Power Spectrum Estimation of Randomly Sampled Signals

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The random, but velocity dependent, sampling of the LDA presents non-trivial signal processing challenges due to the high velocity bias and the arbitrariness of particle path through the measuring volume, among other factors. To obtain the desired non-biased statistics, it has previously been shown analytically as well as empirically that residence time weighting is the suitable choice (Buchhave 1979, Buchhave et al. 1979, Velte et al. 2014). Unfortunately, due to technical problems related to the processors providing erroneous measurements of the residence times, this previously widely accepted theory has been questioned and instead a wide spectrum of alternative methods have been invented and tested (c.f. Albrecht et al. 2003). The objective of the current study is to create a simple computer generated signal for baseline testing of residence time weighting and some of the most commonly proposed algorithms; sample-and-hold and the direct spectral estimator without residence time weighting.

The computer generated signal is a Poisson process with a sample rate proportional to velocity magnitude that consist of well-defined frequency content, which makes bias easy to spot. The idea is that if the algorithms are not able to produce correct statistics from this simple signal, then they will certainly not be able to function well for a more complex measured LDA signal. This is, of course, true also for other methods that are based on the tested algorithms. The extremes are tested by increasing, e.g., the ‘turbulence intensity’ and the ‘shear’.

In the paper, we have generated and tested these algorithms on synthetic data generated for the following signals: a Gaussian pulse, a single and multiple sine waves and a Gaussian random signal. In the abstract we have chosen to show results only for the sum of five sine waves. The primary signal and the corresponding power spectrum are shown in Figure 1.

The corresponding randomly sampled signal and the sampled-and-held signals are displayed in Figure 2. Note how the signal overlaps zero velocity and how the modulation of the randomly sampled signal produces lower data rates around zero. Figure 3 shows the power spectra computed using the three algorithms investigated.

Fig. 3 Power spectra obtained using conventional averaging (upper left), residence time weighting (upper right) and sample-and-hold (bottom).

The conventional spectrum shows multiple erroneous mixing frequencies and the peak values are too low. The residence time weighted spectrum is correct. The sample-and-hold spectrum has lower power than the correct spectrum, and the f−filtering effect appearing for low data densities is evident (Adrian and Yao 1987). The remaining tests also show that sample-and-hold and the free-running processor perform well only under very particular circumstances with high data rate and low inherent bias, respectively. Residence time weighting provides non-biased estimates regardless of setting. The free-running processor was also tested and compared to residence time weighting using actual LDA measurements in a turbulent round jet. Power spectra from measurements on the jet centerline and the outer part of the jet illustrate a distinct difference between the residence time weighted and the non-weighted spectra, in particular for positions far off the jet center axis where the bias increases.


