Fluctuating Flow Acceleration in a Heated Supersonic Jet

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Abstract Advancing the understanding of the noise generation mechanisms in hot supersonic jet flows is of pressing interest for health and environmental reasons. In the work reported, new insights in some of the fundamental mechanisms underlying the behavior of noise sources are presented by virtue of novel measurements of fluctuating flow accelerations in the developing shear layer of an ideally expanded hot jet. A high resolution two-velocity/acceleration component laser Doppler velocimeter was used to acquire velocity measurements in the region from the nozzle exit to 8 diameters downstream of the exit of a 1.65 design Mach number, bi-conic nozzle operated at a total temperature ratio of 2, resulting in a diameter Reynolds number of 1.1M and a convective Mach number of the shear layer of approximately 0.8. Discrete chirp Fourier transform processing was used to obtain time-rate-of-change of frequency for individual particle signals, providing particle acceleration measurements. The mean velocities and Reynolds stresses (axial, radial and shear) measured along radial profiles at x/D = 4 and 8 collapsed well using shear layer growth coordinates, \( \eta = (r - r_{0.5u_j})/x \) where \( r \) is the radius from the centerline, \( r_{0.5u_j} \) is the radius at which the shear layer velocity is half the exit velocity and \( x \) is the distance downstream of the nozzle exit, validating the flow quality and instrumentation. Acceleration results indicate that the same coordinate provides radial collapse of the data; and interestingly, the root-mean-square (RMS) of the stream-wise and radial acceleration fluctuations are nearly equal at all shear layer stations bounding the potential core—a likely result of isotropy of pressure gradient fluctuations that dominate the acceleration fluctuations in high Reynolds number shear layers. The peak magnitude of RMS accelerations were scaled relative to incompressible results from the literature to exhibit compressibility effects in this term which are due again to the influence of pressure on accelerations. The results indicate the value of acceleration measurements in high speed shear layers as an additional means for understanding the role of pressure and compressibility in high Reynolds number flows.

1. Introduction

The noise produced by jet engines has always presented a challenge to aero-acousticians, while interest in reducing this noise is growing. In the present work, an aspect of the behavior of equivalent noise source quantities are examined by leveraging the information contained within Lagrangian accelerations. Specifically, the contribution from pressure and viscous stresses to Lighthill’s (1952) stress tensor is directly related to flow accelerations, \( \frac{\partial u_i}{\partial t} \)

\[ \rho \frac{\partial u_i}{\partial t} = \frac{\partial p_{ij}}{\partial x_j} \]  \hspace{1cm} (1)

Where \( \rho \) is the flow density and \( p_{ij} \) is the pressure/viscous tensor defined by Lighthill. As such, one may write the gradient of Lighthill’s stress tensor as

\[ \frac{\partial T_{ij}}{\partial x_j} = \frac{\partial \rho u_i u_j}{\partial x_j} + \left( \rho \frac{\partial u_i}{\partial t} + \frac{\partial p}{\partial x_i} \right) \]  \hspace{1cm} (2)

making clear that the particle acceleration provides a contribution to the instantaneous gradient of Lighthill’s stress tensor.

As a jet is heated, both the velocity and the turbulence intensities increase, while convective Mach numbers of the turbulent eddies become supersonic relative to the ambient medium. The supersonically-converting eddies have increased efficiency of flow energy conversion to acoustic energy but reduced efficiency of turbulence production and redistribution mechanisms (Frowcs Williams 1963, Vreman et al. 1996). Thus, studies of hot jets are crucial for simulating the key physics of the aeroacoustics problem.

Interest in Lagrangian acceleration measurement in turbulent flows has been growing as capabilities for optical measurements of the term have become available. Published techniques for making such measurements include indirect measurement via the isotropy assumption by measuring the fourth-order
velocity structure functions (Hill and Thoroddsen 1997), particle tracking velocimetry techniques (Virant and Dracos 1997; LaPorta et al. 2001; Voth et al. 1998, 2002), particle image velocimetry (PIV) (Christensen and Adrian 2002), and laser-Doppler velocimetry (Lehmann et al. 2002, Lowe and Simpson 2006). It should also be pointed out that DNS has been used to study accelerations (e.g. Vedula and Yeung 1999).

Significant progress has been made on this subject using variants of particle tracking velocimetry (PTV). In particular, improved two-dimensional photodetectors have allowed important advances in measuring three-dimensional particle trajectories. Virant and Dracos (1997) presented PTV in the more traditional sense using CCD cameras as photodetectors for measurements of particle trajectories. Perhaps the most impressive work utilizing particle tracking has been done by a group at the Laboratory of Atomic and Solid State Physics at Cornell University (LaPorta et al. 2001; Voth et al. 1998, 2002). This group has utilized instrumentation developed for the study of high-energy particle physics to obtain resolved particle trajectories in quasi-homogeneous mixing flows. Two silicon strip detectors are used to obtain two-components of particle position each. The measurement region was projected onto the strips such that each strip represented about 7.8 x 7.8 μm² in the flow. The position could be interpolated to about an order of magnitude better than that. Though these studies have yielded some of the best data ever obtained for resolved particle trajectories, the technique is primarily limited to somewhat homogeneous flows with small mean velocities where particle residence times are large.

PIV has successfully been used to evaluate particle acceleration (Christensen and Adrian 2002, Liu and Katz 2006, Pinier and Glauser 2007). Similar to stereo-PIV methods for velocity gradient tensor measurements, these measurements require two PIV systems to work together in order to construct two sets of velocity measurements from which a first-order accurate estimate of acceleration may be determined via finite difference \[ \frac{\partial U}{\partial t} = \frac{U(t+\Delta t) - U(t)}{\Delta t}. \] Liu and Katz in particular noted the value of the technique for accessing the turbulent pressure via integration of the momentum equation, since

\[ \nabla p = -\rho \left( \frac{\partial U}{\partial t} - \nu \nabla^2 U \right) \] (3)

Further, for many flows, particularly free shear flows, the value of the particle acceleration is much greater than the viscous diffusion terms, enabling one to neglect the viscous term when carefully justified, particularly at high Reynolds numbers. Pinier and Glauser point out the validity of the assumption of negligible effects of \( \nu \nabla^2 U \) in high speed shear layers and also recognized the value in accelerations for assessing the fundamental questions posed in the current study.

In the current study, laser Doppler accelerometry, the variant to dual beam laser-Doppler velocimetry capable of sensing particle accelerations, is chosen over other techniques such as particle tracking or dual-plane PIV due to its very fine spatio-temporal resolution as well as the ability to leverage statistical signal processing in estimating the actual particle acceleration. Previous work has shown the potential for estimating instantaneous particle accelerations using LDV, the seminal being Lehmann et al. (2002), and including several efforts conducted by the co-author (Lowe and Simpson 2006, 2008, Ecker et al. 2012).

The goal of the present work is to begin to quantify the magnitudes of fluctuating accelerations in hot supersonic jet flows simultaneously with Reynolds stress values in order to develop fundamental insights on the role that these fluctuations play in the generation of flow noise.

2. Hot Supersonic Jet Facility

The supersonic hot jet facility at Virginia Tech is depicted in Figs. 1 and 2. The facility may be continuously run for a range of Mach numbers from low subsonic to 2. A Sylvania 192kW Flanged Inline Heater (Model 073153) which has a nominal pipe size of 203mm (8”) and an overall length of 584mm (23”) is used to heat the flow to total temperatures at the exit up to 922K (1200°F) for up to 0.25kg/s mass flow rates. The sections downstream of the heater are devoted to flow conditioning (Fig. 2) based on principles presented by Mehta and Bradshaw (1979). It features three 20 mesh stainless steel screens sandwiched between high temperature gaskets and a 50.8mm (2”) thick piece of honeycomb with a cell length/width ratio of 8. The converging/diverging nozzles at the exit are interchangeable.
Fig. 1 Photo of the Virginia Tech supersonic hot jet facility.

Fig. 2 (Left) Overview diagram of the supersonic hot jet facility, detailing the flow conditioning used. (Right) Cross-section diagram of the Mach 1.65 bi-conic nozzle used for the current study (dimensions in inches).

3. Laser Doppler Velocimeter

For this investigation, an advanced two-velocity/acceleration component, single transceiving lens, laser Doppler velocimeter (LDV) was used to acquire measurements within the heated supersonic jet. The LDV has a measurement volume diameter of 60µm and a fringe spacing of approximately 2µm. This probe has a random single-sample uncertainty of $\delta u / U = 0.33\%$ which was calculated using the methods described by Brooks and Lowe (2012). Detailed specifications for the LDV are given in Table 1. A photo of the two-component LDV used in this study can be seen in Fig. 3. It utilizes one beam pair of 532 nm laser beams to measure the velocity in stream-wise direction and another beam pair of 514.5 nm laser light to measure the radial component. The LDV is fiber-optically coupled to a state of the art laser photonics system (Fig. 4). The cart uses 4 Coherent Genesis 1W optically pumped solid state lasers.

<table>
<thead>
<tr>
<th></th>
<th>Streamwise Pair</th>
<th>Radial Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (nm)</td>
<td>532</td>
<td>514.5</td>
</tr>
<tr>
<td>Fringe Spacing (µm)</td>
<td>1.78</td>
<td>1.7</td>
</tr>
<tr>
<td>Probe Volume Diameter (µm)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Number of Fringes</td>
<td>34</td>
<td>35</td>
</tr>
<tr>
<td>Focal Length (mm)</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Beam Half Angle (deg)</td>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Uncertainty ($\delta u / U$) (%)</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Bragg Cell Freq. (MHz)</td>
<td>Unmodulated</td>
<td>120</td>
</tr>
</tbody>
</table>
The raw burst signals were acquired using a National Instruments NI5154 digitizer cards, set to 500 MHz bandwidth and 1 GS/s acquisition rate. All data were stored directly to disk for later processing and analysis, in particular, to apply the chirp Fourier transform processing discussed in the following section.

4. Signal Processing

The principle for acceleration extraction is based upon the fringe model for dual beam LDV,

\[ f_D = \frac{U_\perp}{d} \]  (4)

where \( f_D \) is the Doppler frequency normally measured in LDV, \( U_\perp \) is the velocity of the particle perpendicular to the fringes, and \( d \) is the interference fringe spacing.

In order to find the acceleration of a particle as it passes through the measurement volume, the raw burst must be post-processed to find the time-rate-of-change in Doppler frequency, as is evident in the equation relating the acceleration to the instrument signal obtained by differentiation of equation (4),

\[ \frac{D U_\perp}{Dt} = \frac{D f_D}{Dt} + U_\perp f_D \frac{D d}{D x_\perp} \]  (5)

where \( x_\perp \) is the coordinate measured perpendicular to the fringes. The seminal work on this technique was reported by Lehmann et al. (2002). For a sufficiently large frequency change due to acceleration and correspondingly small fringe spacing variation, the acceleration takes a form very similar to the dual beam LDV velocity equation, \( \frac{d U_\perp}{dt} = d \frac{df}{dt} \).

In this study, a discrete chirp Fourier transform (DCFT) is used to find the change in Doppler frequency. The DCFT is a method for examining signals with quadratic phase evolutions (Jenet and Prince 2000). The transform is directly analogous to the Fourier transform, but with an additional phase term for the chirp rate,
\[ CF[s(t)](l, m) = \sum_{k=0}^{N-1} s_k e^{-i\left(\frac{\pi m}{N} \left(\frac{k-N/2}{N}\right) + 2\pi i \frac{k-N/2}{N}\right)} \]

where \( l \) is the frequency spectral line number and \( m \) is the chirp spectral line number. Past works have successfully used this method to extract the acceleration from LDV data (Ecker et al. 2012).

In fig. 5 are example LDV bursts measured at 1 nozzle diameter downstream of the jet exit for hot conditions. The DCFT is computed for these bursts to produce the frequency/chirp rate spectra shown in figs 5(c) and (d). The maximum power spectral density in these maps are interpolated to determine the burst Doppler frequency and Doppler chirp which are used in equations (4) and (5) to obtain the particle velocity and acceleration, respectively.

Lowe and Simpson (2008) and Lowe (2006) presented signal processing variance results for the DCFT which indicated the root-mean-square (RMS) error in chirp rate estimation scales on the square of the particle velocity such that

\[ \delta \left( \frac{\partial U}{\partial t} \right) = \frac{\|\vec{U}\|^2}{D_{mv}} \frac{d}{\partial_m \frac{d}{\partial_m}} \]

where \( D_{mv} \) is the measurement volume diameter. In all RMS acceleration presented in section 6.2, the values were corrected according to the effective measurement volume diameter and actual fringe spacing by subtracting the evaluation for equation (7) given the local mean value of \( \|\vec{U}\|^2 \),

\[ a_{t}^r = a_{t,measured} - \|\vec{U}\|^2 \frac{d}{\partial_{mv} \frac{d}{\partial_{mv}}} \]

The effective measurement volume diameter was determined for each fringe pattern by evaluating equation (8) with varying \( D_{mv} \) while seeking to minimize \( a_{t}^r \) and \( a_{t}^r \) at the centerline of the \( x/D = 4 \) profile and with the constraint that no values for corrected \( a_{t}^r \) and \( a_{t}^r \) may be negative. The correction in equation (8) and the free parameter \( D_{mv} \) is necessary due to the large variance of the signal estimator, as predicted by the Cramér-Rao Lower Bound (Lehmann et al. 2002), and receiver and alignment efficiency which sets \( D_{mv} \). It is emphasized that a single value for \( D_{mv} \) was used for each measurement volume, one value for the streamwise sensitive and one value for the radial sensitive volume, for all the results presented.
Fig. 5 Example burst measured on the nozzle lip-line at x/D = 1.0. (a) stream-wise and (b) radial burst signals, DCFT spectral distribution maps for (c) stream-wise and (d) radial component bursts. The frequency scale shown is relative to the burst frequency line as determined from the fast Fourier transform.

5. Experimental Setup

For the current study, measurements are made with a design Mach number 1.65 bi-conic nozzle (Fig. 2, right) with an exit diameter, $D_e$ of 38.1 mm (1.5"), an exit-to-throat area ratio of 1.295, and a design nozzle pressure ratio (NPR, total pressure to ambient pressure ratio) of 4.58. The jet was run at a total temperature ratio (TTR) of 2. The complete conditions for the data of the run are presented in Table 2.

<table>
<thead>
<tr>
<th>$M_J$</th>
<th>$1.66$</th>
<th>$P_0$</th>
<th>$439$ kPa</th>
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<tr>
<td>$U_J$</td>
<td>$641.9$ m/s</td>
<td>$P_a$</td>
<td>$94.6$ kPa</td>
</tr>
<tr>
<td>$Re_d$</td>
<td>$1.1$ M</td>
<td>$T_0$</td>
<td>$575$ K</td>
</tr>
<tr>
<td>$T_J$</td>
<td>$372$ K</td>
<td>$T_a$</td>
<td>$294$ K</td>
</tr>
</tbody>
</table>

Streamwise and radial velocity measurements were acquired at a total of 30 points in the flow at locations shown in Fig. 6. Nine points were taken in a streamwise profile from x/D = 0 to 8 at the lip line of the nozzle. Two radial profiles were taken, one at x/D = 4 (10 locations) and one at x/D = 8 (11 locations).

Due to the high temperature of the flow, ceramic solid particle seeding is used in the facility. For this study Alumina (Al$_2$O$_3$) seed from Inframat Advanced Materials with a 0.5-1.0 µm diameter was chosen. The seeding was added to the flow using a fluidized-bed seeder connected to the port denoted in Fig. 2. The particle response time for this seed in a heated flow at 600 K is estimated to be approximately 3.2 µs using
the first order time constant estimation (e.g., Hjelmfelt and Mockros 1966),
\[ \tau_{St} = \frac{\rho_p d_p^2}{18\mu} \]  
(9)
where \( \rho_p \) is the density of the particle (3600 kg/m\(^3\)), \( d_p \) is the diameter of the particle, and \( \mu \) is the dynamic viscosity.

![Diagram of measurement locations within the first 8 diameters downstream of the nozzle. Lip line locations are large black circles and radial locations are small black circles.]

6. Experimental Results

6.1 Mean flow and turbulent stresses

Before investigating the acceleration measurements, the data were reduced to yield mean velocities \((\overline{U_y}, \overline{U_r})\) and Reynolds stresses \((\overline{u_y^2}, \overline{u_r^2}, \overline{u_y u_r})\). For an absolute sense of scale, dimensional flow statistics along the lip line and radial profiles at the two axial stations are presented in figs. 7 and 8, while all further data are normalized on conventional scales. Various scaling of the mean velocities are examined in figs. 9-12. As is evident from the data in fig. 10, the \( x/D = 4 \) station is still within the potential core of the jet, while \( x/D = 8 \) is outside of it. Thus, the flow region under study contains multiple scaling regimes—the shear layer growth regime for the potential core region and the transition to similarity regime for points downstream of the potential core. The classical self-similar jet scaling on the centerline velocity, \( U_0 \), and the half-centerline velocity radius, \( r_{0.5} \), exhibit relatively poor collapse of the stream-wise velocity profiles (fig. 10), although behavior of the radial velocity is similar in magnitude and profile as the incompressible results of Hussein et al. (1994). As expected, however, the collapse of the stream-wise velocity profile in the shear layer similarity coordinate, \( \eta \ast = (r - r_{0.5 U})/x \), which accounts for linear shear layer growth and was shown by Lau et al. (1979) to be adaptable from planar to annular shear layers, is much better (fig. 12). The current data taken at hot conditions compare favorably with the results extracted from the works of Lau et al. and Kerhervé et al. (2004) for cold jets at exit Mach numbers of \( M_j = 1.37 \) and 1.2, respectively. As the flow currently under study has density variation due to both total temperature variation and compressibility, the authors believe that the quality of comparison with cold data is a significant result. Also to note of significance is that the present data were acquired with a nozzle designed to simulate common tactical nozzles rather a method of characteristics nozzle designed for uniform exit flow. The potential core non-uniformity that has resulted (see, e.g., Powers et al. 2013 or Cadel et al. 2014 for non-uniformity characteristics) appears to little affect the fundamental behavior of the shear layer, bolstering the argument of broad applicability of the current results.

Turbulent velocity statistics results are presented in figs. 13-15 on self-similarity scaling (fig. 13) and shear layer scaling (figs. 14-15). The individual Reynolds stresses collapse well on the shear layer scaling, with the \( x/D=8 \) station exhibit slighter elevated scaled values of each stress. The stream-wise normal stress is compared with the values given by Lau et al. in fig. 14, and show collapse with their data at \( x/D = 4 \). It is unclear what may have caused the difference between current data and those of Lau et al. at \( x/D = 8 \); however, the current results show expected trends among the stresses, including correlation coefficients,
\[ \tilde{R}_{u_{x}u_{r}} = \frac{\bar{u}_x \bar{u}_r}{\sqrt{\bar{u}_x^2 \bar{u}_r^2}}, \] of approximately 0.4 and ratios between the radial and stream-wise normal stresses of approximately 0.5. The latter ratio and ratio of the radial normal stress to the shear stress are both greater than the values computed by Freund et al. (2000) via direct numerical simulation for similar convective Mach numbers, \( M_c \equiv \frac{u_j}{a_\infty + a_j} \) where \( a_\infty \) is the ambient air sound speed and \( a_j \) the jet core sound speed. The peak turbulence intensity values of fig. 15 are consistent with results presented by many authors for cold and hot jets, including Kerhervé et al. (2004) and Wernet (2006). The dimensional results for Reynolds stresses along the lip line (fig. 7 at right) indicate that the lip line station corresponds to approximately the radial position of peak-turbulence for at least the range \( 4 \leq \frac{x}{D} \leq 8 \). Thus, the lip line represents a snapshot of the turbulence magnitude throughout this axial range.

![Graph 1](image1.png)

**Fig. 7** Mean velocity (Left) and Reynolds stress (Right) development along the lip line of the nozzle \((r/D = 0.5)\).

![Graph 2](image2.png)

**Fig. 8** Mean velocity (Left) and Reynolds stress (Right) radial profile at \(x/D = 4 \& 8\).
Fig. 9 Mean velocity profiles normalized by $U_j$ (jet velocity) (Left) Lip line profile (Right) Radial profiles

Fig. 10 Mean velocity profiles normalized by $U_0$ (local centerline velocity), plotted against the radial coordinate normalized by the radial coordinate where the velocity is half of the local centerline velocity (Left) Streamwise velocity (Right) Radial velocity.

Fig. 12 Mean streamwise velocity profiles normalized by $U_j$ (jet velocity), plotted against the radial coordinate minus the radial coordinate where the velocity is half of the jet velocity, normalized by the streamwise position. A collapse of radial velocity profiles from Lau, Morris, & Fisher (1979) taken in a jet of $M_j = 1.37$ is plotted as a solid line. Data from Kerhreve et al. (2004) taken in a jet of $M_j = 1.2$ with a 50 m/s coflow at a streamwise position of $x/D = 5$ is plotted as x’s.
Fig. 13 Reynolds stress profiles normalized by $U_0^2$ (local centerline velocity squared), plotted against the radial coordinate normalized by the radial coordinate where the velocity is half of the local centerline velocity.

Fig. 14 Reynolds stress profiles normalized by $U_j^2$ (jet velocity squared), plotted against the radial coordinate minus the radial coordinate where the velocity is half of the jet velocity, normalized by the streamwise position. Data from Data from Lau, Morris, Fisher (1979) taken in a jet of $M_j = 1.37$ are plotted as triangles.
6.2 Acceleration results

Acceleration fluctuation statistics are presented in figs 16-18. Due to the success achieved in collapsing data using the shear layer growth similarity parameter $\eta^*$, the same independent variable is used for presentation of the radial profile results. A significant result in fig. 16 is the collapse of axial and radial acceleration fluctuations for all except the peak at the x/D = 8 station. The same collapse of axial and radial acceleration fluctuations is obtained throughout most of the range of lip line measurements in fig. 17, although the results diverge at x/D = 7 and 8—a finding that is corroborated by the peak stream-wise fluctuation value in the x/D = 8 radial profile. The isotropy of the turbulent acceleration contrasting the anisotropic Reynolds stresses deserves further examination. According to equation (1) in the limit of high Reynolds numbers, the turbulent acceleration is precisely related to the pressure gradient fluctuation,

$$a_t^2 = \frac{1}{\rho^2} \left( \frac{\partial p}{\partial x_j} \right)^2$$

where Favre averaging is assumed and the density/acceleration correlation is neglected. In incompressible flows, it has been pointed out that the inertial subrange dominates pressure gradient fluctuations, in contrast to Reynolds stresses (Batchelor 1951, George et al. 1984)—a result that would lead to greater isotropy. That the anisotropy enhancement due compressibility may not affect the isotropy of pressure gradient fluctuations is an interesting observation which may lead to better understanding equivalent noise source behaviors, especially considering the link between acoustic noise power spectra, fourth-order velocity correlations, and fluctuating pressure gradient variance (Ffowcs Williams 1963, Batchelor 1951).

The magnitude of the acceleration variance is compared with the published data from an incompressible round jet due to Lehmann et al. (2002) at x/D = 1 in fig. 18. The magnitude of the variance of the comparator data is much greater than current results when scaled on the jet exit velocity and diameter. Given that the prior results were obtained within the potential core of the jet, the same presumption described in section 4 that the acceleration variance should tend toward zero at the centerline was used to correct the data with a simple subtraction of the centerline variance value, plotted as curve LNT (b) in fig. 18. The maximum value of the acceleration variance for the two new profiles measured in the hot supersonic jet and the corrected incompressible profile were then compared in relative magnitude. Again, recalling the relationship between pressure and acceleration, the DNS findings of Freund et al. (2000) for pressure fluctuation suppression with increasing convective Mach number are replicated in fig. 19, in direct comparison to the present data scaled...
according to the corrected incompressible result. Although not identical in value to the Freund et al. curve, the present results are within the spread of typical compressibility results reported in the literature such as variation of shear layer growth rate with convective Mach number (e.g., see data compiled by Fu and Li 2006). The current results are at considerably higher Reynolds numbers than other similar compressibility results, especially those related to pressure fluctuations, providing new information about the behavior of compressible eddies at high Reynolds numbers. Future efforts may be undertaken to achieve a broad range of convective Mach numbers by varying the total temperature of the flow.

![Graph 1](image1.png)

**Fig. 16** Root-mean-square acceleration fluctuations scaled using the jet exit velocity and diameter and plotted on shear layer growth-scaled radial profiles at x/D = 4 and 8. Symbols according to the legend.

![Graph 2](image2.png)

**Fig. 17** Lip line development of the of the RMS acceleration.
Fig. 18 As fig. 16 but including the results of Lehmann et al. (2002, LNT). LNT (a): results as presented in the reference; LNT (b): results corrected to obtain zero RMS acceleration fluctuations at the centerline.

Fig. 19 Present results of peak stream-wise RMS acceleration as a ratio of the incompressible results of Lehmann et al. (2002) compared with convective Mach number trends for pressure fluctuations produced by Freund et al. (2000).

Lowe and Simpson (2006) first reported results on the correlations between velocity fluctuations and acceleration fluctuations, which may be related directly to Reynolds stress transport. In incompressible flows, the velocity-acceleration correlation is equal to the sum of the velocity-pressure gradient correlation, the dissipation rate and the viscous diffusion. In free shear flows, only the velocity-pressure gradient correlation and dissipation rate are non-negligible. The low Reynolds number DNS results of Freund et al. (2000) for velocity-pressure gradient correlation of the stream-wise Reynolds normal stress are compared to the $\overline{\alpha_z u_x}$ correlation in the present results (fig. 20). Noting that the velocity-acceleration correlation measured also contains a contribution from dissipation rate, not included in fig. 20 since this was not independently presented by Freund et al., the behavior of scaled magnitude and location of maximum magnitudes are similar. As pointed out by Lowe and Simpson, the value of the velocity-acceleration
correlation in experiments is the ability to measure terms of significance to stress transport with a single-point measurement and no need for a measurement grid for computing gradients. The results in fig. 20 indicate that this value exists for the high speed flow application, as well; although, it is clear from the apparent scatter near the centerline that acceleration uncertainties become a challenge for the method at higher velocities. It is likely that larger data sets could reduce statistical uncertainties of these correlation measurements.

![Graph](image)

**Fig. 20.** Comparison of the stream-wise velocity/stream-wise acceleration correlation scaled using the jet exit velocity and diameter to the velocity-pressure gradient correlation computed by Freund et al. (2000) at a convective Mach number of 0.99 in a developing annular shear layer. The results due to Freund et al. include a ratio of local mean density to jet exit density as a product with the velocity-pressure gradient correlation, while the present results neglect that ratio by assuming it to be unity.

7 Conclusions

Results for turbulent acceleration statistics measured using laser Doppler velocimetry in a hot supersonic jet at an exit Mach number of 1.65 and shear layer convective Mach number of approximately 0.8 have been presented. The study is motivated by the need to better understand the behavior of turbulence mechanisms and their role in noise generation.

The measurements were considered in the context of published experimental and simulation results in compressible shear layers by comparing single-point velocity statistics (mean and Reynolds stresses). The mean stream-wise velocity radial profiles at both the x/D = 4 and 8 stations collapsed well with prior results in cold supersonic jets using radial profile scaling based upon shear layer growth coordinates. All Reynolds stresses measured collapsed between the x/D = 4 and 8 stations on the same coordinate scaling. Interestingly, the relative magnitudes of the ratios of radial normal stresses and shear stresses on stream-wise normal stress were greater than ratios obtained in low Reynolds number DNS at similar convective Mach numbers.

Several important observations have been made based upon the acceleration results as well. Strikingly, the root-mean-square stream-wise and radial acceleration fluctuations are nearly equal in virtually all measurements, excluding the region around peak turbulence near the nozzle lip line radius downstream of the potential core at measurements for x/D = 7 and 8. This term is closely linked to the fluctuating pressure gradient via the momentum equation, and theoretical results indicate that isotropy is promoted in this parameter due to dominance of the inertial subrange of scales on the single-point pressure gradient variance value. Additional measurements are needed to determine whether the breakdown of isotropy that seems to occur downstream of the potential core is significant. The radial profiles for fluctuating acceleration were scaled and compared with
incompressible results, indicating peak values occurring at nominally the same location in the shear layer coordinates. The incompressible comparison data were used to normalize the current compressible data and results were compared with variation of root-mean-square pressure fluctuations with convective Mach number, further bolstering the importance of the link between the acceleration measurements and fluctuating pressures. Finally, it was demonstrated that streamwise Reynolds stress transport terms measured via the velocity-acceleration correlation exhibited similar magnitudes and peak locations as DNS data reported by Freund et al. (2000) at convective Mach numbers of 0.99, indicating promise for applying the technique for fundamental understanding of turbulence transport in harsh flows without the need for refined measurement grids to obtain mean statistics gradients.

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