Time-resolved tomographic PIV investigation of turbulent flow control by vortex generators on a backward-facing step

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Abstract An experimental study is presented for turbulent flow control in backward-facing step (BFS) flow using time-resolved tomographic particle image velocimetry (TR tomo-PIV) and high-resolution 2D-2C- PIV in the 1-m low speed wind tunnel at DLR in Göttingen, Germany. Vortex generators were designed and implemented on a BFS with an oncoming zero pressure gradient turbulent boundary layer flow. The investigated passive flow control devices were wedge-shaped, backward-oriented vortex generators which were transversely mounted upstream of the step. The Reynolds number was $Re_h = 2.0 \times 10^5$, based on the free stream velocity $U_e = 10$ m/s and the step height $h = 30$ mm. TR tomo-PIV was employed to measure the separated shear layer flow within a measurement volume of $50 \times 60 \times 10$ mm$^3$ in $x$, $y$- and $z$-directions respectively, which was located right behind the step and above the plate. Four high-speed CMOS cameras were used to record image pairs at a sampling rate of 1 kHz. In order to evaluate the effect of the given flow control devices on the reattachment length, additional high-resolution 2D-2C- PIV was applied to a streamwise-vertical measurement plane downstream of the step.

The time-averaged reattachment length was reduced by 29.1%, which proved that the vortex generators maintain the ability to attenuate separated flow regions. For analysis of spatial and temporal evolutions of the vortices, two-point cross-correlation functions revealed scales of turbulence and identified a relative constant convective velocity $U_c \approx 0.5 \cdot U_e$. Reynolds shear stress and turbulent kinetic energy were increased correspondingly where streamwise vortices were induced, entrained and dissipated. Furthermore, Proper Orthogonal Decomposition (POD) was applied to the experimental data to extract coherent structures and the corresponding frequency bandwidth. Periodic travelling waves were visualized by reconstruction of the first two POD modes. Hence, the POD analysis provided clear insights into the evolution of the coherent structures as well as into the interaction between the mean and quasi-periodic flow fields. These coherent structures eventually increased the momentum transfer and consequently conveyed kinetic energy from the mean flow to the random fluctuating motions.

1. Introduction

The study of flow separation control has been an attractive subject of fluid mechanics researches since the early 1900s. As an undesired phenomenon, flow separation causes large energy loss and affects aerodynamic performance of, for instance, aircraft wings and turbine blades. Therefore, reducing or even eliminating flow separation can result in a higher lift coefficient of an aircraft wing or a higher efficiency of turbo machinery. Among various kinds of passive flow control devices, vortex generators (VGs) were first introduced by H. D. Taylor [69] and have been widely used in flow separation control. As economical and reliable passive devices VGs are simply designed of various types, such as vane, ramp, doublet, wishbone and wedge, and do not introduce external energy into the flows [34]. Conventional VGs have a device height $H$ in the order of the boundary layer thickness $\delta$, and are able to increase the near-wall momentum through momentum transfer from the free stream flow to the near-wall region [63]. Since the 1970s, low-profile VGs have been developed and applied in flow separation control [32]. These low-profile VGs have a device height ($H < \delta$) of only a fraction of conventional ones and are submerged within the turbulent boundary layers. Further investigations [1, 35, 36, 37, 51, 77] show that low-profile VGs are capable of providing sufficient vertical momentum transfer over a region several times higher than their own height, and moreover, causing less device drag. In early literature, low-profile VGs have other referred names, such as micro VGs, sub-boundary-layer VGs and submerged VGs [34].

The flow over a backward-facing step (BFS) is a multi-scale flow case containing a separated shear layer,
recirculation flows, and reattachment area. The BFS flow provides a typical separated flow case with very simple geometry and a fixed separation point, which has been extensively studied as a fundamental research case for flow separation control. With increasing Reynolds numbers, small- and smaller-scales of vortices get involved in addition to large-scale structures, which make the flow field even more complex. Although numerous experimental and numerical investigations have been carried out on this problem, a complete understanding of the physical mechanism of the turbulent shear flow has still not been obtained. Armaly et al. [2] studied BFS flows extensively in experiments, including laminar, transitional and turbulent flows in a two-dimensional channel between \( 70 < Re < 8000 \), which presented the typical variations of separation lengths with Reynolds numbers. Ötügen [44] examined different expansion ratio effects on the separation and reattachment of BFS flow at \( Re = 16600 \) by hot wires and laser Doppler anemometry. Hasan [25] introduced controlled perturbations into BFS flow at \( Re = 11000 \) and investigated instability modes of reattaching shear layer. Scarano et al. [53] studied large-scale coherent structures of turbulent BFS flow by applying pattern recognition analysis to PIV data. Schröder et al. [62] investigated transitional BFS flow with external acoustic excitation between \( Re = 1420 \) and \( Re = 3000 \) by 2D-2C- and high resolution 3D-3C-tomo- PIV and detected hairpin-like, streamwise elongated vortices on spanwise waves. Further studies came to instantaneous measurements of BFS flow fields based on PIV techniques [11, 24, 30, 31, 46, 70, 71]. Besides various experimental investigations, numerical simulations have also been widely carried out [8, 10, 33, 43, 50]. Another motivation of studying the backward-facing step flow comes from the presence of separated and reattached flows in many technical applications. This phenomenon usually results in undesired unsteadiness, vibrations and noise. For instance, a series of vortices shed from an afterbody of a launch vehicle and consequently hit nozzles behind, which endangers the whole system [38, 64, 65]. Moreover, flow separation in a joint between two coaches of a high-speed train as well as at a sunroof of a car results in extra drag and noise. Therefore, there is great demand of an economical and reliable solution of the problem not only in aerodynamic research but also in engineering applications.

Tomographic particle image velocimetry is a volumetric flow measurement technique that is able to obtain a three-component velocity vector fields within a three-dimensional measurement volume [49]. This 3D-3C-PIV technique was first developed by Elsinga et al. [16]. Being different from previous techniques, such as standard, stereoscopic PIV or PTV, tomographic PIV provides three velocity components in an Eulerian view as well as the 3D velocity gradient tensor, which makes it suitable for analyzing complex flows. With the fast development of 3D reconstruction [5, 16, 17, 21, 57, 74, 75], laser and high-speed recording techniques, tomo- PIV has been improved during the past decade towards a more robust system providing high spatial and temporal resolution. It has been applied in a wide range of turbulent flow cases, for instance, the investigations of turbulent wakes [13, 16, 23, 26, 42, 55, 56], jets [29, 40, 54, 59, 72] and boundary layer flows [6, 7, 12, 14, 15, 19, 60, 61, 66, 76]. Comprehensive reviews on the development of tomo- PIV have been contributed by Kitzhofer et al. [30], Scarano [52] and Gao et al. [20].

There are mainly four parts in the present paper. The first section presents a brief introduction and research background of vortex generators and backward-facing step flows as well as the recent developments of tomo- PIV technique. In the following section a detailed description of the experimental set-up is given. In the third section the measurement results are analyzed from different views, including Reynolds stresses, turbulent kinetic energy, spatial and temporal cross-correlation functions and POD method. Finally, the fourth section gives a conclusion of the performed work and a brief outlook for future study.

2. Experimental apparatus and procedure

2.1 Flow facility

The experiments were carried out in the 1-m wind tunnel at DLR in Göttingen, Germany, which is low-speed, closed circuit with an open test section and a contraction ratio of 4.8. The open test section has a 1050×700 mm² rectangular cross-section and is 1400 mm long (see Fig. 1). The free stream velocity was \( U_∞ = 10 \) m/s with a turbulence level of 0.15%. A BFS model was mounted horizontally on a flat glass plate with an elliptical leading edge and the flow along the 400 mm long plateau of the plate was close to a zero pressure gradient. The step height was \( h = 30 \) mm which was used to normalize the length scales in x-, y-
and z-axis. Therefore, the Reynolds number was equal to \( \text{Re}_h = 2.0 \times 10^4 \), based on the free stream velocity \( U_\infty \) and the step height \( h \). The BFS was 1300 mm wide resulting in an aspect ratio of the width to the height of 43.3 that was considered to be sufficiently large to provide two-dimensional flow in the measurement domain. The oncoming boundary layer was tripped at the leading edge by spanwise zigzag bands to generate a turbulent boundary layer whose thickness was approximately \( \delta \approx 15 \text{ mm} \) at the BFS. Therefore the ratio of the thickness of the boundary layer to the step height was \( \delta/h \approx 0.5 \).

![Experimental set-up of the backward-facing step in the 1-m wind tunnel](image1)

### 2.2 Vortex generators

Low-profile wedge-type vortex generators (VGs) have been designed and implemented on the flat plate upstream of the step. Generally a set of parameters of a wedge-type VG includes a non-dimensional height \( H/\delta \), a non-dimensional device length \( c/H \), an angle of incidence \( \beta \), a non-dimensional spanwise spacing \( \Delta Z_{VG}/H \), a non-dimensional streamwise distance between trailing edges and the step \( \Delta X_{VG}/H \). The first three parameters describe the device’s geometry and size, and the following two describe its orientation and location. In the present paper, the design of VGs has three characteristics for effective flow separation control:

1. Due to the symmetric shape the VGs are able to introduce counter-rotating vortices which are more effective in controlling 2D flow separation, whereas co-rotating vortices perform better in 3D flow separation [34].
2. The backward wedges enable the streamwise vortices to embed more closely to the wall than other types of VGs do, for instance vane-type VGs. These embedded vortices experience stronger influence of wall shear and consequently result in greater decay.
3. With the height of only a fraction of the thickness of the local turbulent boundary layer (\( H/\delta < 1 \)), these low-profile VGs are submerged within the inner part of the turbulent boundary layer and consequently generate flow perturbations in the region with higher wall-normal mean velocity gradient, which results in greater production of turbulent kinetic energy for flow separation control purpose. The detailed discussion is given in Section 3.1.

In the present paper the VGs have the parameters of \( H/\delta = 0.67 \), \( c/H = 10 \), \( \beta = 14.0^\circ \), \( \Delta Z_{VG}/H = 6 \), \( \Delta X_{VG}/H = 10 \), which are referred to previous experimental researches [3, 4]. Fig. 2 shows the mechanical drawing of the low-profile wedge-type VG. Three VGs were backward-oriented and transversely mounted upstream of the BFS covering a spanwise width of 170 mm (see Fig. 3(b)). The measurements were performed with and without the VGs, which corresponded to the controlled and clean cases respectively.

### 2.3 Measurement techniques

The flow fields downstream of the BFS were measured by time-resolved tomographic PIV and high-resolution 2D-2C- PIV separately. The measurement procedure included two steps. In the first step TR tomo-PIV was used to measure a three-dimensional volume downstream of the BFS and over the glass plate. In the second step 2D-2C- PIV was used to measure a two-dimensional streamwise-vertical plane that covered the turbulent boundary layer in the step region, the separated shear layer and the reattachment area.

![Mechanical drawing and model of the low-profile wedge-type vortex generator](image2)
2.3.1 Time-resolved tomographic PIV

Fig. 3 Time-resolved tomographic PIV set-up (a): sketch of the laser light beam and four high-speed cameras; (b): Zoomed-in on the measurement volume

Double-frame particle images were recorded at 1 kHz by four Photron high-speed CMOS cameras (1024×1024 pixels, 10 bits) with Nikon Micro-Nikkor 105 mm lenses at \( f_p = 8 \). The four cameras were mounted in a pyramidal configuration under the glass plate (see Fig. 3). A diode-pumped double-cavity Nd:YAG laser from LEE with a high-repetition rate at 1 kHz delivering energy of 17 mJ/pulse was used to illuminate a measurement volume of 50×60×10 mm\(^3\) whose center was located 45 mm downstream of the step and at 28 mm height over the plate. The laser light beam was aligned in spanwise direction and reflected back and forth between two mirrors on both sides of the flow in order to increase the light intensity within the volume and generate homogenous light scattering of particles for all camera viewings. The time delay between double pulses was \( \Delta t_{3D} = 60 \, \mu s \). The system was synchronized by TTL signals controlled by 16 channel sequencer from Hardsoft. The flow was homogeneously seeded by DEHS (Di-ethyl-hexyl-sebacate) droplets with a mean diameter of approximately \( d_p = 1 \, \mu m \).

The TR tomo- PIV evaluation was performed by a DLR own SMART algorithm for tomographic volume reconstruction and a 3D cross-correlation scheme of DaVis7.3. First, a mapping function corresponding to the relation between image planes and physical space was obtained by a 3D calibration procedure using a precision-machined twin level calibration target. Second, sparse particle image distributions of the four simultaneous camera images were used for a volume-self-calibration step in order to enhance the accuracy of the reconstruction step [74] and then an array of 865×996×228 voxels have been reconstructed by means of a SMART algorithm including the determination and application of the optical transfer function (OTF) as weights [57]. Third, the particle image volume was analyzed by direct three-dimensional cross-correlation with an iterative multi-grid volume deformation scheme. Finally, with a final 483 voxels interrogation box size at 75\% overlap an instantaneous three-dimensional velocity vector field was achieved which consisted of 69×80×16 (88320 in total) measurement points with a grid spacing of \( \Delta x_{3D} = \Delta y_{3D} = \Delta z_{3D} = 0.752 \, \text{mm} \approx h/40 \). The origin point of the three-dimensional coordinate system located at the bottom of the step. The \( x\)-, \( y\)- and \( z\)-axis corresponded to the streamwise, spanwise and vertical directions, respectively. In each of the clean and controlled cases, a time sequence of 3000 instantaneous snapshots at 1 kHz was obtained during a period of approximately 3 seconds.

2.3.2 2D-2C- PIV
In the 2D-2C PIV measurement the BFS flow was illuminated by a streamwise-vertical laser light sheet with a thickness of 1 mm from downstream direction (see Fig. 4). The laser light sheet was generated by a Big Sky Ultra CFR Nd:YAG laser system delivering energy of 30 mJ/pulse with the time delay of $\Delta t_{2D} = 150 \mu s$. The particle images were recorded by a high resolution camera pco.4000 camera (4000×1000 pixels, 14 bits) with a Nikon Nikkor 85 mm lens at $f_0 = 4$ mounted outside of the flow field with its optical axis perpendicular to the laser light sheet covering a field of view of $310 \times 70 \text{mm}^2$.

In the evaluation process, each of double-frame particle images were computed by a multi-grid cross-correlation algorithm with image deformation and a final interrogation window size of 16×16 pixels at 75%, resulting in a grid spacing of $\Delta x_{2D} = \Delta y_{2D} = 0.25 \text{mm} \div h/100$. A two-dimensional coordinate system had the same origin point as that of the three-dimensional coordinate, and its x-, y-axis corresponded to the streamwise and vertical directions, respectively. In both the clean and controlled cases, 1000 statistically independent instantaneous snapshots were obtained at a sampling rate of 1.4 Hz during a period of approximately 12 minutes. The detailed parameters of the two PIV systems are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Parameters of TR tomo- and 2D-2C- PIV systems</th>
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<tr>
<td><strong>Time-resolved tomo- PIV system</strong></td>
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<tr>
<td><strong>Recording medium</strong></td>
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<tr>
<td><strong>Recording method</strong></td>
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<td><strong>Resolution</strong></td>
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<td><strong>Evaluation algorithm</strong></td>
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<td><strong>Initial window/box size</strong></td>
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**Fig. 4** High-resolution 2D-2C- PIV set-up. (a): sketch of the streamwise-vertical laser light sheet and one high-resolution camera; (b): Zoomed-in on the laser light sheet
Final window/box size 48×48×48 voxels 16×16 pixels
Overlap 75% 75%
Vector spaces \( \Delta x_{3D} = \Delta y_{3D} = \Delta z_{3D} = 0.752 \text{ mm} \) \( \Delta x_{2D} = \Delta y_{2D} = 0.25 \text{ mm} \)

3 Results and analysis

In this section the measurement results are analyzed. There are mainly two objectives in the following analysis. One objective is that the effectiveness of the VGs as flow separation control devices has to be evaluated. The other one is to analyze coherent structures and their temporal evolution as carrier of important Reynolds stress events.

3.1 Instantaneous and time-averaged velocity vector fields

First of all the flow field of the clean case (without VGs) is presented to provide an overview of a turbulent BFS flow. Fig. 5 shows the characteristic flow structures that contains the turbulent boundary layer, separated shear layer, main and secondary recirculation regions and reattachment area. The reattachment length \( L \) is defined as the distance from the step to where the mean flow reattaches the downstream plate surface. In this flow control study, reduction of the reattachment length is a primary goal for flow separation control.

![Fig. 5 Sketch of turbulent backward-facing step flow](image)

After the turbulent boundary layer separates from the BFS, the velocity profile has an inflectional point causing vortices to be developed due to the Kelvin-Helmholtz instability. In the meantime, various vortex interactions of paring, tearing and breaking down are involved as well. Moreover, these above-mentioned vortices are superimposed on three-dimensional turbulence that makes the flow field more complex. Although the largest structure scales, such as the recirculation regions, are given by the BFS geometry, the smallest scales of vortices are determined by the Reynolds number. With increasing Reynolds numbers, the range of vortex scales increases and consequently even smaller scales vortices appear. Thus, the \( \lambda_2 \)-criterion [28] is a used to identify vortices in this region of high velocity gradient and high vorticity. The \( \lambda_2 \)-criterion depends on calculation of the discriminant of non-real eigenvalues of the velocity gradient tensor which consists of:

\[
\lambda_2 = \left[ \text{trace} \left( \frac{\partial U}{\partial X} \right) \right]^2 - 4 \times \text{det} \left( \frac{\partial U}{\partial X} \right), \text{with } \frac{\partial U}{\partial X} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}
\]

(1)

![Fig. 6 Instantaneous velocity vector field (top) and contour of signed-\( \lambda_2 \) (bottom) of the clean case. Vectors are color-coded by streamwise velocity \( u \).](image)
Negative values of $\lambda_2$ correspond to vortices in distinction to velocity gradient or vorticity. In order to distinct the directions of the vortices, “signed-$\lambda_2$” is introduced:

$$signed - \lambda_2 = sign(\omega_x) \times \lambda_2, \text{with } \omega_x = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$  \tag{2}

A 2D instantaneous velocity vector field and the contour of signed-$\lambda_2$ of the clean case are shown in Fig. 6. The view focuses in the separated shear layer and the near wake $0 < X/h < 5$. As above-defined, the red regions (signed-$\lambda_2 > 0$) indicate clockwise vortices while blue regions (signed-$\lambda_2 < 0$) indicate anti-clockwise ones. It can be identified that the quasi-periodic Kelvin-Helmholtz vortex structures roll up to from the step and then form a series of self-similar vortices, and finally breakdown due to strong turbulent mixing. As the vortices grow and move downstream, induced anti-clockwise vortices appear after $X/h > 1$ and merge with other vortex structures. Furthermore, 3D instantaneous flow vector fields present whole view of the separated shear layers in Fig. 7. By the comparison, more vortex structures are identified by iso-surfaces of $\lambda_2$ downstream of the VGs. However, there is hardly any visualized regularity of the vortex structures because the large-scale vortices tend towards cascading down to the small-scale ones due to the turbulence.

![Instantaneous flow field of clean case](image)

![Instantaneous flow field of controlled case](image)

**Fig. 7** Three-dimensional instantaneous velocity vector fields of the clean (left) and controlled (right) cases. Iso-surfaces of $\lambda_2 = -1 \times 10^6$ s$^{-2}$ are shown. The vectors and iso-surfaces are color-coded by streamwise velocity $u$.

Time-averaged flow fields of the clean and controlled cases are calculated by 1000 statistically independent 2D-2C- PIV snapshots. A comparison of the average velocity vector fields of the clean and controlled cases is shown in Fig. 8. In the clean case, the main recirculation region plays a dominant role and the secondary can also be identified near the step, which agrees with previous literature [2]. In contrast, in the controlled case the main recirculation region is clearly suppressed and its center position relocates upstream and lower from $X/h = 3.78$ and $Y/h = 0.44$ to $X/h = 3.11$ and $Y/h = 0.32$. Moreover, the secondary recirculation region disappears probably due to three-dimensional-induced spanwise flows that entrain neighboring flow into the separation region near the step. The reattachment areas are zoomed-in to order to show the near-wall velocity profiles more clearly. The normalized reattachment length in the clean case is equal to $L_0/h = 7.1$ ($L_0 = 213$ mm) while the one in the controlled case is equal to $L_c/h = 5.02$ ($L_c = 151.0$ mm). The reduction rate $R$ is defined as:

$$R = \frac{L_0 - L_c}{L_0} \times 100\%$$  \tag{3}

Therefore the reattachment length is reduced by 29.1% with the VGs.
Reynolds shear stresses are the most important quantities because they contribute to a major part of momentum transfer in turbulent shear layers. It represents the quantity of fluid particles which fluctuate upward and downward between the shear layers and results in additional stresses to the flow fields. These fluctuating velocities are time-varying quantities that are superimposed on time-averaged flow based on the Reynolds decomposition $U = \langle U \rangle + u'$. Reynolds stresses are components of the turbulent or Reynolds stress tensor, which is defined as [58]

$$
\begin{bmatrix}
\sigma_x' & \tau_{xy}' & \tau_{xz}' \\
\tau_{xy}' & \sigma_y' & \tau_{yz}' \\
\tau_{xz}' & \tau_{yz}' & \sigma_z'
\end{bmatrix} = -\rho \begin{bmatrix}
\langle u' u' \rangle & \langle u' v' \rangle & \langle u' w' \rangle \\
\langle u' v' \rangle & \langle v' v' \rangle & \langle v' w' \rangle \\
\langle u' w' \rangle & \langle v' w' \rangle & \langle w' w' \rangle
\end{bmatrix}
$$

(4)

where $\sigma'$ and $\tau'$ are normal and shear stresses, while $u', v'$ and $w'$ are the fluctuating components of the velocity, and $\langle \cdot \rangle$ denotes their time-averaged products. Fig. 9 shows the overall contours of Reynolds shear stresses $(u'w')$ (deviated by negative density $-\rho$) in the volume of the clean and controlled cases. It is revealed that when the VGs are applied, the Reynolds shear stress $(u'w')$ is increased in particular regions in red color, which have much higher values than the neighboring areas. This region has one period in the volume corresponding to the VG spanwise distribution, and on the other hand, the VG-induced streamwise vortices have considerable decay within 15 times of the VG height downstream due to the interference between the two counter-rotating vortices. This result agrees well with previous researches [3, 34].

**Fig. 8** Time-averaged velocity vector fields of the clean (left) and controlled (right) cases. The vectors are color-coded by streamwise velocity $u$. The reattachment areas are zoomed in for clarity.

**Fig. 9** Three-dimensional Reynolds shear stress $(u'w')$ of the clean (left) and controlled (right) cases.
Turbulent kinetic energy (TKE) is an important quantity in turbulent flows, which describes the energy transport within it:

\[
\frac{Dk}{Dt} + \nabla \cdot T' = P - \varepsilon
\]

(5)

\(k, T', P\) and \(\varepsilon\) correspond to TKE, transport, production and dissipation, respectively. On the right-hand side of Equation (5), the “sink” quantity \(\varepsilon\) is turbulent dissipation that transfers TKE to internal energy, whereas the “source” quantity \(P\) is responsible for the generation of TKE, which is given as

\[
P = -(u_i u_j) \frac{\partial (U_i)}{\partial x_j}
\]

(6)

In principle, there are two major factors that contribute to the generation of TKE: one is the Reynolds stresses \((u_i u_j)\) and the other one is the mean velocity gradient \(\frac{\partial (U_i)}{\partial x_j}\). In other words, the perturbations (Reynolds stresses) will be amplified by a higher mean velocity gradient or attenuated by a lower or zero gradient. Therefore the key point of the flow control in the BFS flow is to introduce perturbations into the regions with high velocity gradients in order to achieve the best effectiveness.

In the present work, the low-profile VGs are submerged within the turbulent boundary layer, which introduced streamwise vortices embedded near the wall. Fig. 10 shows the comparison of the TKE in clean (top) and controlled (bottom) cases. It is apparent that the TKE is increased downstream in the controlled case. The perturbations from the VGs are amplified by the high velocity gradient in the turbulent boundary layer and resulting in greater TKE values downstream in the separated shear layer.

![Fig. 10 Comparison of the turbulent kinetic energy of the clean (top) and controlled (bottom) cases](image_url)

**3.2 Spatial and temporal cross-correlations**

Spatial and temporal cross-correlation functions, first introduced by G. I. Taylor [67, 68], have been frequently used to reveal coherent quantities in turbulent flows and also to identify characteristic lengths of turbulent structures. The cross-correlation coefficient \(SC_{ij}\) for two-point spatial cross-correlation at the reference point \(\mathbf{r}_0 = (x_0, y_0, z_0)\) at the same measurement time \(t\) is defined as:

\[
SC_{ij}(\Delta \mathbf{r}) = \frac{\langle u_i' (\mathbf{r}_0, t) \cdot u_j' (\mathbf{r}_0 + \Delta \mathbf{r}, t) \rangle}{\sqrt{\langle u_i'^2 (\mathbf{r}_0, t) \rangle} \cdot \sqrt{\langle u_j'^2 (\mathbf{r}_0 + \Delta \mathbf{r}, t) \rangle}}
\]

(7)

Furthermore, the cross-correlation coefficient \(TC_{ij}\) for two-point temporal cross-correlation at the reference point with \(\mathbf{r}_0 = (x_0, y_0, z_0)\) and with time difference \(\delta t\) is defined as:

\[
TC_{ij}(\Delta \mathbf{r}, \delta t) = \frac{\langle u_i' (\mathbf{r}_0, t) \cdot u_j' (\mathbf{r}_0 + \Delta \mathbf{r}, t + \delta t) \rangle}{\sqrt{\langle u_i'^2 (\mathbf{r}_0, t) \rangle} \cdot \sqrt{\langle u_j'^2 (\mathbf{r}_0 + \Delta \mathbf{r}, t + \delta t) \rangle}}
\]

(8)

where \(u_i'\) is the fluctuation of velocity components, and \(\Delta \mathbf{r} = (\Delta x, \Delta y, \Delta z)\) is the displacement vector. In the following cross-correlation functions, one point at \((x_0, y_0, z_0) = (1.43, 0, 1)\) (normalized by the step height \(h\)) is chosen as reference point. The time difference between each temporal cross-correlated flow fields is \(\delta t = 1\)
ms. Then approximately 10 temporal evolutions are captured due to the length of the measurement volume and the convective velocity.

Fig. 11 shows the iso-surfaces of spatial cross-correlation of the velocity fluctuations in the clean (top row) and controlled (bottom row) cases. In the clean case, the ellipsoidal region of positive correlation (red) locates at the reference point which is equivalent to its autocorrelation. Two adjacent regions of negative correlation (blue) are aligned in spanwise directions for \( \langle u'u' \rangle \) and in streamwise directions for \( \langle w'w' \rangle \), whereas four adjacent regions surround for \( \langle v'v' \rangle \). Moreover, the cross-correlation regions of \( \langle v'v' \rangle \) are clearly inclined toward the free stream. Besides the qualitative descriptions of the vortex structures in Fig. 11, it is necessary to calculate the exact changes of the characteristic lengths of the turbulent structures between the clean and controlled cases. It is defined as “scale of turbulence” by G. I. Taylor [68, 58] to investigate one-dimensional pipe flow and to measure the extent of mass that moves as a unit with an average size of turbulent structures. In the present study, the one-dimensional integral is extended to a three-dimensional integral throughout the measurement volume. Therefore the characteristic volume \( V \) is given as:

\[
V = \sum_{\text{volume}} SC_{ij} \cdot \Delta V
\]

where \( \Delta V = \Delta x \cdot \Delta y \cdot \Delta z \) is the basic element volume with the unit \( \text{mm}^3 \). Then the characteristic length \( l \) can be derived by the volume of a sphere:

\[
V = \frac{4}{3} \pi l^3
\]

Table. 2 shows the results of the characteristic lengths in x-, y- and z-directions, notated by \( l_0(x, y, z) \), \( l(x, y, z) \). It is clear that all of the characteristic lengths are in the same order with small differences, which is mainly because of the same Reynolds number. In the clean case, the \( l_0(x) \) and \( l_0(y) \) are very close and \( l_0(z) \) is relatively smaller due to the vertical shear layer. In the controlled case, however, \( l_c(x) \) is mainly influenced by the spanwise spacing of the VGs, and \( l_c(y) \) and \( l_c(z) \) are reduced due to the strong perturbations of the streamwise vortices induced by the VGs. Therefore the VGs are able to influence the spanwise flow structures and reduce the characteristic lengths in streamwise and vertical directions.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Clean case / ( l_0 )</th>
<th>Controlled case / ( l_c )</th>
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<tr>
<td></td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>Characteristic volume / ( \text{mm}^3 )</td>
<td>320.9</td>
<td>325.3</td>
</tr>
<tr>
<td>Characteristic length / mm</td>
<td>4.25</td>
<td>4.27</td>
</tr>
<tr>
<td>Rate of change</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Fig. 12 and Fig. 13 shows the two-point temporal cross-correlation at a fixed upstream reference point with the time step \( \delta t = 1 \) ms. It reveals the temporal evolution of the flow structures in the clean (see Fig. 12) and controlled (see Fig. 13) cases. At \( t = 1 \) ms, the cross-correlation is equivalent to the spatial cross-correlation. As the flow structures travel downstream, the regions of positive correlation decrease gradually due to the mixing and decay of the vortices. Also the convective velocity \( U_c \approx 0.5 \cdot U_\infty \) can be identified which agrees well with the mean flow in the shear layer.
and Euclidean Third, and covariance which where the represented Thus, complex deeply separated The 3.3 clean of Fig. 12 Iso-surfaces of temporal cross-correlation TC_{ij} of \langle u'u' \rangle, \langle v'v' \rangle, \langle w'w' \rangle from t = 1, 2, ..., 5 ms of the clean case.

3.3 Proper Orthogonal Decomposition

The coherent structures play a predominant role, described as “preferred modes” by Hussain [27], in the separated shear layer, but they are totally superimposed on three-dimensional random fluctuations and deeply buried in fully developed turbulence. It is necessary to extract these dynamic flow structures out of the turbulent background. Proper Orthogonal Decomposition (POD) is a useful method to characterize such complex flows by linear decomposition and reconstruction based on Singular Value Decomposition [39, 9, 22, 45, 41]. All of the POD modes, which are orthogonal to each other, are ranked by their kinetic energy. Thus, if predominant large-scale structures exist in the shear layer, they can be extracted by POD and be represented in first few modes. In this paper, the mathematical background of POD is briefly presented. First, the fluctuating velocity components are arranged in a matrix U as:

\[
U = [u_1 \ u_2 \ \ldots \ u_N] = \\
\begin{bmatrix}
  u_1^1 & u_2^1 & \ldots & u_N^1 \\
  \vdots & \vdots & \ddots & \vdots \\
  u_1^M & u_2^M & \ldots & u_N^M \\
  v_1^1 & v_2^1 & \ldots & v_N^1 \\
  \vdots & \vdots & \ddots & \vdots \\
  v_1^M & v_2^M & \ldots & v_N^M \\
  w_1^1 & w_2^1 & \ldots & w_N^1 \\
  \vdots & \vdots & \ddots & \vdots \\
  w_1^M & w_2^M & \ldots & w_N^M \\
\end{bmatrix}_{M \times N}
\]

where M = 50400 is the number of spatial discrete points and N = 3000 is the number of the snapshots, which represent the spatial and temporal resolutions of the PIV data respectively. The subscript outside of the matrix [\cdot]_{M \times N} indicates the dimensionality of the matrix. Second, the eigenvalues of the autocovariance matrix are calculated as:

\[
\tilde{C} \cdot A_i = \lambda_i \cdot A_i, \text{ with } \tilde{C} = U^T \cdot U
\]

and ranked in a descending order as:

\[
\lambda_1 > \lambda_2 > \ldots > \lambda_N = 0
\]

Third, each mode POD_i is obtained by projecting matrix U onto each eigenvector and then normalized by its Euclidean norm (or 2-norm) as:

\[
POD_i \equiv \phi_i = \frac{\sum_{n=1}^{N} (A_i^T \cdot u_n)}{\left\| \sum_{n=1}^{N} (A_i^T \cdot u_n) \right\|}, i = 1, 2, \ldots, N
\]

and

---

Fig. 13 Iso-surfaces of temporal cross-correlation TC_{ij} of \langle u'u' \rangle, \langle v'v' \rangle, \langle w'w' \rangle from t = 1, 2, ..., 5 ms of the controlled case.
\[ \Phi = [\phi_1 \phi_2 \cdots \phi_N]_{3M \times N} \]

then the coefficients of the modes can be obtained:
\[ a_n = \Phi^T \cdot u_n \]

Last, a snapshot can be reconstructed as:
\[ \hat{u}_n = \sum_{i=1}^{N} a_i \cdot \phi_i = \Phi \cdot a_n \]

It can be proven that each eigenvalue is proportional to its kinetic energy of velocity fluctuations. Thus, the descending rank of the eigenvalues makes sure that the most important modes containing higher energy are in the first few modes, which correspond to large-scale flow structures, whereas other further modes containing lower energy are less dominant [18].

In this part, POD is applied to the three-dimensional flow fields. Fig. 14 shows the comparison of the POD eigenvalues in the clean and controlled cases, which correspond to the energy distributions. It can be seen that the first few modes contain the most part of fluctuation energy and the further modes decay logarithmically. There are more energetic fluctuating motions in the controlled case than those in the clean case, and especially the first two modes are approximately on the same order of energy.

![Fig. 14 POD eigenvalue distributions. Only the first 1000 eigenvalues are plotted for clarity.](image)

In order to analyze the interrelation between the first two modes POD\(_1\) and POD\(_2\) (marked by an ellipse in Fig. 14) the corresponding coefficients \(a_1\) and \(a_2\) are plotted in Fig. 15 (left column). The coefficients are determined by projecting the original fluctuating velocity vector fields onto each POD mode, which present the temporal evolution of the normalized modes. It is shown in Fig. 15 (bottom left) that the coefficients \(a_1\) and \(a_2\) of the controlled case have similar frequencies and approximately fixed phase difference, which indicates the POD\(_1\) and POD\(_2\) are highly coherent. On the other hand, those in the clean case are less similar. Here “coherence” is a statistical quantity used to examine the relation between modes POD\(_1\) and POD\(_2\), which is defined in signal processing as:
\[ C_{12} = \frac{|P_{12}|^2}{P_{11} \cdot P_{22}} \]

where \(P_{11}\) and \(P_{22}\) are the auto-spectral density of coefficients \(a_1\) and \(a_2\) respectively, while \(P_{12}\) is the cross-spectral density of them. The resulting coherence \(C_{12}\) is positive and satisfies \(0 \leq C_{12} \leq 1\). The auto- and cross-spectral densities are estimated by Welch’s method and the resulting spectral coherence are shown in Fig. 15. It is revealed that the modes POD\(_1\) and POD\(_2\) of the controlled case are highly coherent in the particular frequency bandwidth approximately on the order of 100 Hz (marked by an ellipse in Fig. 15), outside of which are less coherent motions or random turbulence.
Fig. 15 Temporal evolutions and coherence of the coefficients of $a_1$ and $a_2$ of the clean (top, left) and controlled (bottom, left) cases.

To visualize the temporal evolution of the coherent structures the flow fields of the controlled case are reconstructed by the modes POD$_1$ and POD$_2$. Thus, the reconstructed flow fields are the visualization of the coherent structures educed by VGs in the separated shear layer in Fig. 16. It is shown that there is approximately a half period of a traveling wave in the volume, which agrees well with the above-discussed spatial and temporal cross-correlation results. Furthermore, it can be estimated that the frequency bandwidth of the coherent structure is on the order of 100 Hz.
Fig. 16 POD reconstructed coherent structures downstream of the VGs. The flow fields are reconstructed by POD$_1$ and POD$_2$ at $t = 1, 2, ..., 9$ ms in the time sequence. The contours are color-coded by the streamwise vorticity $\omega_x$.

4 Conclusions and outlook

The low-profile wedge-type vortex generators have been applied in the backward-facing step flow for separation control at the Reynolds number $Re_h = 2 \times 10^4$ based on the free stream velocity $U_{\infty} = 10$ m/s and the step height $h = 30$ mm. Counter-rotating, embedded and submerged vortices have been generated in the inner part of the turbulent boundary layer with high mean velocity gradient. The complex separated shear layer flow downstream has been investigated by time-resolved tomographic PIV and high resolution 2D-2C-PIV techniques. The TR tomb-PIV results give valuable insights into the complex vortex structures as well as their temporal evolution within the separated shear layer, while the high-resolution 2D-2C-PIV provides a whole view of the time-averaged flow fields and the turbulent kinetic energy. Spatial and temporal cross-correlation and POD methods are applied to analyze coherent structures and their characteristic features. Three main points can be concluded:

1. As passive flow control devices, the low-profile wedge-type vortex generators are able to attenuate the flow separation downstream of the backward-facing step and reduce the reattachment length by 29.1%. The main recirculation region is suppressed and the secondary one is replaced most probably due to three-dimensional injection.

2. The turbulent kinetic energy is increased downstream in the controlled case because the perturbations from the VGs are amplified by the higher velocity gradient in the turbulent boundary layer and the separated shear layer. Therefore the key point of the flow control in the BFS flow is to introduce perturbations into the regions with the high velocity gradient to achieve the best effectiveness.

3. The streamwise vortices generated by the VGs can enhance the turbulent mixing and energy transfer from the mean flow to the fluctuation motions by reducing the scales of turbulence in slantwise and vertical directions.
4. The coherent structures, detected by the cross-correlation and POD, act as traveling waves in the streamwise direction and are highly coherent in the frequency bandwidth from 100 Hz to 200 Hz. Furthermore, the coherent structures entrain high momentum fluid from the mean flow into the separated shear layer and consequently enhance the momentum transfer, which are effective for the flow separation control.

Although the main coherent structures have been extracted from the flow field, more vortex formation and breakdown buried in the turbulent background are still unknown. In future studies, time-resolved high-resolution measurements might be used to reveal the complete flow topology as well as the temporal development. It might help to improve the understanding of the mechanism of turbulence.

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