PIV and LDA velocity measurements for the characterization of the TKE budget in flows within arrays of emergent cylinders

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Abstract The flow within and above plant canopies in estuarine wetlands is normally turbulent in the inter-stem space. Turbulent kinetic energy (TKE) is mainly produced due to vortex shedding from individual stems. Due to the complex spatial distribution of the flow field, the relative magnitude of the terms of the TKE budget is not well known in the inter-stem space. This work is aimed at the characterization and quantification of the terms of TKE equation so as to advance the understanding of globally non-homogenous flows. The work is fundamentally experimental and is based on the analysis of 3D Laser Doppler Anemometry (LDA) and 2D Particle Image Velocimetry (PIV) databases, for which the spatial resolution is of the order of magnitude of Taylor's microscale. The compatibility of the two databases proved to be sufficiently good. This work leads to the conclusion that turbulent production is mainly wake production. There are important turbulent fluxes of TKE and interactions between the mean and turbulent flow fields. Convective terms are important in the vicinity of the stems while turbulent transport is mostly associated to the von Kármán vortex streets. The dissipation rate of TKE presents smaller spatial variations and its magnitude increases with the stem areal number-density.

1. Introduction

The flow within plant canopies in estuarine wetlands determines fluxes of suspended sediment, pollutants and nutrients, thus constituting the physical stratum upon which biological and ecological strata are formed (Tanino and Nepf, 2009; Nepf, 2012). In the inter-stem space, the flow is normally turbulent and vortex shedding from individual stems is the main source of turbulent kinetic energy (TKE). The spatial distribution of flow variables is complex. The velocity field is non-homogeneous, being mostly determined by the interaction of vortexes shed by individual stems (Sumner et al., 2005). The spatial characterization and quantification of the terms of TKE conservation equation may allow progresses on the understanding of the nature of turbulence generated in these conditions. In particular, it is necessary to know in detail the turbulence in the inter-stem space to attempt closures to time- or double-averaged conservation equations (Raupach and Thom, 1981). For steady flows, the equation of conservation of TKE is written

\[
\frac{1}{2} \frac{\partial u_i u_i}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \frac{\partial u_i}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right] = \frac{1}{\rho} \frac{\partial \rho' u_i}{\partial x_i} + 2\nu \frac{\partial s_{ij}}{\partial x_j} - \overline{\varepsilon} 
\]

where \( \overline{\varepsilon} \) stands for time averaged velocity, \( u'_i \), and \( \rho' \) are the fluctuation of velocity and pressure fields, respectively, \( \nu \) is kinematic viscosity, \( s_{ij} \) is the symmetric part of turbulent strain tensor and \( \overline{\varepsilon} \) is the dissipation rate of TKE. The TKE per unit mass, \( \overline{u'_i u'_i} \) (summation of repeated indices intended) is hereinafter referred simply as TKE.

The aim of this work is the characterization of the terms of the TKE equation, namely the convective term, production term, turbulent transport and dissipation rate of TKE, terms I, II, III and VI, respectively, in equation (1). The work, fundamentally experimental, consisted on the combination of a 3D Laser Doppler Anemometry (LDA) and a 2D Particle Image Velocimetry (PIV) databases, for which the spatial resolution is of the order of magnitude of Taylor's microscale.

LDA technique allows acquisition of instantaneous velocities at high sampling frequencies. If the frozen turbulence hypothesis is applicable, LDA measurements allow long velocity series in streamwise direction with good spatial resolution. However, a good spanwise and vertical resolution is extremely time consuming

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(and possible only for time-averaged variables). On the other hand, a 2D PIV allows, in general, good spatial resolution and it is much cheaper than LDA, in terms of laboratory time to obtain spatial variability. Therefore, these two techniques combined can render very good results within a feasible time. For an introduction to these two techniques see Tropea et al.(2007), pp. 287-342, or other more specialized literature (Raffel et al., 1998; Durst et al., 1976).

An important advantage of PIV and LDA systems is its relatively low intrusiveness, limited to the introduction of the solid targets (seeding) in the flow that may affect flow or fluid particles; and, in some free-surface flows, transparent water surface stabilizers. In general, no material parts of the instrumentation are needed in the flow region under measurement. PIV allows for relatively simple acquisition of raw data is relatively simple, but it relies heavily on software for the analysis of that data. A weak point of PIV is the large processing time required for the first step of raw data analysis. The process of converting the acquired images into instantaneous velocity fields conveniently calibrated in metric units and referenced to the channel coordinate system is, normally, very slow. Concerning LDA systems, the directional-sensitivity, the high temporal resolution, the high accuracy and little software complexity can be listed as important advantages. However, LDA techniques are based on (normally costly) complex chemical and electronic units and very sensitive optical systems.

Regarding the symbols used throughout this work, overlines represent the time-average operator, primes indicate time fluctuations and angular brackets stand for space-average operator. A Cartesian referential system is considered, where $x$, $y$ and $z$ correspond to the streamwise, spanwise and vertical directions, respectively, and $u$, $v$ and $w$ are the corresponding velocity components.

### 2. Experimental setup

#### 2.1. Laboratory facilities and experimental test

The experimental work was carried out in two laboratory facilities: Laboratory of Hydraulics and Environment of Instituto Superior Técnico (IST) in Lisbon, Portugal, acquiring 2D velocity maps with a PIV system; and Laboratory of Leichtweiß - Institute for Hydraulic Engineering and Water Resources at Technische Universität Braunschweig (LWI) in Germany, where 3D point-wise velocity series were acquired with a LDA system.

The two laboratory tests were carried out with identical experimental conditions. They were performed in a 10.1 m long and 0.40 m wide tilting flume at LWI and in a 12.5 m long and 0.408 m wide recirculating tilting flume at IST. Both flumes have glass side walls, enabling flow visualization and laser illumination. In both cases, the flume bottom was covered with a thin horizontal layer of gravel and sand and arrays of rigid, vertical and cylindrical stems were randomly placed along 3.5 m, simulating emergent vegetation conditions. The diameter of the cylindrical elements was 1.1 cm. Downstream the reach covered with stems, a coarse gravel weir controlled the flow.

To enable the velocity measurements, gaps (narrow regions without stems in the spanwise direction) were enforced, whose width is equal to the mean inter-stem distance of the upstream reach. These gaps will herein be designated by “measuring gaps”. The stems were placed in order to create a pattern with several wavelengths, each 0.5 m long, with varying stem areal number-density, $m$, defined, herein, as the number of stems per unit of plan area. Eight measuring gaps were considered, distributed along two wavelengths - P1 to P4 (first) and P5 to P8 (second) as shown in Fig. 1. Each wavelength features:

- a 15 cm long patch with $m=1600$ stems/m$^2$ (dense patch, herein); this is the case of patch $p_{4,5}$ in the second wavelength;
- a 10 cm long transition patch with an average $m$ of 980 stems/m$^2$, divided into two 5 cm reaches with 1200 stems/m$^2$ and 800 stems/m$^2$, respectively from upstream to downstream; it is the case of patches $p_{1,2}$ and $p_{6,6}$, respectively in the first and second wavelengths;
- a 15 cm long patch of $m=400$ stems/m$^2$ (sparse patch, herein); this is the case of patches $p_{2,3}$ (first wavelength) and $p_{6,7}$ (second wavelength);
- an ending 10 cm long transition patch with an average $m$ of 980 stems/m$^2$, divided into two 5 cm reaches with $m=800$ stems/m$^2$ and $m=1200$ stems/m$^2$, respectively from upstream to downstream; it is the case of patches $p_{3,4}$ (first wavelength) and $p_{7,8}$ (second wavelength).

The flow was subcritical both downstream and upstream the vegetated reach. A coarse gravel weir downstream of the array of cylinders controlled the flow depth.
Fig. 1 - Plan view of the stem covered reach. The solid lines aligned with flow direction indicate the location of the vertical planes measured with PIV. The rectangles point out the regions where horizontal velocity maps were acquired. The blue arrow indicates the flow direction. The points along lines perpendicular to the flow direction represent the location of LDA measurements (P3 to P8).

2.2. Particle Image Velocimetry (PIV)

The PIV system used in the present experimental work (Fig. 2) consisted of an 8-bit 1600 × 1200 px$^2$ CCD camera and a double-cavity Nd-YAG laser with pulse energy of 30 mJ at wavelength of 532 nm. This PIV systems was commercialized by Dantec®, therefore the software used to control the data acquisition and to process the raw data was the DynamicStudio®. This software allows the user to control the acquisition mode, to set the time between consecutive pulses and the acquisition time. It also offers several control variables for the data processing. The CCD sensor is only sensitive to light intensity, i.e., the amount of energy. To maximize the contrast between the bright illuminated particles and the black background in the recorded images, the flume was covered with black fabrics. The camera was positioned perpendicularly to the laser sheet in order to acquire images of the region lightened by the laser minimizing the distortion effects.

Fig. 2 – Pictures of the flume at IST and the PIV during the experiments.
PIV image pairs were acquired at a frequency of 15 Hz with a time delay of 1500 µs between frames. For the present work a polymerized material, commercially named Decosoft 60, with a mean diameter of 60 µm in a range from 50 µm to 70 µm was used as seeding. It is a white material with round shaped particles and its chemical composition consists in 73% of polyurethane and 27% of titanium dioxide. The density of this material is 1.31g/cm³. According to Melling (1997), it is possible to evaluate the suitability of the seeding particles to follow a specific flow and, hence, to determine the smallest turbulent eddies that can be identified, evaluating the ratio \( r_p \) defined by \( r_p = \frac{V_p^2}{V^2} \), where \( V_p \) is the modulus of seeding terminal fall velocity and \( V \) is the modulus of flow velocity. The seeding particles are able to perfectly track the flow when \( V_p^2 = V^2 \), however Melling (1997) proposed a threshold of acceptability of \( r_p = 0.95 \). Fig. 3 shows the ratio \( r_p \) as function of the frequency \( f_c \), according to the solution proposed by Hjemfelt and Mockros (1996), for the seeding particles used in the present work. The graph represents the curves for the mean as well as the limits of the range (50 and 70 µm) of particles diameters. As it can be observed, \( r_p = 0.95 \) corresponds, in the graph for \( d_p = 60 \) µm, to \( f_c = 30 \) Hz meaning that the employed seeding is suitable to detect turbulent structures with frequencies lower than 30 Hz. Since the PIV was operated at 15 Hz, the Nyquist frequency is equal to 7.5 Hz. Therefore, it can be concluded that the seeding particles used ensure the quality of the data acquired in the time domain. In the space domain, applying Taylor’s frozen turbulence hypothesis (Tennekes and Lumley, 1972, p. 253) and considering a mean velocity of 0.10 m/s, the frequency \( f_c = 30 \) Hz corresponds to a turbulent length scale \( \lambda_c = 3.3 \) mm. This means that the velocity of eddies smaller than 3.3 mm may be measured with less than 95% confidence employing this material as seeding.

The data acquired in the scope of this research project was processed with adaptive correlation and a validation method based on the median of neighboring vectors was applied. The size of the interrogation area was chosen to start at 128x128 px² and to finish at 16x16 px² after 3 iterations on the correlation process. This choice intended to maximize the spatial resolution of the velocity field. No overlap of the interrogation area was considered, since it would significantly increase the computational time without improving the results.

Vertical (\( u \) and \( w \)) and horizontal (\( u \) and \( v \)) maps of instantaneous velocity were acquired for each measuring gap (Fig. 1). The vertical measurements were performed in 9 lateral planes, for each position, acquiring 10x573 image couples for each plane which correspond to \( \approx 6.5 \) minutes of acquisition time. Regarding horizontal measurements, maps of \( \approx 9.5 \times 12.5 \) cm² (length \times width) were acquired covering the entire flume width. The spatial resolution yields interrogations volumes of 1.3x1.3x2.0 mm³. Each dataset consisted in 5000 image couples performing 5.5 minutes of consecutive data. The horizontal planes were located at about 60% of the flow depth of each measuring gap.

2.2. Laser Doppler Anemometry (LDA)

The LDA-system was mounted on an automated transverse system. It featured a 5W Argon-Ion laser, a F80 flow processor and two watertight probes with focal length of 198 mm in water (Fig. 4). The used system...
works in back scattering mode. A two-component probe transmitted two orthogonal pairs of beams (wavelengths 488 and 514.5 nm) while the one-component probe, positioned at 30° to the two-component probe, transmitted the third pair of beams (wavelength 476.5 nm). Due to the focal length larger than the flow depth, an acrylic case with a glass bottom was built to submerge the probes in water (Fig. 4 - middle) and ensure that the laser beams travel in water from the probe casing to the measuring volume, except in the 2 mm glass plate. The dimensions of the measuring volume in the directions normal and parallel to the probe axis were 0.10 mm and 0.38 mm, respectively.

In the present experimental work, at LWI, titanium dioxide, commercialized by AppliChem® under the designation of Titanium (IV) oxide pure, was used to seed the flow. The diameter of the particles ranges from 5 to 50 µm and the specific gravity of the titanium dioxide is 4.2 g cm⁻³. Considering a flow velocity of 0.10 m/s, the smallest turbulent scale susceptible to be measured with titanium dioxide particles of 50 µm is 4.4×10⁻⁵ m, according to Hjemfelt and Mockros (1996). For the present experiments this value is much smaller than the spatial resolution of the acquired velocity series, not causing significant errors.

The sampling time at each point was 3 minutes with sampling frequencies between 50 and 110 Hz. The 3D LDA velocity series were measured at 27 points along spanwise direction with a spatial resolution of 5 mm, for 6 measuring gaps (P3 to P8 - Fig. 1) approximately at the center of each measuring gap, at 60% of the flow depth.

**2.4. Flow properties**

| Table 1 - Flow properties and features of experimental measurements for each measuring gap. |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                 | P1   | P2   | P3   | P4   | P5   | P6   | P7   | P8   |
| x (m)           | 6.680 | 6.780 | 6.935 | 7.036 | 7.192 | 7.293 | 7.446 | 7.545 |
| m (stems/m³)    | 1600  | 980  | 400  | 980  | 1600 | 980  | 400  | 980  |
| h (m)           | PIV 0.065 | 0.064 | 0.063 | 0.062 | 0.057 | 0.056 | 0.054 | 0.052 |
| dh/dx (-)       | LDA 0.063 | 0.063 | 0.061 | 0.056 | 0.056 | 0.054 | 0.054 | 0.052 |
| z/h (-)         | PIV -0.020 | -0.017 | -0.002 | -0.012 | -0.031 | -0.018 | -0.010 | -0.017 |
| w (m/s)         | LDA -0.001 | -0.018 | -0.033 | 0.000 | -0.010 | -0.028 |
| Re_p (-)        | PIV 0.57 | 0.61 | 0.61 | 0.62 | 0.67 | 0.69 | 0.55 | 0.59 |
|                 | LDA 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 | 0.60 |
| w (m/s)         | PIV 0.086 | 0.093 | 0.090 | 0.091 | 0.092 | 0.108 | 0.106 | 0.100 |
| Re_p (-)        | LDA 0.109 | 0.115 | 0.123 | 0.112 | 0.127 | 0.130 | 0.130 | 0.130 |
| w (m/s)         | PIV 1161 | 1282 | 1233 | 1302 | 1266 | 1514 | 1469 | 1338 |
| Re_p (-)        | LDA 1025 | 1081 | 1156 | 1053 | 1194 | 1222 |
For all tests, the flow was gradually varied, accelerating in the downstream direction as the flow depth decreased. The discharge was 2.33 l/s and the water depth at the end of the vegetated reach was 4.2 cm (in both tests). The free surface exhibited an oscillating behavior, being the amplitude of those oscillations larger for dense patches.

Table 1 summarizes the main variables for each measuring gap, where \( x \) is the longitudinal coordinate of the measurement section relative to the inlet channel, \( m \) is the stem areal number-density, \( h \) is the mean flow depth, \( dh/dx \) is the gradient of the mean flow depth, \( z \) is the elevation of the measurement planes, \( \langle \bar{u} \rangle \) is the mean longitudinal velocity, and \( Re = \langle \bar{u} \rangle d / \nu \) is the stem Reynolds number, \( d = 0.011 \) m being the stem diameter and \( \nu \) the kinematic water viscosity.

3. Compatibility of LDA and PIV databases

To use LDA and PIV databases complementarily one must first argue for its compatibility. In the present case, the argument relies on the comparison of flow variables calculated from both databases, namely time-averaged longitudinal and lateral velocities and two components of the turbulent production of TKE, \(-u'v' \partial \bar{u} / \partial y\) and \(-v'v' \partial \bar{v} / \partial y\). The PIV data used to compare with LDA data was subsampled from the 2D horizontal maps at the line where LDA data is available. The LDA time series were converted in longitudinal series applying the frozen turbulence hypothesis proposed by Taylor (1938).

![Comparison of time-averaged longitudinal velocity](image)

**Fig. 5** - Comparison of the time-averaged longitudinal velocity, computed from PIV and LDA databases for measuring gaps P3 to P8. Vertical dotted lines identify the \( y \)-coordinate of centers of stems close to the upstream limit of the measuring reach.
Fig. 5 presents the comparison, for all measuring gaps, of the time-averaged longitudinal velocity measured with LDA and with PIV, showing a good agreement for all the different stem areal number densities tested. The largest differences are found at the wake of the stems, where good measurements are difficult to obtain, especially for PIV acquisitions due to the out-of-plane loss of pairs. Furthermore, LDA velocity measurements are likely to be biased to large velocities, which might have impact on these wake regions where the longitudinal velocities are relatively small.

The comparison for the lateral velocity component, $\bar{v}$, is shown in Fig. 6, for measuring gaps P3 to P8. The agreement is good for patches with lower stem areal number-density (P3, P4, P6 and P7), however in dense patches (P5 and P8) LDA velocities tend to be larger than those measured with PIV. This may be attributed to the reasons above mentioned.

![Fig. 6 - Comparison of the time-averaged lateral velocity, computed from PIV and LDA databases for measuring gaps P3 to P8. Vertical dotted lines identify the y-coordinate of centers of stems close to the upstream limit of the measuring reach.](image)

After confirming the similarity of the time-averaged flow field of both databases, variables more complex should also be compared. Fig. 7 and Fig. 8 present $-\bar{u}'\bar{v}'\frac{\partial \bar{u}}{\partial y}$ and $-\bar{v}'\bar{v}'\frac{\partial \bar{v}}{\partial y}$, respectively, which represent two relevant contributions of the turbulent production of TKE (term II in equation 1).

Turbulent production of TKE results from the product of Reynolds stresses and time-averaged velocity derivatives. The spatial resolution of LDA database is low and, hence, the spatial derivatives were computed by means of backward differences, $\frac{\partial \bar{u}_i(y_n)}{\partial y} \approx \frac{\bar{u}_i(y_n) - \bar{u}_i(y_{n-1})}{y_n - y_{n-1}}$, where $n = 1, 2, ..., 27$ is the index of the time-average velocity vector, $\bar{u}_i$ and the y-coordinate, being $y_n - y_{n-1} = 5$ mm. In the case of PIV database,
\( \frac{\partial \bar{u}}{\partial y} \) and \( \frac{\partial \bar{v}}{\partial y} \) are approximated by central differences for time-averaged velocity maps with the resolution of the PIV interrogation areas without superimposition. It should be noted that the spatial resolution of PIV database is about 5 times larger than that of the LDA database.

Globally, there is a good agreement between the LDA and PIV databases. In the sparser regions the results are almost coincident. In denser regions both databases agree qualitatively and, in the case of the space between stems, where the production terms are small, also quantitatively. The quantitative disagreement registered at the wakes of cylinders can be attributed mainly to the low spatial discretization of the LDA database, which tends to exacerbate spatial finite-differences. Bias-to-zero characteristic of PIV data and bias to large velocities, characteristic of LDA data may also play a role in such disagreement.

Based on the comparisons presented in Fig. 5, Fig. 6, Fig. 7 and Fig. 8 it can be assumed that there is a good accordance between the PIV and LDA databases. Hereafter, the two databases are used complementarily.

![Fig. 7 - Comparison of \(-\bar{u}'\bar{v}' \frac{\partial \bar{u}}{\partial y}\) component of the turbulent production of TKE, computed from PIV and LDA databases for measuring gaps P3 to P8. Vertical dotted lines identify the y-coordinate of centers of stems close to the upstream limit of the measuring reach.](image-url)
Fig. 8 - Comparison of \(-\overline{\nu'\overline{v'}} \partial \overline{v'}/\partial y\) component of the turbulent production of TKE, computed from PIV and LDA databases for measuring gaps P3 to P8. Vertical dotted lines identify the y-coordinate of centers of stems close to the upstream limit of the measuring reach.

4. Results

4.1. Computed terms

This subsection is aimed at a description of the methodology for the calculation of the terms of the TKE budget equation (equation 1). Dissipation rate of TKE (term VI) is computed with the LDA database while the convective term, production and turbulent transport, terms I, II and III respectively, are computed with the horizontal PIV measurements. It should be noticed that only x- and y-derivatives and correlations of \(u\) and \(v\) components of time averaged velocities and fluctuations were considered. The vertical components are much smaller than longitudinal and lateral ones.

The convective term is calculated from the maps of time-averaged velocity multiplied by derivatives of TKE. Production results from the work of Reynolds stresses against the shear rate. Maps of production are thus result of the product of Reynolds stresses and time-averaged velocity derivatives. Turbulent transport consists on gradients of turbulent fluxes, i.e. derivatives of triple correlations of velocity fluctuations.

To compute dissipation rate of TKE, Kolmogorov’s (1941) equation was applied. This equation relates the third order structure function, \(S_{LL}^{(3)}\), the derivative of the second order structure function \(S_{LL}^{(2)}\) and the dissipation rate of energy, \(\overline{\varepsilon}\), by:

\[
S_{LL}^{(3)}(r) = -\frac{4}{5} \overline{\varepsilon} r + 6\nu \frac{\partial S_{LL}^{(2)}(r)}{\partial r}
\]

where \(r\) stands for the longitudinal increment and \(\nu\) is the kinematic viscosity. This equation is valid for the
range of scales where the flow exhibits local isotropy in the sense of Monin and Yaglom (1975).

To evaluate the existence of a range of isotropic scales, and therefore, the validity of equation (2), second and third order structure functions and the correlation-coefficient spectra were computed for each measuring gap. It was observed in the third order structure function a clear linear reach, at small scales, identifying the inertial range of scales. Concerning the second order structure the inertial range was also easily identified for all the measuring gaps (reach of 2/3 slope). The correlation-coefficient spectra presented nearly zero values at the scales identified as scales within inertial range in the structure functions. Therefore, local isotropy in the sense of Monin and Yaglom (1975) can be assumed.

Then, the dissipation rate of energy, $\bar{\varepsilon}$, corresponds to the first plateau of the compensated version of equation (2).

4.2. Rate of production of TKE

Spectral analysis of time series of velocity fluctuations obtained in the inter-stem space has revealed that vortex shedding from individual cylinders is the main source of TKE. Also the maps of production presented in Fig. 9 show a strong production downstream of the stems due to vortex shedding. The maximum of production is not immediately downstream of each stem but about 1d further downstream. For large enough inter-stem distances, the magnitude of the productive term decreases in the downstream direction until nearly zero (P6 and P7). For cases with small inter-stem distances, as P5 and P8, the maximum of production is felt very close to the next array of stems since there is no space for its magnitude to decrease.

Fig. 9 - Maps of turbulent production (m²s⁻³) for P5, P6, P7 and P8 (top to bottom). The dot align horizontally identify the position of LDA measurements.
Negative rate of production of TKE is observed at the beginning of the measuring area in regions with stems placed very close to each other. Flows exhibiting negative production of TKE are relatively rare. The negative production areas are located at the upstream end of the measuring gap in the spaces between adjacent cylinders, which are regions where the flow is strongly accelerated. Therefore, in this region, the work of the strong positive shear rate $\partial \overline{u}/\partial x$ against the positive $\overline{u}'\overline{u}'$ is more important than the work generated by other components of shear rate, namely $\partial \overline{u}/\partial y$ against the positive $\overline{u}'\overline{v}'$.

4.3. Convective rate of change of TKE

Fig. 10 presents the spatial distribution of the term I of equation (1), the convective rate of change of TKE, for the measuring gaps on the second wavelength of the stem distribution. The importance of this term is higher in the vicinity of the array of stems once they express the interaction between mean and turbulent flow fields. Maps of the convective rate of change of TKE evidence negative values on the upstream vicinity of the cylinders (downstream of the measuring gap) which are linked to the reduction of the longitudinal velocity. Within the inner part of the cylinders wake, the convective term of TKE presents generally small magnitude with negative values, once the longitudinal velocity often shows negative values herein. While in the outer part the convective term presents its highest positive values. Hence, in the vicinity of the cylinders, both upstream and downstream, the convective term has magnitudes of the same order as the productive term, whereas in the regions between cylinders it is nearly zero.

![Fig. 10](image)

Fig. 10 – Maps of the convective term of TKE ($m^2 s^{-3}$) for P5, P6, P7 and P8 (top to bottom). The dot align horizontally identify the position of LDA measurements.
4.4. Turbulent transport of TKE

Flows within arrays of cylinders exhibit spatial gradient of second and third order moments of instantaneous velocities. They are therefore examples of globally non-homogeneous turbulence for which turbulent transport may be relevant. Maps of turbulent transport of TKE are presented in Fig. 11. Turbulent transport is the gradient of the flux of turbulent kinetic energy and it is observed that it is more important in the von Kármán vortex streets, regions where TKE is itself more important. It was observed that the flux of TKE seems to depend on the gradient of TKE therefore possibly configuring a Fickian process, although not necessarily an isotropic one.

Globally, turbulent transport is large and positive at streaks that loosely reproduce the path of vortexes shed behind cylinders, i.e. the von Kármán street. Large and negative values are found adjacent to the streaks with positive values, both at the wake behind cylinders but mostly at the “outer” interface of the von Kármán street, confining with the faster flow regions between cylinders. In measuring gaps representative of lower stem areal number-densities, the magnitude of turbulent transport decreases significantly in the downstream direction, before any interaction with the downstream neighboring stems.

The main difference between sparse and densely populated patches is that at the latter the positive transport of TKE streaks and adjacent negative regions are spatially less coherent. In some cases at P5 and P8, they are superimposed, which is due to the flow patterns imposed by cylinders further upstream and the interaction of the vortex streets of neighboring cylinders. The magnitude of both positive and negative values is larger in the measuring gaps representative of denser regions.

**Fig. 11** – Maps of turbulent transport of TKE (m$^2$s$^{-3}$) for P5, P6, P7 and P8 (top to bottom). The dot align horizontally identify the position of LDA measurements.
4.5. Rate of dissipation of TKE and overall budget of TKE

Since the rate of dissipation of TKE was computed with LDA data and, hence, results are available at only 27 locations approximately in the center of each measuring gap. Thus, to allow for comparison, the maps of the remaining computed terms are subsampled on the line, extending spanwise, where $\varepsilon$ was calculated (shown as dotted lines in Fig. 1, Fig. 9, Fig. 10 and Fig. 11).

Spanwise profiles of productive, convective, turbulent transport and dissipative terms of equation (1) are presented in Fig. 12 and Fig. 13, for all measuring gaps with data available from both databases.

Concerning the dissipation rate of TKE (red markers), it was observed that this term presents smaller spatial variations in comparison with other terms. However, the dissipation rate is higher in the wake regions than in the regions between stems. One also observes that the dissipation rate of TKE is higher in patches with higher stem areal number density.

For variables computed with PIV data, Fig. 12 and Fig. 13 show that all the terms are larger in patches with higher $m$. This might be a consequence of the fact that the terms of TKE budget have greater absolute value in the wake of cylinders.

Fig. 12 – TKE terms measured with PIV and LDA for measuring gaps P3, P4 and P5 (top to bottom). Vertical dotted lines identify the $y$-coordinate of centers of stems close to the upstream limit of the measuring reach.
Fig. 13—TKE terms measured with PIV and LDA for measuring gaps P6, P7 and P8 (top to bottom). Vertical dotted lines identify the $y$-coordinate of centers of stems close to the upstream limit of the measuring reach.

A comparison of all directly computed terms shows that production and dissipation rates are not in equilibrium, except in the regions between cylinders where both are negligible. It is also noteworthy that the balance of all directly computed terms is not zero, i.e., the rate of dissipation is not balanced by the sum of the production rate, transport and convective rate of change of TKE (with respective signs). This is particularly evident in positions with high number of stems per unit area, like P5 or P8. This highlights the role of pressure diffusion (term IV in equation 1), i.e., the transport by turbulence of pressure fluctuations. The imbalance seems more relevant for the measuring gaps characterized by larger stem areal number-density.

5. Conclusions

This paper was aimed at the characterization of the terms of the TKE equation, namely production, the turbulent fluxes, the convective term and the rate of dissipation of TKE, for flows within an array of stems, analyzing experimental databases acquired with laser based techniques: a 3D LDA and 2D PIV systems.

The compatibility of the two databases proved to be sufficiently good by leading to similar values of two components of the time-averaged velocity field and two contributions of the turbulent production of TKE.

This work leads to the conclusion that within the inter-stem space there are important turbulent fluxes of
TKE and interactions between the mean and turbulent flow fields. Therefore, production is not in equilibrium with dissipation, generally.

Turbulent transport is, generally, smaller than production and the convective term. Its magnitude is higher within the von Kármán vortex streets, where TKE is itself more important. It thus appears that the flux of TKE depends on the gradient of TKE, therefore possibly configuring a possible Fickian process, although not necessarily an isotropic one.

The turbulent production proved to be mainly of wake nature. In the region downstream of a stem, production increases longitudinally until reaching a maximum value about $1d$ further. Then, if the inter-stem distance is large enough, it decreases towards downstream. Regions with exhibiting negative production of TKE were identified in the beginning of the measuring gap between close cylinders, associated with strong accelerations that the flow field is here subjected to.

Expressing the interactions between mean and turbulent flow fields, the convective term reveals itself in the vicinity of the array of stems. Due to the spatial distributions of the production and turbulent transport of TKE, it is expected that the convective term is balanced by pressure diffusion.

Concerning the mean dissipation rate of TKE, this term exhibits a pattern of spatial variation close to that of the areal number-density of stems.

The balance of the main TKE terms lead to the conclusion that dissipation is not, in general, in equilibrium with production, which shows that these flows are examples of globally non-homogeneous turbulence. Furthermore, dissipation seems not in equilibrium with the sum of production, turbulent transport and convective rate of change of TKE. Hence, the role of pressure diffusion may thus be relevant in some regions of the inter-stem space.

Acknowledgements

This research funded by FEDER, program COMPETE, and by national funds through Portuguese Foundation for Science and Technology (FCT) projects PTDC/ECM/117660/2010 and RECI/ECM-HID/0371/2012. Last author acknowledges FCT for his sabbatical grant SFRH/BSAB/1291/2012.

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