3D path tracking of particles in a swirling flow using a light-field camera

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Abstract A Particle Tracing measurement technique is presented. Images of a rising particle are captured by a single camera equipped with a Plenoptic lens, consisting of a 3x3 doublet micro lens array in combination with a shadowgraphy technique. Aim of this setup is to triangulate the time resolute 3D position of the particle by using the small stereobasis of the induced by the micro lens array. For camera calibration a three dimensional polynomial approach is used and the resulting errors are examined. For 3D triangulation a center of mass-based method is applied and adapted to handle the occurring dragging slices, caused by the optical setup.

1. Introduction

It is well known that bluff bodies in flows at higher Reynolds numbers (Re>10⁵) induce wakes. In case that the body is free to move such as for a particle in flow and that its density is low compared to the surrounding fluid, the particle path will be affected by the wake instabilities. For bubbles rising in liquids the trajectory can be straight but also be in the form of a zigzagging or spiraling path, depending on the Weber- and Reynolds number (Brücker 1999). However, not much is known in the case that the particle is moving in a vortical flow, especially the influence of the vorticity field on the wake and the instabilities therefore. First investigations in our group focused on particles in swirling flows, these studies already showed the stabilization of the wake instabilities if the particle is rising along the axis of a vortex (Rudert A. 2006). Despite these findings, the forces could not be directly measured within these experiments. In principle, from measurements of the particle rise trajectory it is possible to reconstruct the temporal evolution of the forces acting on the particles. This requires measuring the path in all three coordinates to calculate the momentum equations in the Lagrangian form (Mougin G., 2002). For that reason a 3D particle tracking measurement is needed.

3D PTV is nowadays a standard procedure with multi-view recordings to capture the 3D motion and has a long history. However, in special cases as in the present experiments on a large swirl tank the optical access is limited to only a single view and also illumination must be along the same optical axis as the viewing direction. In addition, other components such as mirrors etc. might not be easy to be implemented within the flow system. Furthermore, if the particle path is investigated over longer time-spans, the whole setup needs to be portable in a sense of weight and geometrical dimension thereby it could be moved upwards with the rising bubble. Finally, it requires a sufficient aperture to capture the whole trajectory. In the last years, the method of light field recordings using Plenoptic cameras has grown in interest for these reasons. Herein we use such a method in combination with shadowgraphy. A Plenoptic lens consists of an array of micro lenses orthogonally to the main optical axis of the imaging optics. Depending on the number of micro lenses several sub-images are generated on the camera chip, see Fig 1,2. The sub images could be understood as a different section of the light field spreading out as spherical wave originating from any surface in the object space. These sub-images distinguish each other through a slightly differing angle of view, resulting in a small parallax in the imaging plane which arises from the lateral shift of the micro lenses within the lens array (Adleson Edward H. 1992). Using this technique it is possible to gather information on the spatial position by using only one imaging device. The accuracy is however affected largely by the pixel resolution and the number of sub-images, thus there is always a bottleneck between angular resolution and image resolution. Herein this accuracy is investigated on a single object in the measurement space represented by a light particle rising in water.
2. Experimental Setup

For the study of 3D trajectories of particles in liquids at low density Ratio $\sigma_{\text{Particle}}/\sigma_{\text{Fluid}}$ two experimental setups are available. First there is a rotating vertical pipe with a diameter of $D=500$ mm and a length of $L=5$ m. This device offers the opportunity of long observation times through a vertical movable sled, capable of following the rising particle on its way up. So it could be assured to observe the wake instabilities over a long time-span. The second setup is a non-rotational vertical pipe of the same diameter and material with a height of about 0.6 m that is used for camera calibration purposes.

The particles under investigation are polystyrene spherical particles of about 1-5 mm in diameter and a density of 1.05 g/cm³. Prior to the experiments they are loaded into a particle ejector, consisting of a particle magazine. The particles can be released as a single object or as a swarm via an infrared trigger signal that starts the actuator from outside the tank.

The imaging device designed for the purpose of particle tracking is, as mentioned above, a light field shadowgraphy optics. A LED light source emits from the receiving optics light into the measuring volume in
direction of the view. The light travels through the measurement volume, is reflected behind the volume by a retro reflective layer back into the measurement volume and enters again into the receiver.

The optical design was simulated using the optical design Software RADIANT ZEMAX©. Due to the character of the retro reflective layer light rays are reflected back in the exact angle of impact, in reality they are scattered under an angle of about 10°. For the simulations we assume in first approximation an ideal working retro layer with the consequence, that the beam path for illumination has to correspond the beam path of observation exactly. For that purpose the light is emitted into a beam splitter cube of 25 mm edge length in the observed ray path. According to the supposed ideal condition of the retro layer, the angular light field has to be created artificial. So a number of light sources equal to the number of micro lenses are needed. Any of the non-centered light sources is tilted about 5° towards the center light source (see Fig. 5).

The micro lens array consists of 3x3 5 mm Doublet lenses with a focal length of 10 mm. In the Simulation the image is directly captured by an 2000x2000 Pixel ray detector with an Area of 1 Square inch. The Wall of the Vortex channel is assumed to be made of PMMA with an Refractive Index n of 1.49 and is filled with water (n=1.33) (See figures 5 and 6).

During conversion to the real measuring system some modifications of the theoretical composition had to be carried out. Figure 7 shows the final version of our Plenoptic camera. The chosen imaging device is a PointGrey GS3 CMOS camera with a 1” chip and a resolution of 2048x2048 pixels capable of capturing 90 fps. Since the active area of the chip is a bit smaller than 1” two further lenses have to be installed, one additional field lens with 50 mm focal length and a converter lens (f = 30 mm). Furthermore the beam splitter cube was replaced by a beam splitter plate to avoid surface reflections. For contrast and focus enhancement an additional Iris diaphragm was installed in front of the CMOS chip. Illumination is provided through 9 high power single chip LED’s with peak wavelength of 530 nm, connected to the optical system via a bundle of 9 optical fibers of 3 mm diameter. The conduction of the light through optical fibers enables the periphery devices, like power supply and trigger unit to be placed spatially independent of the imaging device, which is a great advantage with view to the camera mounted on a sled moving up- und downwards.

The nine optical fibers are polished on both sides. The one end is mounted to the LED via fiber coupling and the other end is arranged according to the simulation tilted about 5° towards the center fiber. Light source and camera are synchronized via 5V TTL. Since the arrangement guarantees a moderate loss of light energy, because of the retro reflective sheet, the camera shutter time of the camera is limited to 100 µs.
3. Camera Calibration Procedure

A common way of calibrating a Plenoptic camera is via the use of so called shift maps (Skupsch Christoph 2013). Here estimation of the depth position (along the optical axis) is done by correlating the shift of certain objects with respect to the sub image center. So the obtained shift vectors of any sub image are distinguishing a depth position. These models assume that there is no influence of distortion in the sub images or at least that the level of distortion is equal in all sub images. The simulation of a checkerboard lying in the center of the tank using the presented optical model as well as the captured calibration images shows that there is a considerable amount of distortion caused by the micro lenses although the lenses are duplets of high quality (see figures 8 and 9). For that reason it appears to be necessary to apply a calibration procedure known from standard stereo camera systems. In the present work the calibration method after Soloff et. al. (Soloff S. M. 1997) is adopted because of its comparatively easy implementation and its high accuracy (Joshi Basanta 2013).

Figure 8 Simulated checkerboard consisting of 2mm plates (simulated with RADIAN ZEMAX)

Figure 9 Image of checkerboard with pitch length of 5mm

Figure 10 Detected corners in upper left sub image

Corner detection was carried out via a Harris corner Finder method (Harris C. 1988). Figure 10 shows the
result of such a corner detection (using MATLAB® Camera Calibration Toolbox). It can be seen that despite of severe image quality all 196 corners are detected and ready for use into the calibration routine. For the calibration of our measurement volume a set of 21 images with a spacing of 5 mm in the coordinate of depth was used. In the Imaging plane (X-Y-plane) a set of 14x14 points ranging from 0 to 65 mm (spacing 5 mm) was employed. The accuracy of the target position detection is a critical issue in the present method and from the right-hand side line of the blue boxes it is seen that the detection is far from being perfect, otherwise it would be a rather smooth variation. An estimation of the overall position error yields a value of roughly 0.2px that is due to the relative low image quality.

The calibration method after Soloff et. al. uses a set of points and their corresponding coordinates in object space to interpolate two 3-dimensionel polynomials with cubic dependence in X and Y direction and square dependence in direction of the depth coordinate (Z), as a functions of the correspondent points in the image space (xp, yp).

\[
F(x_p) = a_{x0} + a_{x1}X + a_{x2}Y + a_{x3}Z + a_{x4}X^2 + a_{x5}Y^2 + a_{x6}XY + a_{x7}XZ + a_{x8}YZ + a_{x9}X^3
\]
\[+ a_{x10}Y^3 + a_{x11}X^2Z + a_{x12}XY^2 + a_{x13}XYZ + a_{x14}X^2Y + a_{x15}Y^2Z + a_{x16}Z^2 + a_{x17}XZ^2 + a_{x18}
\]

\[
F(y_p) = a_{y0} + a_{y1}X + a_{y2}Y + a_{y3}Z + a_{y4}X^2 + a_{y5}Y^2 + a_{y6}XY + a_{y7}XZ + a_{y8}YZ + a_{y9}X^3
\]
\[+ a_{y10}Y^3 + a_{y11}X^2Z + a_{y12}XY^2 + a_{y13}XYZ + a_{y14}X^2Y + a_{y15}Y^2Z + a_{y16}Z^2 + a_{y17}XZ^2 + a_{y18}
\]

The unknown polynomial coefficients could be found via QR decomposition or a comparable fitting technique. The overall accuracy of this method depends on the number of points used for correlation and the amount of distortion occurring. Figure 11 shows the vectors of the back-projection error and figure 12 the Euclidian norm of the error vectors.

![Figure 11](image1.png)  
**Figure 11** rear projection error of center image

![Figure 12](image2.png)  
**Figure 12** Euclidean Norm of rear Projection of center image (error in pixels)

From figure 11 it is seen, that the back-projection error is statistically distributed and does not seem to have systematical contributions. The Euclidean Norm of this error shows a range from 0 to approximately 0.5 pixels. The mean error over all examined pixels over all depth positions is about 0.19 pixels which is the
minimum value of all sub images. Whereupon the mean error of the center image is slightly lower than in the rear images whose error ranges are between 0.19 and 0.21. Considering the error dimensions, it could be stated, that the accuracy of the measurement system has sub pixel accuracy, however it is considerably below the level needed for e.g. tomographic PIV. However, with the information of the particle shape and size we have additional information that helps us to improve the accuracy later by using pattern matching methods in object space. This is subject of ongoing work.

3. Position Triangulation

The camera calibration procedure uses the information about corresponding markers in the image and world coordinates and enables to formulate the transfer functions and line-of-sights for each pixel in each sub-image. In senses of triangulation accuracy the stereo basis between the different perspectives (Lawson N. J. 1997) shows that the error ratio between the depth coordinate and the object plane coordinates parallel to the image plane increases dramatically with decreasing camera angles. This error ratio reaches a factor 6-7 for a stereo basis of about 10°. This fact shows the basic problem of Plenoptic systems which got a by far smaller stereo basis. The idea behind the Plenoptic reconstruction techniques lies in the fact, that a large number of cameras could compensate this deficiency through a much larger data base. The indicated way would be to use 3D reconstruction techniques as the MART (“Multiplicative Algebraic Reconstruction Technique”) as suggested by Elsinga et. al. (Elsinga G.E. 2005), considering all pixels, reproject them into the object space solving the minimization problem between projection and reproduction iteratively. Given that this type of algorithms got a large computational expense, techniques are needed which limits either the area that has to be reconstructed or to create a good initial guess as a starting point for the MART reconstruction.

In the present work a Center-of-Mass based algorithm was used to estimate the particles center into object space. In each sub-image we determined the centroid by an image based center of mass method. For this purpose the raw images are splatted into the sub images, executing a coordinate transformation and determine the centroids. Here additionally a bounding box around the interesting area is calculated for later reconstruction (see fig. 13).

![Figure 13 Results of Center-of-Mass determination of all sub images](image)

Based on this nine centroid positions in the image plane the calculated calibration polynomials could be used to calculate the object space coordinates affecting these pixel values. For that a nonlinear target value search is necessary to determine the line-of-sight polynomials for each pixel. Due to the high computational effort of calculating an arbitrary number of space coordinates via Newton-Raphson Algorithm an approximation is
adopted. These is done by calculating just several points and approximate an inverse function with respect to the depth coordinate as described in (Kühn 2011):

\[
F(X) = b_{x0} + b_{x1}Z + b_{x2}Z^2 + b_{x3}Z^3
\]

\[
F(Y) = b_{y0} + b_{y1}Z + b_{y2}Z^2 + b_{y3}Z^3
\]

Using these functions (3) and (4) the lines of sight could be projected back into the object space. Figure 14 shows the nine lines of sight travelling through the measurement volume for a single particle in the flow tank.

Figure 14 Lines of sight with equal scaled axes

It could be seen, that there is no clear intersection point which could be used for determining the spatial position of the center of mass. Due to the presence of dragging slices, which means lines of sight with a vanishing relative angle to each other, a method for determination of the center of mass has to be found. A common non-voxel-based approach for systems without dragging slices, would be to calculate the distance of all lines for arbitrary points of Z, considering the minimum of this function with respect to Z the position of the center of mass. This approach is insufficient due to near parallel path of the lines of sight. For that reason a slightly other method for estimation had been found.

The assumption is, that in the striking distance of the center of mass in object space the number of intersections (in the X-Z and the Y-Z plane) becomes a maximum with respect to the depth coordinate (Z) (see figures 15 (a) and (b)). So the depth position is discretized in a number of intervals and the probability distribution of ray intersections per depth-interval is calculated. Figure 17 shows one of the probability distribution charts.
Figure 15 Ray intersections in the X-Z plane (a) and the Y-Z plane (b) and the Frequency distribution (c) of ray intersections with respect to the Z coordinate (Z interval=10mm). The line labels are referring to the sub image position, so ‘ul’ means the upper left sub image, ‘mm’ means the center sub image and ‘dr’ means the downer right sub image.

Figure 15 (c) shows that the probability distribution of intersections along the Z-direction is non-Gaussian. For that reason the use of arithmetic mean values or standard deviations cannot give any reasonable results. Therefore we used the maximum peak of the probability distribution as an estimator for the particles depth position. It is clear from this method that the accuracy increases with the number of micro lenses and the number of correlated sub-images, respectively.
4. Results and Discussion

The presented algorithms reaching form calibration over center-of-mass detection to position triangulation had been applied to several particle lift experiments. These were recorded using an in-house developed lightfield-camera shadowgraphy system with a 3x3 micro lens array at a frame rate of 90 fps. The calibration results indicate accuracy below typical levels of stereo camera systems with a view to the rear projection errors of around 0.19 pixels. The algorithm used for center of mass triangulation shall be understood as a fast approximation procedure to be either used to get an idea of the experimental conditions or as first guess for further computations as a 3D reconstruction in connection with 3D cross correlation of the center of mass within the voxel volume, which is considered to be more accurate than the presented way. It may be expected, that the accuracy of the presented way of triangulation could be increased by the use of a larger lens array in a sense of the number of used micro lenses. Figures 17-20 show different pro one exemplarily particle Trajectory smoothed with sectional splines. It is seen that the particle trajectory is rather smooth in the X-Y-projection while it shows considerable discontinuities in the Y-Z-projection. The latter is due to the remaining position-error in the Z-coordinate.
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6. References


