

Direct Estimation of PIV Uncertainty from Correlation Plane Analysis

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Particle Image Velocimetry (PIV) is an established tool for non-invasive, quantitative measurement of fluid velocity. Particle images captured at subsequent time frames are cross correlated and the cross correlation peak denotes the most probable displacement. Modern PIV algorithms are optimized for great accuracy such that typical error in measurement is of the order of hundredth of a pixel. However, the interplay of several complex factors involved in the process poses a challenge in quantifying the uncertainty of the measurement. The first attempt to tackle this problem employed an “uncertainty-surface” (Timmins et al. 2012) which was constructed by mapping the effects of selected primary error sources. Later studies resulted in an *a posteriori* “image-matching” (Sciacchitano et al. 2013) method where the distribution of disparity vectors due to incomplete matching of particle image pairs was used to predict the uncertainty. Charonko et al. proposed an uncertainty model based on the ratio of correlation plane primary peak height to secondary peak height (Charonko and Vlachos 2013). More recently, a method for estimating uncertainty from the correlation peak shape has been developed for Digital Image Correlation (DIC) (a closely related method for displacement estimation) which ought to be easily adaptable to PIV (Wieneke and Prevost 2014). Each of these models has advantages and limitations. The present work improves upon these methods by estimating the uncertainty of a measurement from first principles by calculating the second order moment of the probability density function (PDF) of all possible displacements.

Method and Results

The cross-correlation plane represents the probability distribution of all possible particle image pattern displacements between consecutive frames, combined with the effect of the number of particles, mean particle image diameter and effects that contribute to loss of correlation. In other words, the correlation plane is a surrogate of the combined effects of the various sources of error that govern the accurate estimation of a particle pattern displacement. It has been previously shown in single-pixel ensemble correlation that the cross correlation plane is essentially the PDF of displacement convolved with the particle image diameter (Scharnowski et al. 2011). Here also we use the Fourier shift theorem to obtain the Generalized Cross Correlation (GCC) plane by deconvolving the particle image information from Standard Cross Correlation (SCC) plane. Since the standard uncertainty, u_x , is typically defined as the standard deviation of all possible measurement values, we believe it is possible to directly estimate the uncertainty of each PIV measurement by the second order moment of the correlation plane defined as follows:

$$\begin{aligned} u_x^2 &= I_{xx}^2 = \int_x (x - X_p)^2 p(x) dx \\ &= \int_x (x - X_p)^2 G(x) dx / \int_x G \cdot dx \end{aligned}$$

Here X_p is the primary peak location and $G(x)$ is the Generalized Cross Correlation plane (GCC). Here the moment is taken over the primary peak area assuming the true measurement lies within 4 standard deviations of the

primary peak. Estimation of the GCC peak diameter has its own uncertainty which leads to a bias in the estimated uncertainty. Thus, convolving the GCC with a known Gaussian function increases the accuracy of the estimated diameter from which the PDF diameter can be found analytically. The peak diameter is estimated by a least squares fit and the possibility that the primary peak shape is a tilted elliptical Gaussian included. The contribution of the background noise level is eliminated to reduce the bias in estimation. Once the convolved peak diameter is known, the standard uncertainty can be easily obtained provided the PDF can be approximated by a Gaussian distribution.

Conclusions

This method thus provides a general framework for direct calculation of PIV uncertainty. The main advantage of this method is that the predicted uncertainty can be directly obtained without using requiring calibration or much additional post-processing. The method is tested with simulated images from the 2003 PIV challenge case B (boundary layer) (Stanislas et al. 2005), PIV challenge 2005 case B (Stanislas et al. 2008), (laminar separation bubble) and an experimental image set for stagnation flow (Charonko and Vlachos 2013). For each of the above cases the estimated uncertainty is compared with the RMS error distribution. The results show that I_{xx} is larger than the true uncertainty by a factor proportional to the square root of the number of correlated particles, NFF . This reflects that the measured velocity is a weighted mean of the particle displacements, and the uncertainty of a mean is the variance of the measurements scaled by the square root of the number of samples. However, the determination of the precise scaling for all cases is work in progress.

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