Multiple-eye PIV

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ABSTRACT

This paper describes the new method of PIV to reduce error associated with measurement of particle displacement by computing ensemble average of \( n = 7 \) sub-images projected on a single image sensor through microlenses. Stereo PIV method was applied to evaluate instantaneous velocities based on combinations of sub-images, and its ensemble average was computed at each instance. The system was applied to a fully developed pipe flow, and statistics were compared to DNS results. The random errors, estimated from the discrepancy of the measured rms velocity to the DNS, was successfully reduced by a factor of \( \frac{1}{\sqrt{n}} \).

1. Introduction

Typical velocity dynamic range of particle image velocimetry (PIV) is approximately 1:100 (Adrian 2005), which is far below conventional velocimeter such as hot-wire anemometry or laser Doppler velocimetry. This limitation mainly comes from error associated with measurement of particle displacement, which is typically 0.1 pixels. Further reduction of such error can be achieved by a method called pyramid correlation (Sciacchitano et al. 2012), which uses linear combination of the correlation functions computed at different time intervals. Evaluation of ensemble averaged correlation functions reduces random error of particle displacement by a factor 3. While pyramid correlation uses ensemble average of correlation functions computed from temporal series of images, we propose an alternative way that computes ensemble average of particle displacements measured by multiple cameras viewing simultaneously from different directions. If the random error involved in particle displacements evaluated from images of each camera are mutually independent, the random error in the ensemble average of them reduces \( n^{-n} \), where \( n \) is the number of cameras, according to central limit theorem. In this paper, we describe a PIV system having optical equipment named multiple-eye camera, which captures the images viewing from multiple directions. Stereo PIV method was applied to evaluate instantaneous velocities based on combinations of images captured by each ‘eye’, and its ensemble average was computed at each instance. Application of this system to a turbulent pipe flow demonstrated the reduction of the random errors in the velocity signals.
2. Method

2.1 Optical design of multiple-eye camera

Optical configuration of the multiple-eye camera is shown in Fig.1. First, the ray of light from objects is concentrated by an objective lens, and then the ray is collimated and expanded by a collimator lens and beam expander. Finally, the ray is focused on a C-MOS image sensor (Fastcam SA3, 1024x1024 pixels, pixel size 17x17µm; Photron) through micro lens array. By this configuration, each micro-lens projects a same object, but viewing from slightly different direction, on the image sensor. An image projected by a single micro-lens is hereafter referred ‘sub-image’. An iris installed between the objective lens and collimator lens eliminate the overlap of adjacent sub-images on the sensor. The sub-image has a dimension of 2 mm in diameter on the sensor and corresponding diameter in the object plane is 4 mm. Total of 25 sub-images are projected. A snapshot of actual camera is shown in Fig.2.

![Fig. 1 Optical design of multiple-eye camera.](image-url)
Fig. 2 Snapshot of multiple-eye camera. From the left, the camera consist of objective lens (85mm F1.4 EX DG HSM, Sigma), collimator lens (50mm Dia., 0.66 NA, Uncoated, Calibration Grade Aspheric Lens, Edmund Optics), plane – concave lens (SLB-50-80NM, Sigma Koki), plane–convex lens (SLB-50-100PM, Sigma Koki) and microlens array (Fly-eye-lens 730, Koyo, focal length 10mm).

2.2 Flow apparatus and instruments

Fig.3 and Fig.4 show schematic of flow apparatus and instruments. Water pumped by an centrifugal pump (MDH-401SE5-D, 200/200-12.0L/min-m, 0.75kW, Iwaki) flows through a circular pipeline. This pipeline has straight section of Plexiglass circular pipe having diameter of D(=2R)=50 mm and length of L=123D. The pipe was surrounded by a Plexiglas rectangular container filled with water, i.e., a water jacket, to minimize the distortion of the image observed across the round surface of the pipe. Tracer particles (Silvercoat hollow sphere, 10µm, Dantec) was seeded in the water. The temperature of water was maintained at 20±0.1°C by use of a heat exchanger and a chiller (RKS750F, 1.97kW, Orion) controlled by a digital temperature controller (E5CN, Omron). Cartesian coordinate system has been applied with its origin set at the center of the circular cross-section of the pipe at the inlet of the straight section. The axes x is streamwise, y is vertical and z is perpendicular to both x and y. Velocity components along x, y and z directions are represented by u, v and w, respectively. Bulk mean velocity $\bar{U}$, of the pipe flow was 1.065m/s based on electromagnetic flowmeter (AXF050G, Yokogawa), and corresponding Reynold’s number was $Re=UD/\nu=4.86 \times 10^4$, where $\nu$ is kinematic viscosity.

A test section was located at $x=84D$ where the flow reaches fully-developed turbulent. A laser light sheet created by single cavity Nd-YLF laser with double-pulse option (DM-10, 10 mJ/pulse, Photonics Industries) with a laser light sheet optics illuminated a planar volume parallel to x-y plane through $z=0$. Time interval between double pulses of the laser was varied in a range from $\Delta t=30 \mu s$ to 500 $\mu s$, and particles illuminated by each pulses were exposed onto two
successive image frames by use of a delay pulse generator (9600, Quantum Composers). The frame rate of the camera was set at 125Hz, and corresponding data rate of PIV output was 62.5Hz.

![Fig. 3 Schematic of the pipe flow facility.](image)

![Fig. 4 Schematic of the PIV setup.](image)

### 2.3 Stereo PIV and calibration

Since the viewing direction for different sub-image is not identical, in-plane particle displacement computed from individual sub-image does not coincide if non-zero out-of-plane displacement exists. This implies that the stereo-PIV method can be applied to extract whole
three-components of velocity vector based on any combination of two different sub images. Here, we define the \(i\)-th component of velocity vector evaluated from a combination of \(j\)- and \(k\)-th sub-image taken at coordinate \(x\) and time \(t\) as \(v_{i,j,k}(x,t)\), named hereafter as 'elementary velocity'.

Interrogation window size was 50 x 50 pixels, which corresponds to 2.2 x 2.2 mm² in object plane. Direct cross-correlation of images in interrogation window of the first frame and search area in the second frame was computed to evaluate two component displacement vectors of the particles, and whole three-component velocity vector was estimated by stereo PIV algorithm (Sakakibara et al. 2004). Since the stereo-PIV requires precise image calibration, a calibration plate, where a regular grid of markers of 0.3mm in diameter and 0.6mm interval was printed in terms of laser marking technology, was placed on the light sheet plane with a slight angle respect to the \(x\) direction. Calibration image was captured at two different locations of the plate, i.e. the distance between the locations were set at 2 mm, which was achieved by shifting the plate in \(x\) direction. Displacement of particle, which travels at \(U_i\) in \(x\) direction, in the image plane is summerised in Table 1.

<table>
<thead>
<tr>
<th>(\Delta t [\mu s])</th>
<th>particle displacement [pixel]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.7252</td>
</tr>
<tr>
<td>60</td>
<td>1.4504</td>
</tr>
<tr>
<td>125</td>
<td>3.0216</td>
</tr>
<tr>
<td>250</td>
<td>6.0432</td>
</tr>
<tr>
<td>500</td>
<td>12.0864</td>
</tr>
</tbody>
</table>

Table 1 Displacement of particle travels at bulk velocity.

2.4 Ensemble average of elementary velocities

After evaluating elementary velocities based on all combination of sub-images, its ensemble average, named hereafter as 'ensemble-averaged velocity' is calculated by

\[
\bar{v}_i(x,t) = \frac{1}{n} \sum_{j=1}^{n} \sum_{k=j+1}^{n} v_{i,j,k}(x,t)
\]

where \(n\) refers number of sub-images used.

3. Result and discussion

3.1 Image of particles and calibration plate
Fig. 5 shows a raw image of the calibration plate captured by this system. The image involves 7 sub-images, which is perfectly in round shape, surrounded by another 23 sub-images that has some deficit in its shape. Here we use the perfectly-round sub-images \((n=7)\) to compute elementary velocities and other defected sub-images are masked out. Fig. 6 shows particle image after applying the mask. Typical particle image size was approximately 5 pixels.

![Fig. 5 Raw image of calibration plate. Image size is 512x512 pixels.](image1)

![Fig. 6 An example of raw particle image. Image mask was applied to eliminate defected sub-images. Contrast was adjusted for clarity.](image2)

### 3.2 Temporal development of instantaneous velocity

Fig. 7 shows temporal series of instantaneous elementary and ensemble-averaged velocity measured at \(y=0\) with two different \(\Delta t\). Elementary velocities represented by blue marker were all scattered around ensemble-averaged velocity indicated by black solid line. The scattering of elementary velocity at each instant reflects purely random error of each elementary velocity, while fluctuation of the ensemble-average velocity reflects both random error and actual fluctuation of velocity due to turbulent motion of the flow. Standard deviation of elementary and ensemble-averaged velocity indicated respectively by green and red dashed lines shows significant difference of amplitude of the signals at \(\Delta t=30\mu s\). Here the typical particle mean displacement is 0.7 pixels referring Table.1, which is comparable in order of magnitude to the subpixel random error, \(\sim 0.1\) pixels, of the particle displacement. Note that rms of out-of-plane \(w\) component is significantly larger than that of in-plane \(v\) component. By symmetry, both standard deviations should be equal, but the error of out-of-plane component evaluated by stereo PIV algorithm might be augmented by use of narrow angle, such as 7 degrees, of the view axis of two sub-images. In contrast to \(\Delta t=30\mu s\), difference of rms of elementary and ensemble-average velocity is reduced at \(\Delta t=500\mu s\), where the random error of the elementary velocity is relatively
small compared to the larger particle mean displacement such as 12 pixels. Significant error of the out-of-plane component is still observed in $\Delta t=500\mu s$.

Fig. 7 Temporal series of instantaneous $u$, $v$ and $w$ components of velocity measured at the center of the pipe. Symbol and solid line indicates elementary and ensemble-averaged velocity,
respectively. Black and green dash indicates mean and standard deviation of ensemble-averaged velocity, and red dash indicates standard deviation of elementary velocities at each instance.

### 3.3 Mean and RMS velocity

Fig. 8 shows radial distribution of streamwise mean velocity $U^+$ calculated from ensemble-averaged velocity, where superscript denotes the velocity and length being normalized by friction velocity $u_\tau$ and viscous wall unit $v/u_\tau$, respectively. The friction velocity was estimated based on Blasius friction formula (Schlichting 1979). As a reference, a DNS result obtained at $Re=44000$ by Wu & Moin (2008) is overlaid in a dashed line. The value under $\Delta t \geq 60 \mu s$ have agreement with the DNS results, while $\Delta t=30 \mu s$ is overestimated. Mean velocity does not affected theoretically by pure random error in the measured velocity, but it is sensitive to bias error such as peak-locking, which is a tendency of the measured particle displacement to be biased towards integer pixel values. Maximum bias error in measured mean velocity increases as decreasing $\Delta t$ by Angele & Muhammad-Klingmann (2005) due to the peak-locking. Discrepancy of measured value to the DNS arising in a case of short $\Delta t$ is also found in radial profile of RMS velocity profiles shown in Fig. 9. Both $u_{+\text{rms}}$ and $v_{+\text{rms}}$ under the case of $\Delta t=250 \mu s$ and $500\mu s$ shows good agreement with the DNS, while other cases having smaller $\Delta t$ shows larger discrepancy, which was observed in instantaneous velocity shown in Fig. 7.

![Fig. 8 Streamwise mean velocity profile.](image-url)
Fig. 9 RMS velocity profile; (a) streamwise component; (b) radial component.

3.4 Estimation of random error associated with particle displacement
By assuming that the DNS result always indicates true value, the discrepancy found in Fig.9 might give an estimate of magnitude of random error of velocity measurement. The measured rms value of the streamwise velocity without normalization, \( u_{\text{rms, measured}} \), can be expressed in terms of squared-sum of true value of rms displacement of particles in image plane and random error, \( \varepsilon \), associated with estimation of particle displacement in the image plane by a relationship;

\[
    u_{\text{rms, measured}} = \left( u_{\text{rms,DNS}}^+ u_x \Delta t / \alpha_x \right)^2 + \varepsilon^2 \right)^{1/2} / \Delta t. \tag{3}
\]

Here \( \alpha \) represents the physical particle displacement in the object plane corresponding to a particle displacement of one pixel in image plane. The \( \alpha \) has been known through the calibration procedure. Note that the error in \( \Delta t \) and \( \alpha \) was so small that their contribution was neglected in (3). Furthermore this formula does not account for the reduction of random error due to the lack of spatial resolution, which acts like low pass filter. The error \( \varepsilon \) obtained from (3) is plotted against \( n \) in Fig.10. In this figure, the \( \varepsilon \) estimated from \( u_{\text{rms}} \) agreeing with that of DNS, i.e. the case of \( \Delta t \geq 250 \mu s \), were eliminated. Also \( \varepsilon \) being complex value is not plotted. The markers at \( n=2 \) (\( n=7 \)) are the \( \varepsilon \) evaluated from elementary velocity (ensemble-averaged velocity). At a glance, the error \( \varepsilon \) decays as increasing \( n \). Based on central limit theorem, the random error of ensemble-averaged velocity is expected to be proportional to \( \frac{1}{\sqrt{n}} \), if the all of the elementary velocities are mutually independent. This is evident in Fig.10, where the data agree with solid curves expressed by

\[
    \varepsilon = \frac{\varepsilon_{n=2} \sqrt{2}}{\sqrt{n}}. \tag{4}
\]

Here \( \varepsilon_{n=2} \) denotes ensemble average of \( \varepsilon \) estimated from elementary velocities, which was evaluated from \( n=2 \) sub-images, at specific \( \Delta t \). In other words, the solid lines were drawn through the mean error of elementary velocities (\( n=2 \)). It is clear that the error \( \varepsilon \) decays proportionally to \( \frac{1}{\sqrt{n}} \), and that implies the all elementary velocities are mutually independent.
Fig. 10 Estimated random error of streamwise component of particle displacement vector. Type of symbols are all identical to that of Fig. 9.
4. Conclusion

We developed a new PIV system, named multiple-eye PIV, which reduces error associated with measurement of particle displacement by computing an ensemble average of velocities evaluated from \( n=7 \) sub-images captured on a single image sensor through micro-lens array. The system was applied to a fully developed pipe flow, and mean and rms velocity profiles were compared to that of DNS. The random errors were estimated from the discrepancy of the measured rms velocity to the DNS. The random error in the ensemble-averaged velocity was successfully reduced by a factor of \( 1/\sqrt{n} \). Further refinement of imaging optics to increase \( n \), and consequently reduction of the error, is a challenge for the future.

5. Reference