\sqrt{\Delta S_{LL}(r/\eta=4.2)/u'}$</td>
<td>4.6%</td>
<td>4.2%</td>
<td>11%</td>
</tr>
</tbody>
</table>

**Table 1** Differences in $S_u(r)$ for the smallest $r/\eta$ measured.

The proximity of the values for $\Delta S_u/S_u$ indicates coherence of the results. The fact that the PIV Simulator case shows errors in the range of 45% for $a = 1.6$ in respect to its DNS flow field (see Fig. 7-Left) is coherent with the hypothesis that the mixing of small scales along the interrogation volume thickness generates larger errors, but there is a low pass effect on the synthetic images coming from particle cross talk and other correlation background effects. These low pass effects conceal the error, but do not necessarily mean a better measurement.

The enlarged value of $\sqrt{\Delta S_{LL}/u'}$ for the real case due to the presence of smaller intensity on large scales indicates that this effect is uncoupled from large scales features.

5. Conclusions and future work
The PIV Simulator has evidenced the following capabilities:

- To uncouple outlier occurrence and correlation background and cross-talk noise from errors coming from the velocity gradients within the interrogation volume.
- To provide estimations of the error generated by those velocity gradients at each scale.
- To explain the relative error increment produced by $\Delta t$ enlargement for fixed interrogation volume and spatial gradients.

Thanks to including the PIV Simulator in the error analysis process, it has been possible to clarify that:

- Without outliers occurrence, there is an effective influence of the spatial gradients in inducing differences in $S_u(r)$ which increase with $\Delta t$.
- This error seems to be linked to peak-splitting and convergence errors occurring.
- Errors due to variations in the non-dimensional gradient parameter of the interrogation volume have different behavior if the variation is due to changes in the time delay between laser pulses or to changes in the interrogation volume depth.
• Errors induced by the spatial gradients easily reach significant values compared with \( S_{,r} \) (45% for example for the mimic of the real case), but are strongly concealed by averaging mechanisms that do not necessarily improve the measurement. These effects should be taken into account, especially if the characterization of the velocity fluctuations is the measurement objective.

For future works, a DNS database closer to the experimental capabilities would allow sounder conclusions. The results obtained also advice the application of the PIV Simulator to the study of other flows with steady spatial gradients where the low pass effects would not conceal the studied errors.

Additional experimental measurements with laser sheet thicknesses variations should be undertaken, to obtain confirmation on the errors observed for the PIV Simulator and synthetic images results.

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