Stereoscopic Astigmatism Particle Tracking Velocimetry for macroscopic 3D3C flow measurements in air

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Abstract This paper introduces a stereoscopic 3D3C flow measurement technique (SAPTV), which is derived from the single-camera Astigmatism Particle Tracking Velocimetry (APTV) method. Particle locations are first estimated applying APTV algorithms. This is followed by a more accurate particle location using the stereoscopic view. Particle location errors yield values less than 0.1 % of the measurement volume dimensions for xyz. A dt → 0 analysis using DEHS seeding particles shows small statistical measurement errors: \( \sigma_x \approx 5.5 \, \mu m \), \( \sigma_y \approx 3.3 \, \mu m \), and \( \sigma_z \approx 12.1 \, \mu m \). With the combination of APTV and stereoscopic APTV, a redundant measurement scheme is established. Overlapping particle images on one sensor are not likely to overlap on the other sensor. While triangulation is not applicable for overlapping images, the particle location can still be determined using single camera APTV with only slightly lower accuracy. This is advantageous, especially for time-resolved measurements, where complete particle tracks are required to avoid interpolation errors.

1. Introduction

Capturing volumetric velocity fields is essential for understanding complex flows. Thus, different 3D measurement techniques based on Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV) have been developed [1,2]. A well-established 3D3C measurement technique is tomographic PIV, employing multiple cameras [3]. As the evaluation of the results is usually performed with spatial correlation techniques, bias errors appear at low seeding concentrations due to the spatial averaging [4]. With increasing particle image concentration, ghost particles appear, which lower the measurement precision. To avoid the random errors due to the ghost particles and the bias errors due to the spatial averaging, three-dimensional PTV can be used to capture volumetric flow fields with multiple cameras [5]. With increasing seeding concentration, 3D PTV suffers from ambiguous particle image allocation. These ambiguities are largely resolved by taking time-series of images. However, this strongly limits the applicability of the technique as it is not possible to conduct time-resolved measurements in many applications. In microfluidics, the optical access is limited, which allows for single camera recording only. To measure 3D3C velocity fields with a single optical system, the APTV technique has become a standard in recent years [6,7,8].

The APTV measurement principle is also applicable to macroscopic flows. Employing only a single camera, APTV is best suited for measurement domains with limited optical access (e.g. compressor, turbine, combustion research). The velocities in xy direction are measured highly accurate with a spatial resolution of less than 0.5 % of the measurement volume depth \( d_z \) in z direction [9]. I.e. the z location of a particle is estimated with an accuracy of 0.25 mm for \( d_z = 50 \, \text{mm} \). Therefore, using APTV, velocities in xy direction are averaged (in z direction) over a significantly smaller depth compared to 2D PIV, where the in-plane velocities are averaged (in z direction) over the light sheet thickness, which is normally about 1 mm or even larger. When the measurement volume depth is small \( d_z \leq 5 \, \text{mm} \), e.g. gap flows, the APTV particle z location uncertainties yield values less than 0.025 mm. However, using APTV processing algorithms, the accuracy of the velocity component \( w \), in the z direction is significantly improved, compared with \( u \) and \( v \). This is due to the fact that the xy location of a particle is derived from its particle image’s center location XY, which can be determined accurately. The z location, though, is derived from the axis lengths \( \left( a_x, a_y \right) \) of the particle image, where the determination is more strongly affected by e.g. low signal to noise ratios (SNR). This yields lower accuracies in the particle’s z location, hence yielding also larger errors in the estimation of the velocity component \( w \). For highly three-dimensional flows (e.g. flows in cavities of compressor casing treatments), it is desirable to measure \( w \) more accurately - while keeping the experimental setup as simple as possible.
Therefore, this paper introduces a stereoscopic 3D3C flow measurement method, which is derived from the single camera Astigmatism Particle Tracking Velocimetry (APTV) method. Following a first estimate of particle locations using APTV processing, the spatial location is triangulated(mapped), yielding lower location errors, especially in the z direction (see Fig. 1).

In the next section, the experimental methods of this work are introduced. First, the APTV measurement principle is outlined, followed by a detailed description of the two camera triangulation/mapping approaches. Section 3 presents a comprehensive accuracy analysis of particle location using two cameras.

![Fig. 1 Qualitative uncertainty of stereoscopic APTV (red) compared to single camera APTV (gray).](image)

For $\beta = 90^\circ$ the particle location accuracy is equal for $\mathbf{xyz}$.

2. Experimental methods

2.1. Astigmatism PTV

To locate particles with APTV, a cylindrical lens is placed in front of the camera sensor, to elongate the particle images with a certain magnitude and direction depending on the particle’s depth location. Consequently, the analysis of the elliptical shapes with distinct geometry, allows to relate the $\mathbf{xyz}$ location of a particle with the axis lengths and the center location of the particle image [9]. APTV imaging setups require a 3D calibration procedure, which relates the center location ($\mathbf{X} = \mathbf{XY}$, capital letters denote image space coordinates) and the axis lengths ($a_x, a_y$) of an elliptical particle image to the particle location in physical space ($\mathbf{x} = \mathbf{xyz}$, small letters denote physical coordinates). For the calibration, a pinhole matrix (see Fig. 2) is used to simulate particles at well defined locations, as pinholes have similar light emission behavior (Babinet’s principle). This backlight illuminated pinhole matrix is moved through the measurement volume in steps of $\Delta z$, where the magnitude of $\Delta z$ is roughly 1% of the corresponding measurement volume depth $d_z$. At each z position the pinhole matrix is imaged and the axis lengths of the pinhole images are determined (with known physical coordinates $\mathbf{x}$ of the pinholes), resulting in a 3D function,

$$\mathbf{x} = f(\mathbf{X}, a_x/a_y),$$

(1)

which is then used for estimating the particle location. An accuracy analysis of single camera APTV shows its excellent ability to measure macroscopic 3D flows. The particle’s z location error is less than 0.5% of $d_z$, while the xy location error is less than 0.2 pixel. These measurement uncertainty values were derived by means of a $\Delta t \rightarrow 0$ analysis using real seeding particles with a diameter of $d_p \approx 1 \mu m$ in air in a macroscopic measurement volume ($d_x \times d_y \times d_z \text{ mm}^3, 20 \times 20 \times 45 \text{ mm}^3$). For a single particle track, this results in measurement errors of about 2% for the velocities in the xy direction (normalized with the maximum flow velocity). The calculations are based on a maximum displacement of the particles of 15 pixels on the sensor.
In the $z$ direction, $w$ is less accurate, as a direct result of the less precise particle $z$ location determination. However, if a more accurate determination of $w$ is required, it is desirable to extend the single camera APTV approach for more accurate measurements.

![Image](https://via.placeholder.com/150)

**Fig. 2** Section of an intensity inverted gray-scale image of the backlight illuminated pinhole matrix, used for the calibration of single camera and stereoscopic APTV setups.

### 2.2 Stereoscopic methods

Employing a second camera to locate particles using triangulation or coordinate mapping increases the determination accuracy of $xy$ and in particular that of $z$, as illustrated in Fig. 1. For $\beta = 90^\circ$, the particle location accuracy is equal for all three coordinates. Thus, stereoscopic APTV allows for velocity measurements of highly three-dimensional flows with only two cameras. Both triangulation and coordinate mapping require knowledge of the sensor locations $X$ of the corresponding particle images on both sensors. As a stand-alone technique, triangulation suffers from ambiguous particle image allocation at high seeding concentrations. Coordinate mapping of two sensor locations to physical space is possible without knowing the particle image correspondences. APTV provides a means to overcome these limitations, giving a first estimate of the particle locations. Using this information, the sensor coordinates of the two particle images of a particle on both sensors are estimated. This way, corresponding particle images are allocated uniquely, even at higher seeding concentrations. Furthermore, if particle images overlap on one sensor, they most likely do not overlap on the other sensor, so the respective particles can still be located, though with single camera APTV accuracy. This is advantageous, especially for time-resolved measurements, as it is unlikely to loose particle tracks.

The calibration of the stereoscopic APTV system requires a set of point correspondences between two images $X_1 \leftrightarrow X_2$ (image space). These point correspondences are derived from the same pinhole matrix, which is also used for the calibration of the single camera APTV system. Hence, the calibration of the two imaging systems is conducted simultaneously. The sensor locations of the point correspondences are denoted by the center locations of the pinhole images. Physical coordinates $x$ of the points are given by the $z$ location of the calibration target and the $xy$ locations of the pinholes on the calibration target.

### 2.2.1 Triangulation

Now, a more detailed description of the triangulation approach follows. From the point correspondences the fundamental matrix $F$ of the stereoscopic optical system is calculated such that it satisfies (from [10]):

$$X_1^T F X_2 = 0.$$  \hspace{1cm} (2)

The Gold Standard Algorithm is an efficient way to estimate $F$ in two steps [11]. First, an initial linear solution of $F$ is calculated using the normalized 8-point-algorithm [12]. In a second step the camera matrices, $P_1$, and $P_2$, are determined. If $P_1$ is defined as a $3 \times 4$ identity matrix, $P_2$ is computed from the epipolar line and $F$. Using $P_2$, the sensor position, $\tilde{X}$, of the known points is estimated. To optimize $P_2$, the error function,
\[ \sum d\left(\mathbf{x}_{1i}, \mathbf{x}_{1i}\right)^2 + d\left(\mathbf{x}_{2i}, \mathbf{x}_{2i}\right)^2, \]

describing the geometric distance between a point \((\mathbf{x}_1, \mathbf{x}_2)\) and its estimation \((\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2)\), is minimized by means of a non-linear Levenberg-Marquardt algorithm. If vibrations and movement of the camera occurs during measurements, \(P_2\) can be corrected for each frame by point correspondences derived from particles in the flow, which is comparable to self-calibration in tomographic PTV/PIV [13].

Finally, spatial locations \(\mathbf{x}\) (voxel coordinates) are estimated by the optimal triangulation method [11]. These voxel coordinates have to be converted to physical coordinates. Spatial as well as sensor locations of the pinholes and their pinhole images are known. Hence, the conversion is performed by sixth order polynomials relating pixel to metric coordinates, where the coefficient matrix, \(S\), is optimized using a least squares approach with the known pinhole locations. Having determined \(F, P_1, P_2, \) and \(S\), all the required information to locate a particle in a measurement volume is available. Fig. 3 (left) shows an overview of the calibration procedure.

With the information \((P_1, P_2)\) derived from the calibration procedure, particle locations \(\mathbf{x}\) are triangulated from the sensor locations \(\mathbf{X}_1\) and \(\mathbf{X}_2\) of the two particle images. The corresponding particle images are allocated using the preliminary APTV location estimation. Particle locations are then triangulated into non-metric space. The final step is to convert the non-metric coordinates into physical/metric coordinates (particle location procedure: Fig. 3 (right)).

![Fig. 3 (left) Stereoscopic APTV calibration procedure. (right) Particle location scheme.](image)

### 2.2.2 Coordinate mapping

It is also possible to conduct a coordinate mapping of the image coordinates to physical coordinates without performing triangulation. The mapping function consists of third order polynomials [14]. Before estimating particle locations it is necessary to determine the coefficients of the polynomial functions. These coefficients are derived, using the known point correspondences of the calibration target, by means of a least squares approximation. As mentioned above, it is essential to know which two particle images correspond to each other. Unlike with triangulation, where a search corridor - denoted by the epipolar line - on the other sensor is analyzed to allocate particle image pairs, the mapping approach does not have this ability. Instead, APTV gives a first estimation of particle locations and enables the allocation of the particle images. With this knowledge, \(\mathbf{x}\) is determined by simply mapping the coordinates of the particle images to physical coordinates, yielding a simple, fast and accurate 3D particle location approach.
3. Measurement uncertainty

3.1 Calibration error

In a first analysis, the locations of the pinholes used for the calibration of the stereoscopic system are reconstructed and compared to their actual, known locations. Fig. 4 shows the deviations of the estimated pinhole locations for each spatial direction for the triangulation approach. For the $xy$ direction, absolute deviations of the pinhole locations are mostly lower than 10 $\mu$m, while for the $z$ component the deviations are somewhat larger (black scatter, left ordinate). This is due to the fact that the angle between the cameras was $\beta = 35^\circ$ and such is $z$ the coordinate with the lowest accuracy as illustrated qualitatively in Fig. 1. The average deviation at each $z$ position oscillates slightly, with absolute average values below 3 $\mu$m for $xy$ and below 6 $\mu$m for $z$ (right ordinate, red curve and data points). These oscillations show a similar behavior for all coordinates – which is also present for the mapping approach. Thus, the authors assume that the accuracy of the micro stage (6 $\mu$m according to the manufacturer) influences the average deviation values. However, the magnitude of the oscillations is rather low and does not introduce significant uncertainties.
The resulting error of the reconstructed pinhole locations is calculated as follows:

\[ E_x = \sqrt{\frac{\sum \Delta x^2}{N-1}}, \]

where \( \Delta x \) is the deviation of the estimated particle position to the actual particle position, and \( N \) is number of analyzed values. \( E_y \) and \( E_z \) are calculated accordingly. Fig. 5 shows the errors at every pinhole matrix \( z \) position for \( x,y,z \) for three different calibration approaches. First, the triangulation approach (black), has a reconstruction error of roughly \( <5 \) \( \mu \)m for the \( x \) coordinate, while for \( y \) it is slightly lower, around \( 2-3 \) \( \mu \)m. The \( z \) uncertainty is larger yielding \( 10 \) \( \mu \)m (average errors of the triangulation approach along \( z: E_x = 3.8 \) \( \mu \)m, \( E_y = 2.3 \) \( \mu \)m, \( E_z = 9.0 \) \( \mu \)m). The blue plots denote the errors of the mapping approach. Accuracies yield similar values as using triangulation (average errors of the coordinate mapping approach along \( z: E_x = 4.1 \) \( \mu \)m, \( E_y = 2.5 \) \( \mu \)m, \( E_z = 8.0 \) \( \mu \)m). The red plots denote a calibrated triangulation method, using camera models [15]. Errors are then somewhat larger for this setup and increase strongly, if different media and/or large aberrations are present [5]. The methods described in section 2 have the ability to compensate these influences.

![Graph showing reconstruction errors of x,y,z along z for particle location approaches](image)

**Fig. 5** Reconstruction errors of \( x,y,z \) along \( z \) for particle location approaches. (black) Triangulation (section 2); (blue) Coordinate mapping; (red) Triangulation using camera models.

### 3.2 Statistical error

In addition to the particle location accuracy investigation, an analysis of the statistical measurement uncertainty was carried out by means of particle location in a “motionless” flow. I.e. DEHS particles (\( d_p \approx 1 \) \( \mu \)m) in a glass tank were imaged at two instances with a separation of just \( 0.5 \) \( \mu \)s (\( dt \to 0 \)). Except for thermal convection there was no driven flow of the air, hence the displacement of the seeding particles between the two instances approaches zero (\( \Delta x \to 0 \)). Fig. 6 shows the principle experimental setup of the
uncertainty analysis. The measurement volume (40×40×20 mm³) was illuminated by a Spectra Physics PIV laser with a pulse energy of 400 mJ. The particles were imaged using sCMOS cameras with a resolution of 2560×2160 pixel. The angle between the cameras was β = 35°. Astigmatism was induced by a cylindrical lens with a focal length of 1000 mm, placed in front of the camera sensors. The statistical measurement error was calculated as follows:

\[ \sigma_x = \sqrt{\frac{\sum \Delta x^2}{N-1}}, \]

where Δx (or Δy, Δz respectively) is the displacement of the particles between two frames. For the triangulation approach, the statistical measurement errors for x are: \( \sigma_x = 5.5 \mu m, \sigma_y = 3.3 \mu m, \) and \( \sigma_z = 12.1 \mu m. \) The mapping approach yields the following uncertainties: \( \sigma_x = 5.5 \mu m, \sigma_y = 3.6 \mu m, \) and \( \sigma_z = 13.0 \mu m. \) Calibrated triangulation yields similar values: \( \sigma_x = 5.5 \mu m, \sigma_y = 3.4 \mu m, \) and \( \sigma_z = 12.1 \mu m. \)

![Fig. 6 Experimental setup for the measurement uncertainty analysis of stereoscopic APTV.](image)

The measurements were conducted at \( \beta = 35^\circ. \)

### 4. Discussion and conclusion

This paper presented a three-dimensional particle location technique using a stereoscopic imaging setup. The proposed method is an extension of the single-camera APTV technique in order to determine the z coordinate of particle locations more accurately – on the cost of a second camera and more complex image analysis techniques. Particle location accuracies are in the same order of magnitude for all coordinates. The statistical measurement error lies in the range of 3-5 \( \mu m \) for the particle \( xy \) locations and 12-13 \( \mu m \) for \( z \) locations, allowing for accurate measurements of 3D3C flow fields (measurement volume size: 40×40×20 mm³). Unlike standard 3D particle tracking techniques, the method employs only two cameras for unambiguous particle location. Furthermore, measurement in different media as well as strong optical aberrations are accounted for in the calibration functions. Hence, a complex adjustment of the light path is not necessary. Altogether, it has been shown that stereoscopic APTV is a simple and powerful 3D flow measurement technique.
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