Volumetric Vector Velocity Measurements in a Hot Supersonic Jet

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Abstract Cross-Correlation Doppler Global Velocimetry (CC-DGV) is a new laser diagnostic technique for high spatial resolution vector velocity measurements. A full theoretical description of the instrument is presented, including post-processing routines for data quality enhancements and selection of valid data points. An uncertainty analysis combining experimental observations and numerical Monte Carlo simulations is also included. The method is found to be insensitive to vapor absorption variations, relaxing requirements to match vapor limited gas cells or even control their temperature. The CC-DGV instrument is used to measure mean velocities in the Virginia Tech hot supersonic jet facility. The facility as used in this work is a 38.1 mm (1.5") nozzle exit diameter free shear jet, with exit Mach number of 1.65 and total temperature ratio of 1.0 and 2.0. Spatially resolved three-component mean velocity vectors are measured within a volume in the jet at cold, over-expanded conditions (nozzle pressure ratio = 2.7) via several closely spaced planar measurements. Stream-wise planar measurements are also presented for heated, over-expanded conditions. Resulting 2D and 3D velocity contours allow for detection of flow features including shock cells and shear layers, as well as the downstream evolution of the flow.

1. Introduction

Large volumetric mean velocity measurements are important for characterization of complex flow fields (Westerweel et al. 2013). Many optical instruments are well suited for unsteady measurements, such as point measurements by Laser Doppler Velocimetry (LDV) (Yeh & Cummins 1964), planar and holographic Particle Image Velocimetry (PIV) (Sheng et al. 2008), (Willert 1997), and time-resolved Doppler Global Velocimetry (DGV) (Fischer et al. 2007). To obtain a fully spatially-resolved mean velocity map of the flow would require either a large number of test points for a high acquisition rate method such as LDV, or very large ergodic image sets for unsteady measurements, requiring great care for consistent seeding during long acquisitions, and great computational expense to process and then distill data down to means.

To address fundamental sources of uncertainty, a new DGV variant referred to as cross-correlation DGV (CC-DGV) is presented wherein the fundamental measurand is sensed via time-domain differences rather than intensity measurements (Cadel et al. 2014). The CC-DGV instrument provides for rapid volumetric mean velocity vectors.

The Virginia Tech hot jet facility was recently built in support of various research thrusts related to propulsion, aeroacoustics, and fundamental aerodynamics (Brooks et al. 2014, Ecker et al. 2014). The facility features interchangeable nozzles and a wide range of total temperature and pressure ratios. Mean velocity maps are important for this type of facility to complement point measurements and define flow-field development. The value of the mean velocity maps is not trivial and obtaining them has been time consuming, done either using point probes (optical or pressure) or with PIV. In the Virginia Tech facility, high frequency LDV and point-DGV are currently used for this mapping, albeit with qualitative Schlieren images or self-referencing for global flow-field context. CC-DGV offers a quantitative way to locate features such as shocks and shear layers while also providing a baseline velocity for validation of unsteady point measurements. Further, although not presented herein, (Papamoschou & Roshko 1988) and (Freund et al. 2000) have shown that the shear layer spreading rates obtained from mean velocity measurements are well-correlated to both mean and radial turbulence Mach numbers, providing a means for exploratory examination of eddy convection velocities and radial turbulence levels, informing decisions for conditions to obtain detailed measurements.
2. Instrument Background

Doppler global velocimetry (DGV) is an optical technique that relies on the Doppler shift of scattered light as filtered by a molecular vapor cell. A flow of interest is seeded with particles, and illuminated by single frequency laser light. Mie-scattered light from the seed particles experiences a frequency shift according to the Doppler shift equation

$$\Delta \nu = \frac{(\hat{\delta} - \hat{i}) \cdot V}{\lambda}$$

where $\Delta \nu$ is the Doppler shift frequency, $\hat{\delta}$ is the scattered light direction, $\hat{i}$ is the incident laser illumination direction, $V$ is the velocity vector in the global frame, and $\lambda$ is the incident light wavelength. The scattered and incident light directions depend on the geometry of the system, and the incident light wavelength depends on the laser source; the Doppler frequency shift is the direct measurand of the system.

(Meyers 1995) provides a detailed description of the fundamental and practical operating principles of DGV, the reader is referred there for further details. To determine the Doppler frequency shift, molecular vapor cells are used to selectively filter the scattered light intensity as a function of absolute light frequency. The absorption spectra of various gases are well known and found to be a function of temperature, pressure, and cell length only (Forkey et al. 1997). Typically, DGV requires knowledge of the absorption spectrum and the frequency of the incident non-Doppler shifted light. Thus, the relative intensity of the scattered light after passing through the vapor cell will indicate Doppler shifted frequency. To this end, a vapor cell is placed in the scattered light path ahead of each receiving camera.

Many systems employ a secondary light path for each receiving camera in order to obtain a unique reference image for each filtered image. Scattered light is evenly split into each of the two paths, one of which contains a vapor cell, and one that does not. The ratio of the filtered to non-filtered light at each instant of time is used to obtain the light intensity ratio (Meyers 1995). Mean velocity measurements in a laminar jet using long camera exposure times with a continuous wave laser have also been demonstrated by (Jenkins & McKenzie 2009) using this approach, however the authors note that the camera exposure time can be reduced with increased laser power. Another method for obtaining unique reference images, known as 2ν-DGV (“two-frequency DGV”), eliminates the secondary light path by flashing the incident laser light between a frequency well within a full transmission region of the absorption spectrum and a low-transmission region. These image pairs are then used to measure the absorption ratios and thereby the frequency shift (Charrett et al. 2004).

Frequency-modulated DGV (FM-DGV) is a spatially, temporally resolved technique of increased development in recent years (Fischer et al. 2011, 2007). FM-DGV operates by sweeping laser frequency over an absorption line of the vapor cell spectrum, and computing spectral harmonics of the measured intensity signal; Doppler frequency shift is determined by ratios of resulting harmonics, removing dependency on incident and scattered light intensity.

For any DGV method, the resulting velocity value from each detector is the scalar component of velocity in the direction indicated by the $\delta - i$ vector directions. To obtain 3-component vectors, three linearly independent measurements must be made and transformed into the global frame according to the transformation equation (2):

$$\vec{V} = \mathbb{D}^{-1} \hat{\vec{C}}$$

where $\mathbb{D}$ is given by
\[
\mathbf{D} = \begin{bmatrix}
(\hat{\mathbf{v}} - \hat{\mathbf{b}})_1 \\
(\hat{\mathbf{v}} - \hat{\mathbf{b}})_2 \\
(\hat{\mathbf{v}} - \hat{\mathbf{b}})_3 
\end{bmatrix}
\]

and \( \hat{\mathbf{v}} \) is a vector of velocity in the native camera directions. The three measurements may come from any combination of independent \( \hat{\mathbf{v}}-\hat{\mathbf{b}} \) directions; this often is designed as three imaging directions of one laser sheet, however other arrangements are possible as is the case in this work.

Cross-correlation DGV (CC-DGV) was developed by (Cadel et al. 2014) for spatially resolved mean velocity measurements. The system was initially referred to as cross-correlation scan DGV (CCS-DGV).

As with all other DGV methods, CC-DGV relies on the selective absorption properties of iodine vapor at specific incident light frequencies. A crucial difference with CC-DGV is that the Doppler shift for a large portion of the light spectrum is monitored and used to collectively determine the mean velocity at each spatial point.

The paradigm shift in CC-DGV comes in acquisition and processing of data with time as the independent parameter, in contrast to more conventional intensity-based DGV measurements (although similar to the FM-DGV technique). An overview of the system is shown in Figure 1. Intensity values are still directly measured; however, it is the time-based differences between corresponding signals that are used to determine Doppler frequency shift. The incident light frequency must be controllably swept during the data acquisition to obtain a spectral intensity variation. This is done by changing the applied voltage on a piezoelectric element within the laser head, which minutely changes the cavity length. Using this method, it is possible to sweep up to 7 GHz about the 532nm design condition, corresponding to measurements of up to hypersonic regimes. The primitive value, applied voltage on the laser cavity, is controlled linearly in time, making time the independent variable.

![Fig. 1 Cross-correlation DGV overview: Laser light frequency is scanned and the intensity as filtered through an iodine vapor absorption cell is monitored by both a reference photodiode in quiescent conditions and by a camera detector receiving Doppler shifted, Mie-scattered light from seed particles in a flow. The signals are cross-correlated to determine the Doppler frequency shift.](image)

A small portion of the laser (\(~1\%\), further attenuated with a neutral density filter) is separated by a polarizing beam splitter and measured with a photodiode for the non-Doppler shifted reference signal. The reference signal is recorded by a photodiode outside of the flow, along with three iodine cell / camera detector pairs providing the three components of the flow. The signal detectors used here are 5.5 megapixel sCMOS pco.edge scientific cameras, fitted with 532nm narrow bandpass filters.

The processing technique is shown schematically in Figure 2. For each pixel of the camera, a time-history of filtered scattered light intensity is extracted (Figure 2a). The Doppler-shifted signal is then cross-correlated with the reference photodiode time-history signal (Figure 2b). The Doppler shifted pixel signal will have the same shape as the reference; however, corresponding intensity levels will have occurred at different times.
within the frequency scan; cross-correlation of the two signals thus yields the frequency shift between them. The peak value of the correlation coefficient occurs at the shift value corresponding to the statistically strongest correlation for the entire signal. The cross-correlation function is given as:

\[
R(\eta_i) = \frac{1}{\sigma_{S_1} \sigma_{S_2}} \sum_{j=1}^{N} S_1(t_j)S_2(\nu_j - \eta_i)
\]  

(4)

where \( R(\eta_i) \) is the correlation coefficient at a phase shift \( \eta \) between the signals, at phase shift query point \( i \). \( S_1 \) and \( S_2 \) are the mean subtracted Doppler-shifted signal and non-Doppler shifted reference signals, respectively; \( \sigma_{S_1} \) and \( \sigma_{S_2} \) are the corresponding standard deviations of these signals. \( \nu_j \) is the frequency at point \( j \) in the acquisition scan for every data point in the signal, where \( N \) points are taken in total.

The signals are zero padded at the start and end of a scan (regions of full transmission) so as to allow a greater frequency range for the cross-correlation. A peak-finding algorithm is employed to improve the precision of the cross-correlation, such that shifts in smaller increments than the discrete scan frequency steps can be determined (Figure 2c). A parabola is fit around the peak of the correlation function as given by

\[
\hat{R}(\eta) = P_1 \eta^2 + P_2 \eta + P_3
\]  

(5)

where \( P_i \) are the polynomial coefficients of the fit curve. The interpolated peak location, \( \eta_{R_{\text{max}}} \) is then determined as

\[
\eta_{R_{\text{max}}} = -\frac{P_2}{2P_1}
\]  

(6)

A frequency mapping is determined by comparison with the theoretical absorption spectrum of the vapor cell. For a linear scan, since the absorption line center frequency does not change with cell conditions, the same mapping coefficient can be used for both the Doppler shifted and reference signals.

Fig. 2 Doppler shifted and non-Doppler shifted reference signals exhibit a Doppler frequency shift during a frequency scan (a). The signals are cross-correlated to determine the shift between them (b). A parabolic peak finding algorithm is employed to improve the precision of the method (c).
The cross-correlation algorithm is insensitive to absolute intensities as well as the relative intensity between the reference photodiode and camera, and thus is insensitive to common sources of uncertainty, such as dirty optics and spatial variations in scattered light intensity. A good review of error sources in DGV measurements is given by Meyers (Meyers et al. 2001), along with the ways they are typically handled. In CC-DGV, signal noise and speckle noise only correlate at the zero-shift (no flow) condition, and can thus be filtered out by ignoring the zero-shift correlation. Further, only the shapes of the absorption spectra from the signal and reference are needed for the cross-correlation. This allows for different types of sensors to be used for the Doppler-shifted flow signal and the non-Doppler shifted reference signal. Dependence on vapor cell temperature, which affects the depth of the absorption lines, is therefore dropped, although using vapor limited absorption cells can also greatly reduce this effect as well (Elliott & Beutner 1999).

3. Measurement Configuration

The Virginia Tech hot jet facility (Figure 3) is capable of providing continuous supersonic flow at total temperatures ratios up to three times ambient temperature (total temperature ratio $TTR = T_d/T_a$) at 0.25 kg/s mass flow rate for both over- and under-expanded conditions. Flow heating is obtained by a Sylvania 192kW Flanged Inline Heater (Model 073153). The facility is operated from a high-pressure tank supplied by an Ingersoll-Rand Type 4-HHE-4 4-stage reciprocating air compressor driven by a 500 hp, 480V Marathon Electric Co. motor. The plenum features three 20-mesh stainless steel screens sandwiched between high temperature gaskets and a 50.8 mm thick honeycomb with a cell length/width ratio of 8. The flow is seeded with 0.3 µm nominal diameter alumina (Al$_2$O$_3$) particles using a fluidized bed seeder with a high impact vibrator attachment, and introduced through a plenum port. The biconic converging/diverging 38.1mm (1.5”) exit diameter nozzle used for this study yields a design exit Mach number of $M_D = 1.65$, design nozzle pressure ratio, $P_{D}/P_a = NPR_D = 4.58$, and allows a maximum TTR of about 2.5, due to heater capacity constraints. Coordinate system, nozzle schematic and measurement ranges are depicted in Figure 4. The jet was run at a total temperature ratios $(T_d/T_a=TTR)$ of 1 and 2. The complete conditions for the cold and hot runs are presented in Table 1.

![Fig. 3 The Virginia Tech hot supersonic jet facility. A modified configuration including a flow conditioning section just upstream of the nozzle and a larger 1.5” exit diameter nozzle were used in this work. Figure from (Ecker et al. 2014). Reprinted with permission of the author.](image-url)
To achieve linearly independent velocity components, two cameras and two laser sheets are used, resulting in four measured components of velocity, of which one is redundant (Figure 5). Two pco.edge sCMOS cameras with 532nm narrow bandpass filters are aligned with optical axes perpendicular to the flow axis, thereby imaging planes parallel to the jet axis. Laser illumination is supplied by two equal power laser sheets in the same plane as the camera fields of view. Both laser sheet sending optics are located several diameters below the jet exit, one upstream and one downstream of the measurement plane. The cameras and laser sending optics are fixed to a 2-axis traverse system in the streamwise and transverse radial directions. At each station, data are obtained from each of two laser sheets sequentially such that only one sheet is emitting at any time.

A calibration grid target is imaged in no-flow and uniform illumination conditions. From this, the image can be spatially mapped. To convert frequency shifts to velocities, precise knowledge of the direction vectors is also needed. The centerline vector of each camera and laser is measured relative to a known feature on a calibration target. The incident laser light vectors \( \hat{\mathbf{i}}_1 \) and \( \hat{\mathbf{i}}_2 \) were measured as \([-0.758, -0.653, 0.000]\) and \([0.924, -0.382, 0.000]\) respectively; the scattered light observation vectors \( \hat{\mathbf{o}}_1 \) and \( \hat{\mathbf{o}}_2 \) were measured as \([0.004, 0.013, 1.000]\) and \([-0.001, 0.001, -1.000]\), respectively. An offset angle from the centerline direction for each pixel location is then found for all cameras and lasers. The incident laser light direction varies spatially since the laser sheet is diverging.

The Coherent, Inc. Verdi V6 diode-pumped solid-state 532nm laser is placed outside of the test cell in order to reduce the effects of acoustics-induced vibration on the laser stability. Fiber optics cables are used to route laser light into the test cell. The no-flow reference photodiode (ThorLabs, Inc. PDA100A) is also placed outside the test cell so as to isolate it.

Where possible, reflective surfaces in the background were masked with black matte tape. There remained however locations in the images subjected to significant background light, such as the interface of the aluminum housing of the vapor cell with the vapor cell glass; this was seen as an out-of-focus artifact in the opposing camera. Even with correction for background effects, the signal quality of these regions was insufficient for validation and removed from the presented data set. Further work is needed to implement an effective background subtraction method.

### Table 1. Jet conditions.

<table>
<thead>
<tr>
<th></th>
<th>Cold conditions</th>
<th>Hot conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_j )</td>
<td>1.27</td>
<td>1.27</td>
</tr>
<tr>
<td>( U_j )</td>
<td>379 m/s</td>
<td>532 m/s</td>
</tr>
<tr>
<td>( Re_D )</td>
<td>1.5 M</td>
<td>780k</td>
</tr>
<tr>
<td>( T_j )</td>
<td>222 K</td>
<td>435 K</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>257 kPa</td>
<td>575 K</td>
</tr>
<tr>
<td>( P_a )</td>
<td>94.6 kPa</td>
<td>94.6 kPa</td>
</tr>
</tbody>
</table>

**Fig. 4** Diagram of the nozzle geometry, coordinate system and measurement ranges.
Modulation depth was used as a final discriminator for valid data points. Modulation depth is defined as ratio of the minimum intensity value of a pixel time history signal compared to the full transmission intensity value. This full transmission value is determined by taking an average of 10 points, 5 from each end of the signal. Low values of the modulation depth signify that light is being filtered on the same order as the reference signal, whereas high values of the modulation depth imply the presence of large amounts of non-Doppler shifted background light. After camera spatial registration is performed but prior to the final application of equation (2), a threshold value of modulation depth is applied. For the cold flow, locations where the value is above 0.8 for any of the contributing cameras are removed from consideration. With lower seeding densities in the hot flow, validation criteria for modulation depth was set at 0.8 for regions known to have low background light and 0.5 for those regions with greater background. This scheme enables more valid data to be recovered in the shear layers while ensuring valid results for values presented in the core.

4. Uncertainty Analysis

In order to determine the sensitivities of the CC-DGV instrument to various test parameters and environmental conditions, a number of Monte Carlo simulations were performed. The simulations are specifically to study the effect of iodine cell related processing errors on the frequency shift term on the left hand side of equation (1). Velocities are reported instead of frequency shift for ease of interpretation by means of multiplication with the incident light wavelength $\lambda$. These velocities are to be understood as the component of the global velocity vector along the ideal $\hat{a}$ direction of an optimally positioned detector. Portions of the description of the synthetic signal generation are heavily excerpted from (Cadel et al.).

Photon arrival is modeled based on the quantum efficiency (QE) of the detectors. The pco.edge cameras used in this work have QE = 57%; the QE of the reference photodiode is not known, so a matching value of 57% was used. A random vector with uniform distribution was created of length N, where N is a function of the vapor absorption spectrum, simulating the scattering photons from seed particles at a pixel location over the camera integration time. The filtering effect of the vapor absorption cells was applied to the number of scattered photons; the N value at full transmission was proscribed, and then modulated to a minimum value corresponding to the minimum transmission ratio for a given absorption spectrum. The number of photons within the QE threshold was summed, and the process repeated for each data point in a signal acquisition.
An evenly spaced logarithmic range of N full-transmission photon arrivals was studied, in order to assess the SNR response of the signal processing routine. SNR was computed for each iteration of the simulations; the SNR displayed in the results shown are averaged over the number of runs for a given simulation condition. SNR, expressed in decibels, is defined using the mean signal and the root-mean-square background levels.

Absorption spectra used to attenuate the photon arrival signals were created using a program developed by (Forkey et al. 1997) that generates iodine transmission spectrum based on temperature, pressure, and length of the cell, along with a specified incident laser frequency range. The non-Doppler shifted reference is taken directly from the output of the Forkey program, while the Doppler shifted signals are created synthetically, which is to say, shifted by a known value of Δη relative to the reference signal according to equation (1). To create this shift, a large bandwidth of spectrum is generated, and then cropped down based on the simulation conditions. A velocity is proscribed, and the Doppler frequency shift calculated a priori. The spectrum generation program output is cropped based on these shifted bounds relative to the non-shifted reference signal. In this way, there is no “cycling” of the signal, and high shifts can be simulated while maintaining dependence to the theoretical spectrum.

Three simulations were performed to study the effects of iodine vapor cell temperature, absolute velocity scale, and incremental delays between discretely sampled delay times. A new uniform-distribution random vector of length N was generated and applied for each iteration. For each study presented, 101 frequency scan points were used, matching that of the experimental data in this work. The portion of the spectrum selected for the first two studies also matches that used in this work, centered at 563.244 THz. 5,000 independent iterations of each case were performed within each study.

The first simulation studies the effect of changing iodine cell sidearm temperature. This effect is muted in systems utilizing “starved” cells (Elliott & Beutner 1999); however, such cells were not available in the current measurements and the issue must be addressed. The temperature of the reference detector cell is kept at 20°C for all simulations. Nine signal detector cell temperatures are simulated, ranging from 0°C to 40°C. The transmission spectra at the temperatures used are shown in Figure 6. The length of both cells is kept constant at 5cm, and the cell pressure is determined as a function of the temperature. An evenly spaced logarithmic range of nine values of N photon arrivals from 100 to 359381 was studied. Results for the mean error normalized by the standard deviation for each condition are shown in Figure 7. Camera component velocity was set at 50 ms⁻¹. The results show that CC-DGV achieves equivalent insensitivities to starved cells without the use of temperature regulation. At very low temperatures, the amount of light absorbed by the vapor cells is of the same magnitude as random variations from the photon arrival noise, thus accounting for the increased standard deviation at the lowest temperatures studied.

To study the scalability of the technique, a simulation was performed keeping conditions equal between the Doppler shifted and reference cells, but varying the magnitude of the frequency shift. The same values of N photon arrivals was used as in the results of Fig. 6, with the addition of a value of N = 1000000. In a supersonic free shear jet as measured in this work, flow regimes ranging from supersonic down to low subsonic are all to be expected. Each pixel in the camera field of view is analyzed independently, so high spatial velocity gradients can easily be handled. Camera component velocities of 10 ms⁻¹, 100 ms⁻¹, and 500 ms⁻¹ were studied. Results are shown in Figure 8.

Finally, to test to the applicability and accuracy of the parabolic peak-finding algorithm, a simulation was run at a constant N=10000 for a range of camera component velocities in between those corresponding to integer values of the discrete scan points. As with the previous simulations, 101 discrete scan points were used to create the signals. Note that in all of the above simulation cases for temperature and velocity scale effects, the parabolic peak-finding algorithm was used. Results for the sub-integer shift study are shown in Figure 9 as the normalized bias error in velocity compared to the normalized distance between discrete scan points. The absorption line used was located at 563.229 THz, and 101 query points between two discrete scan points were studied.
Fig. 6 Variation of iodine vapor cell transmission with cell temperature. The red curve at 20˚C was used for the reference cell for all iterations in Fig. 6, and for both cells in Fig. 7.

Fig. 7 Effect of temperature mismatch between reference signals and Doppler shifted signals as a function of signal to noise ratio. The red curve with markers at 20˚C corresponds to the condition wherein the cells are at matched temperature. Camera component velocity of 50 ms⁻¹ in the $\delta$-$\iota$ vector direction was used for all Doppler-shifted signals.
Fig. 8 Scalability of the CC-DGV processing routine for different flow velocities, as a function of signal to noise ratio. At higher velocities, the relative error decreases for measurement of the camera component direction velocity. Velocities at all three ranges shown are expected in the flow field of the supersonic jet used in this work.

Fig. 9 Normalized bias error associated with the parabolic peak-finding algorithm. The relative shift refers to the position of the interpolated peak between two frequency scan points.

Beyond the uncertainty in the primary detection of velocities in the measurement directions, the geometry of the system plays a significant role in the uncertainty. Measurement of the $\hat{\phi}$ and $\hat{i}$ direction vectors described by the Doppler shift equation (1) play directly into the final measurement components as related through the rotation matrix $\mathbb{D}$ in equation (2). The measurement variance magnitude is given as

$$
\delta U^2 = \sum_i \left( \frac{\partial U}{\partial x_i} \delta x_i \right)^2
$$

(7)
where $U$ is the global velocity vector and $x_i$ are the individual parameters affecting the final result. Substituting relevant parameters for the current system as described in equation (3) and accounting for individual uncertainties of each input parameter yields a final form for the three-component case of

$$\begin{bmatrix}
\delta u^2 \\
\delta v^2 \\
\delta w^2
\end{bmatrix} = \begin{bmatrix}
\delta u_1^2 \\
\delta v_1^2 \\
\delta w_1^2
\end{bmatrix} = \begin{bmatrix}
\delta c_1^2 \\
\delta c_2^2 \\
\delta c_3^2
\end{bmatrix}$$

(8)

where the delta operator $\delta$ represents the uncertainty in each fundamental parameter. $c$ is the velocity measured by each camera in the $(\delta-i)$ direction, and $\mathbb{D}$ is a matrix containing the vector differences in the observation and incident light vectors for each component, given as

$$\mathbb{D} = \begin{bmatrix}
(\delta_1 - \bar{t}_1)_x \\
(\delta_1 - \bar{t}_1)_y \\
(\delta_1 - \bar{t}_1)_z
\end{bmatrix}$$

(9)

The square power in equation (8) is applied to each matrix element individually after the inverse matrix is computed. As the system was installed, uncertainties in the position vectors were recorded for each measurement and applied as perturbations to the normalized vectors. $\bar{t}$ was taken by observation as the uncertainty for the camera vectors components in x and z, and $0.5^\circ$ in the y component. $0.5^\circ$ was applied for each component of the incident laser light vector.

Camera-specific velocity magnitudes were estimated based on the results of the simulation studies above. The mean error value for camera component velocity using $N=10000$ and $5^\circ$ mismatched temperatures ($T_{\text{camera}} < T_{\text{ref}}$) is determined for both subsonic and supersonic flow regimes. Final uncertainty estimates are shown in Table 1. The $U_c$ values given in Table 2 are for free stream velocity vectors entirely in the $u$-velocity (stream-wise) component.

### Table 2 Uncertainties in velocity components in subsonic and supersonic regimes.

<table>
<thead>
<tr>
<th>$U_c$ (ms$^{-1}$)</th>
<th>$U_c = 10$ ms$^{-1}$</th>
<th>$U_c = 100$ ms$^{-1}$</th>
<th>$U_c = 500$ ms$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS $\delta-i$ component velocity error (ms$^{-1}$)</td>
<td>0.789</td>
<td>0.783</td>
<td>0.711</td>
</tr>
<tr>
<td>$u$-velocity uncertainty (ms$^{-1}$)</td>
<td>0.608</td>
<td>0.786</td>
<td>2.590</td>
</tr>
<tr>
<td>$v$-velocity uncertainty (ms$^{-1}$)</td>
<td>0.890</td>
<td>2.312</td>
<td>10.767</td>
</tr>
<tr>
<td>$w$-velocity uncertainty (ms$^{-1}$)</td>
<td>0.494</td>
<td>0.630</td>
<td>2.036</td>
</tr>
</tbody>
</table>

### 5. Results

The cameras and laser sheet were scanned through the depth of the jet at 8 discrete planes to obtain volumetric data at unheated, over-expanded conditions, nozzle pressure ratio $NPR = 2.7 \pm 0.03$, per Table 1. For each set acquired, the camera exposure time was monitored and set to the maximum allowable without saturating the camera sensor. Exposure times varied from 500 to 1500 milliseconds depending upon seeding levels. Note that exposures were not altered within the acquisition of a single set of data representing one scan of the absorption line.

As a first step to reconstructing the volumetric field, points within the jet profile measured at approximately five nozzle diameters were compared among in-plane and scanned, out-of-plane results. The shear layer profile development is reported in Figure 10, wherein red symbols were obtained from the out-of-plane scans at the centerline and black symbols were obtained in the plane of the laser sheet when positioned on the centerline. Agreement between the in-plane and out-of-plane results at the same radius relative to the centerline confirms the efficacy of the scanning technique in this flow.
This region of the flow contains series of shock cells indicative of the over-expanded condition, as illustrated in Figure 11 for the centerline velocity development. The shock cell features are easily visualized in the volumetric reconstruction, an annotated example given in figure 12. In this plot, the iso-surfaces for stream-wise velocity equal to 90% of the jet exit velocity are presented with shading to indicate the radius at which the surface is encountered. As such, dark regions are nearer the center, while light regions are further from the jet core. The high velocity regions bounded by shocks are clear in this reconstruction and exhibit regular spacing, as already indicated in Figure 11. Also to note is the rapid decay of the potential core in this region of the flow, as evident by the reducing radius of the iso-surface with stream-wise distance. There are artifacts present, as labeled in the figure, which were encountered throughout these measurements due to laser scatter on iodine cell surfaces near the rig. In instances with excessive background scatter, data were discarded, resulting in the voids seen in Figure 12. The structure of the mean velocity is further exhibited in Figure 13 via iso-surfaces of at several mean velocities between 50 – 100% of the jet exit velocity. The shear layer and shock cell structure are visible—lower velocities exhibit a conical behavior in the iso-surfaces, while velocities of 0.8\(U_j\) and greater have structure provided by the shock cells. While the data exhibit artifacts from signal-to-noise issues already mentioned due to background light, the results provide a quantitative visualization of the jet mean velocity structure. Such a capability is especially useful for non-axisymmetric flows.

Stream-wise velocity results for the center-plane development of the jet at hot conditions are provided in Figure 14. For clarity, radial profiles were extracted from the planar data and plotted in Figure 15. A result presented by (Powers et al. 2013) of reduced mean velocities at the centerline of a similar nozzle geometry is also encountered herein, a consequence of the nozzle expansion characteristic lines from the bicone nozzle. As an example of the analysis that may be performed using these data, the shear-layer thickness, \(\delta_{99}\), defined as the difference between the radial locations of 90% of the maximum profile velocity and 10% of that same velocity, is plotted as a function of stream-wise distance from the nozzle exit in Figure 16. Extensive collections of data on shear layer growth, as may be computed from the data in Figure 16, have been reported for shear layers in the absence of shock cells (e.g. Freund et al. 2000). The data in Figure 16, however, exhibit an interesting trait due to the over-expanded condition and non-uniform exit flow in the range of 1.5≤\(x/D\)≤1.7. A radial gradient due to the shock cell structure results in an apparent growth of the shear layer thickness. This is an artifact, however, as review of the development of profiles indicates that diffusion-dominated shear layer growth appears continuous.

![Fig. 10. Shear layer radial profiles obtained in the cold jet conditions. The legend provides the stream-wise station of the measurements. Red symbols indicate measurements from multiple positions of the laser sheet through the depth of the jet; the agreement between in-plane and out-of-plane results provides a confirmation of the ability to perform volumetric reconstruction.](image-url)
Fig. 11. Centerline stream-wise velocity development in the cold jet.

Fig. 12. Detail of the volumetric reconstruction of scanned planes via an iso-surface of stream-wise mean velocity at 90% of the jet exit velocity. The iso-surface is shaded using the local radius of the surface. Shock cells are clearly visible as is the decay of the potential core. Artifacts due to background light are noted.
Fig. 13. Iso-surfaces of stream-wise velocities for the cold jet. Top left is the iso-surface of 50% jet exit velocity, bottom right is 100% of exit velocity, with figures between providing 10% increments.

Fig. 14. Contours of mean stream-wise velocities for the hot over-expanded conditions listed in Table 1.
Fig. 15. Radial profiles of the jet in the range of $0.7 \leq x/D \leq 2.6$. Data are shaded according to stream-wise locations, with darker symbols being upstream.

Fig. 16. Stream-wise development of the shear layer thickness.

6. Discussion

The data presented above shows promise for the CC-DGV instrument application for robust measurements in the Virginia Tech supersonic hot jet facility. Shock cells and shear layers are readily apparent in the jet cross-sections. Since the results are mean velocities, these features appear with significant spatial blurring of the shocks, which would be expected to be very sharp in instantaneous snapshots. Shock unsteadiness of the jet related to screech has been observed in this facility, and, although screech is less pronounced at this low NPR, it is still expected to be present to a measure.

CC-DGV appears well suited for hot flow measurements due to the vapor cell temperature insensitivity. It was observed qualitatively that due to their proximity to the hot flow and facility, the temperature of the signal detector absorption cells increased slightly from prolonged exposure to the heated components. This effect is compatible with the results of the temperature simulation study, in that if a sufficient transmission ratio is initially used, heating will result in increased absorption by the cell. By comparison, “saturated cell” configurations may be sensitive to ambient temperature variations (Elliott & Beutner 1999) and could likely
still benefit from employing the CC-DGV method of data acquisition and processing.

Several enhancements to the procedure can be readily applied to improve data quality. Improved background subtraction techniques such as those described previously (Meyers et al. 2001) must be implemented. The measurable region may also be expanded by using co-flow seeding. The alumina particles added upstream of the nozzle exit provide only minimal seeding density in the outer portions of the shear layer. Oil-based seed particles can be added to the ambient air in the test cell so as to seed to entrained fluid and provide more uniform seed density through the shear layer.

Finally, with the volumetric capability of CC-DGV now demonstrated, it is recommended that a finer test matrix of measurement planes be used. Interpolation was used between measurement planes in the present work; more numerous, closely spaced planed will improve the accuracy of interpolation methods and data representation so that detailed interrogation of the results for flow physics is possible.

7. Conclusions

Cross-correlation Doppler global velocimetry (CC-DGV) is a newly developed technique that yields high spatial resolution mean vector velocities. Monte Carlo simulations were performed simulating the Doppler frequency shift as measured by individual sensors. Absorption cell temperature mismatches are studied for variations of ± 20°C, and insensitivities comparable to existing methods are found without temperature control provided a transmission ratio threshold is met. The technique is further shown to be versatile to measure velocities from low subsonic to supersonic regimes independently within the same field of view. The cross-correlation function is insensitive to absolute intensity mismatches between the Doppler shifted and reference signals. Bias errors in the correlation function peak-finding function have also been investigated.

CC-DGV has been applied to measurements in the Virginia Tech hot supersonic jet facility. An uncertainty analysis was performed for the specific experimental configuration used in this work, with values reported for free-stream flows of 10 ms⁻¹, 100 ms⁻¹, and 500 ms⁻¹ flows. Volumetric data is presented for cold (TTR = 1.0) conditions, and planar data is presented for hot (TTR = 2.0) conditions, both over-expanded (NPR = 2.7). Modulation depth was used as a metric for identification of valid data locations.

The CC-DGV approach will enable quick, robust measurements in the Virginia Tech supersonic hot jet facility in order to study flow at a variety of configurations. These measurements will help characterize key parameters of jet flows at varying thermodynamic total conditions such as spreading rate and potential core length. Due to the ease of application, the technique is a candidate for other facilities, including in situ measurements of full-scale jet plumes, wherein probe measurements are expensive and very difficult.

8. Acknowledgements

The authors wish to acknowledge the support of the Office of Naval Research Hot Jet Noise Reduction Basic Research Challenge and DURIP, grants N00014-11-1-0754 and N00014-12-1-0803 under program managers Drs. Brenda Henderson and Joseph Doychak. Some aspects of the work were also supported by GE Power & Water, program managers Jon Luedke and Christian Carroll.

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