Direct Measurement of Three-Dimensional Velocity by a Doppler-Phase-Shifting Holography

Nao Ninomiya¹*, Yamato Kubo¹, Daisuke Barada¹, Tomohiro Kiire¹

¹: Utsunomiya University, Yoto 7-1-2, Utsunomiya, Tochigi 321-8585, Japan
* correspondent author: nino@utmu.jp

Abstract In order to understand the details of the flow field in the micro- and nano- fluidic devices, it is necessary to measure the three-dimensional velocity field under a microscopy. Thus, the development of a new measuring technique for the three-dimensional velocity by a single camera is strongly needed. One solution is the use of holography, but it has been known that the resolution in the depth direction is very poor for the commonly used in-line holography. Presently, the Doppler-phase-shifting holography has been used for the three-dimensional measurement of an object. This method extracts the signal of a fixed frequency caused by the Doppler howling between the object light and the reference light, either or both of which travels at a constant speed. Applying the shifting to the holography, the three-dimensional shape of an object can be measured very precisely. Here, the frequency of the Doppler howling is determined by the velocity difference between the object light and the reference light. This implies that the velocity of an object can be measured by the Doppler frequency. In this study, a concave mirror was traversed at a constant speed and its holography has been observed by a high-speed camera. By extracting only the first order refraction signal at the Doppler frequency, the 0th order object light can be removed from the hologram and thus a precise measurement of the surface profile has been achieved. At the same time, the lateral velocity of the mirror can be measured by the Doppler frequency. Furthermore, a 5 yen coin has been traversed by different velocities at different angles and its shapes and the three-dimensional velocities have been measured. The transverse velocities are measured by the PIV algorithm and at the same time the longitudinal velocity is measured by the Doppler frequency. The results reveal that the precise measurement of the three-dimensional velocity by a single camera is possible by the use of the Doppler-phase-shifting holography. This method can be applied to the particles motions in the micro- or nano- devices, and thus the three-dimensional velocities will be measured under a microscopy.

1. Introduction

In order to understand the details of the flow field in the micro- and nano- fluidic devices, it is necessary to measure the three-dimensional velocity field under a microscopy. Thus, the development of a new measuring technique for the three-dimensional velocity by a single camera is strongly needed. One solution is the use of holography, but it has been known that the resolution in the depth direction is very poor for the commonly used in-line holography. Presently, the Doppler-phase-shifting holography has been used for the three-dimensional measurement of an object. This method extracts the signal of a fixed frequency caused by the Doppler howling between the object light and the reference light. The frequency of the howling is determined by the velocity difference between the object light and the reference light. This implies that the velocity of an object can be measured by its frequency. In this study, a 5 yen coin has been traversed at different angles and its shapes and its three-dimensional velocities have been measured accurately. If this method is applied to the particles motions in the micro- or nano- devices, the three-dimensional velocities can be measured.

2. Fundamental Methodology

Digital holography observes the interference of the observed light scattered at the surface of an object and the reference light. Then, the three-dimensional shape of an object can be reproduced by calculating the diffraction by the computer. But, in the commonly used in-line holography, the diffraction is contaminated by the 0th order object light and the accuracy in the depth direction is deteriorated. In order to improve this drawback, several phase-shifting methods have been proposed, which can calculate only the 1st order diffraction and thus the longitudinal accuracy is guaranteed. Presently, Doppler-phase-shifting holography is chosen because it is capable of unsteady measurement and does not need multiple cameras. Figure 1 shows the experimental setup of a Doppler-phase-shifting holography.
The complex intensities of the object light and of the reference light are expressed as follows:

\[ E_O(x, y, t) = a_O(x, y) \exp \{i[\phi_O(x, y) - \omega_O(t)t]\} \]
\[ E_R(x, y, t) = a_R \exp \{i[\phi_R - \omega_R(t)t]\} \]  

where \( a_O \) and \( a_R \) represent the amplitudes, \( \phi_O \) and \( \phi_R \) are the phase angles and \( \omega_O \) and \( \omega_R \) are the angular velocities of the object light and the reference light. If the object travels at velocity \( v_O \) and the reference mirror at \( v_R \), the angular velocities are shifted by the Doppler effect of light as:

\[ \omega_O(t) = \omega_0 \frac{1 + 2v_O(t)/c}{\sqrt{1 - 2v_O(t)/c}} \]  
\[ \omega_R(t) = \omega_0 \frac{1 + 2v_R(t)/c}{\sqrt{1 - 2v_R(t)/c}} \]

where \( \omega_0 \) is the angular frequency of the light source and \( c \) is the speed of light. Consequently, the superposition intensity of the holograms detected by the image sensor is expressed as follows:

\[ I(x, y, t) = |E_O(x, y, t) + E_R(x, y, t)|^2 \]
\[ = a_O^2(x, y) + a_R^2 + 2a_O(x, y)a_R \cos[\Delta\phi(x, y) - \Delta\omega(t)t] \]

\[ \Delta\phi(x, y) = \phi_O(x, y, t) - \phi_R \]
\[ \Delta\omega(t) = \omega_O(t) - \omega_R(t) \]

Fig. 1 Experimental setup for Doppler phase-shifting digital holography
The Fourier transformation of eq. (5) in the time domain is

\[
\mathcal{F}_t(l)(x, y, \nu_b) = [a_O^2(x, y) + a_R^2] \delta(\nu_b)
\]
\[
= a_O(x, y) a_R A_1(\nu_b) \exp\{i[\Delta \phi(x, y) + B_1(\nu_b)]\}
\]
\[
+ a_O(x, y) a_R A_2(\nu_b) \exp\{-i[\Delta \phi(x, y) + B_2(\nu_b)]\}
\]

(7)

If the angular frequency of the object light is higher than that of the reference light, the +1st order diffraction is obtained in the second term. If the spectrum of each term can be split, the phase difference \(\Delta \phi(x, y)\) between the object light and the reference light is easily obtained.

The discrete Fourier transformation of the image recorded by the image sensor of the finite size is obtained as:

\[
DFT[l](x, y, \nu_{b,m}) = \sum_{n=0}^{N-1} I(x, y, t_n) \exp(-i2\pi n \nu_{b,m})
\]

\(m = 0, 1, 2, \ldots, N - 1\)

(8)

If the +1st order diffraction spectrum of a fixed frequency is extracted from eq. (8), the complex amplitude of an object can be calculated very accurately.

3. Velocity Measurement

Basically the Doppler-phase-shifting is invented to measure the object profile accurately. But, as the beat frequency of the Doppler howling is proportional to the velocity difference between the object light and the reference light, the longitudinal velocity of an object can be measured by the beat frequency of the Doppler howling. Thus, the longitudinal velocity of a concave mirror travelling at a constant speed is measured by a Doppler-phase-shifting holography in this study.

Ar-ion laser of 514nm is used as the light source, which is introduced to a pin-hole and expanded by the spatial filter and then made parallel by a collimator lens of \(f=100\)mm. A concave mirror of the radius of curvature is \(R=30\)m is traversed at \(v_o=100\)µm/s. The interference images are recorded by a high-speed camera whose resolution is 1024x1024 and its frame rate is 2000fps with a shutter speed of 1/70000s. Figure 2(a) shows the time series intensity fluctuation of the central pixel of the 512 images recorded and the Fig. 2(b) is its spectrum. It is obvious that the clear peaks of Doppler howling is seen and its frequency is

![Figure 2](image_url)

(a) Time series intensity at a pixel of the central pixel (b) Spectrum obtained from 512 images
386.7Hz, which corresponds to $v_0=99.4\mu m/s$ with the error of 0.6%. Now it has been proved that a Doppler-phase-shifting holography can be used for the measurement of the longitudinal velocity. Moreover, the Fig. 3s show the 3D and 2D plots of the surface profile of the concave mirror reproduced by digital holography and the rms to the R=30mm profile is only 3.27nm. This is the evidence that the present technique is capable of measuring the three-dimensional position and the longitudinal velocity.

![Surface shape of concave mirror measured (a)Cross-section view of x-position and fitting curve](image)

4. Three-Dimensional Measurement

As the fundamental measurement technique has been proved to work, the actual measurement of the three-dimensional velocity is done by the experimental setup shown in Fig. 4. The Japanese 5 yen coin is used as a target object and is traversed at a constant speed at a specific angle. The diameter of 5 yen coin is about 22mm and as the reflection from is somewhat weaker than the reference light, the ND filter is placed in front of the reference mirror and the shutter speed is set to 1/2000s in order to match the light intensities.
The resolution of the high-speed camera is set to 2048x2048. The angle of the moving stage is set at $\theta=76$ degree and it travels at $v_0=413.4\mu$m/s, which corresponds to $v_{Oz}=100.0\mu$m/s. Figure 5s show the measured spectrum obtained from 512 images at two different timings of the traverse. Both of them have strong peaks at (a) 396.2Hz and (b) 399.7Hz, which correspond to the longitudinal velocity of (a) $v_{Oz}=101.8\mu$m/s and (b) $v_{Oz}=102.7\mu$m/s.

Figure 6 shows the reconstructed images of the 5 yen coin at two different timing. The former image is colored by red and the latter by blue. The expanded plot of the velocity vectors are obtained by PIV algorithm and the transverse movement by a fixed speed can be observed. By the combination of the longitudinal velocity measurement by the Doppler frequency and the in-plane velocity measurement by PIV, three-dimensional velocity measurement of an object has be achieved.
But as the 5 yen coin is also moving in the transverse direction, the time series intensity used to obtain the spectrum shown in Fig. 5 is not an exact trace of the light emitted from a fixed point. Nevertheless, as the transverse movement of the 5 yen coin has been measured by PIV as shown in Fig.6, the temporal movement of a fixed point can be interpolated from the results in Fig.6. Presently, each frame of the original interference image is shifted by Eq.(9) in order to trace the exact time series of the light emitted from a fixed point.

\[
f(x - x_0, y - y_0) = \frac{1}{N_x N_y} \sum_{k=0}^{N_x-1} \sum_{l=0}^{N_y-1} F(k, l) \times \exp \left[ 2\pi i \left( \frac{k x}{N_x} + \frac{l y}{N_y} \right) \right] \exp \left[ 2\pi i \left( \frac{k x_0}{N_x} + \frac{l y_0}{N_y} \right) \right]
\]  

(9)

Then the beat frequencies obtained from the spectrum are (a) 391.2Hz and (b) 392.6Hz, which correspond to the longitudinal velocity of (a) \(v_{Oz} = 100.5 \mu m/s\) and (b) \(v_{Oz} = 100.9 \mu m/s\). It is quite obvious that the image shifting improves the measurement accuracy of the longitudinal velocity. Figure 7 shows the reproduced images of the 5 yen coin with the image shifting. It can clearly be seen that the shape of the 5 yen coin is more clear with the shifted images and the it is quite a matter of course but the transverse movements are not hardly seen in Fig. 7. This is a proof that the shifting by Eq.(9) has been done properly.

Table 1 summarizes the result of the three-dimensional velocity measurement by a Doppler-phase-shifting holography. The vertical velocity is almost zero as the traversing stage moves in the horizontal direction. Now it can be concluded that this technique is capable of three-dimensional velocity measurement. The lateral velocities are calculated as the sum of the image shift and the measured displacement in Fig. 7, where the size of the pixel is 10 \(\mu m\).
Table. 1 Veocimetry of Japanese 5-yen coin (76degree)

<table>
<thead>
<tr>
<th>Theoretical Value(µm/s)</th>
<th>Ox</th>
<th>Oz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement value(µm/s)</td>
<td>100.7</td>
<td>401.2</td>
</tr>
</tbody>
</table>

Finally, the similar measurements have been repeated with different angles of the traverse. The results are shown in Fig.8 and it is quite obvious that this technique is valid for different angles of the traverse.

5. Conclusion

Presently, a new three-dimensional velocity measuring technique based on a Doppler-phase-shifting holography has been developed. It can measure the three-dimensional velocity of an object very accurately by the single camera observation. This method can be adapted to the particle flows and then the three-dimensional particle velocities can be measured simultaneously. It will sure to contribute to the development of micro- and nano- fluidic devices.

Acknowledgement

A part of this work has been supported by the Center for Optical Research and Education (CORE) at Utsunomiya University, the Project for Bio-imaging and Sensing at Utsunomiya University and the JSPS KAKENHI Grant Number 25420147. All supports are greatly acknowledged.

References
