Coherent Tomographic Laser Interferometry for the Aero-acoustic Characterization of Cold Jets

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Abstract This paper aims to describe the complete characterization of aero-acoustic sources, with specific attention to turbulent jet flows with a new approach. The traditional way to tackle fluid dynamic phenomena is to measure the velocity field by means of Laser Doppler Anemometer, Particle Image Velocimetry with the main drawback due to the need of seeding the flow. To overcome that problem the 3D spatial distributions of flow pressure fluctuation due to the flow turbulence can be measured via an interferometric technique and 3D field reconstruction by tomographic algorithms. That procedure employs a non-contact laser Doppler vibrometer in a non-conventional way. This instrument is able to measure the density oscillation within the medium traversed by the laser beam, with a bandwidth up to 200 kHz and with a very fine spatial resolution. The raw measured data, they being interpreted by the system as velocity of vibration and being due to the integral of the flow density variation over the whole laser optical path, need to be post-processed in order to obtain quantitative data about the 3D distribution of the flow density and, finally, the pressure fluctuation. The method is sensitive to both the aerodynamic “cause” of the noise, such as turbulence or vortex shedding, and to the “effect” of it, i.e. the acoustic waves that propagates from the jet to the far field. In order to distinguish between acoustic and purely aerodynamic phenomena, i.e. pressure fluctuation that generate or not an acoustic field, joined flow and acoustic pressure measurements were performed by using the proposed procedure and an omni-directional microphone. The two phenomena will be separated by applying de-correlation algorithms based on coherence function, it allowing to perform a complete aero-acoustic characterization of the fluid dynamic phenomenon.

1. Introduction

Recently, the limiting of environmental noise due to the acoustic emissions of jets is an important task which addresses the increasing of life quality level. The nature of the noise generated by these devices is fluid dynamic since the acoustic phenomenon is originated by fluidic causes, i.e. turbulences and vortices, producing annoying noise and whistle.

The traditional way to describe fluid dynamic effects is to measure the velocity field by means of Constant Temperature Anemometer (CTA), Laser Doppler Anemometer (LDA), Particle Image Velocimetry (PIV), etc. and then to derive the generated sound field by applying the Lighthill analogy (Fischer and Sauvage, 2008). Amongst the non-invasive methods, the most employed for jets characterization is the PIV (Schram, 2003), (Violato and Scarano, 2011). However that technique has a main drawback due to the fact that the PIV requires seeding the flow in order to measure the particle velocity, and this is not always a trivial task. The inertia of the seeding particle will, in fact, limit both the speed of the jet that could be measured and the frequency content, this being restricted also by hardware performances, specifically CMOS cameras and lasers, for instance.

In recent works several attempts have been done for improving the processing methods and they have been focused on the identification of “cause-effects” relationships by combining fluid dynamic in-flow measurement, and acoustic far-field measurement.

Typical processing algorithms employed are the so-called “causality correlations” based on the cross-correlation between the acoustic and the fluid dynamic data (Henning et al., 2010), (Schaffar, 1979). However the above-mentioned PIV limits persist. The jets aero-acoustic can thus be studied in terms of its effect, the noise, instead of its cause, the fluid dynamic.

The recent advances on phased array techniques, i.e. beamforming (R. Dougherty, 2009), for measuring the spatial distribution of acoustic fields, allowed to measure the noise generated by air jets and to have a
sort of its acoustic image. However beamforming techniques own several limits: (i) the measurement input is the sound field, e.g. the effect of the fluid dynamic phenomenon and not the direct cause; (ii) the spatial resolution is high only at the high frequency range and therefore the method cannot apply for identifying noise sources at the low-medium frequency range, and (iii) the technique is sensitive to errors, since an error of 1% on the aerodynamic pressure field yields an error of 100% on the acoustic field. The causality approach previously described for PIV measurements can apply also to beamforming measurements by inverting its logic: instead of measuring on a grid of points located within the flow and having as reference a far field acoustic sensor, the point grid can be placed in the far field for measuring the acoustic field, by employing phased array microphones and the reference can be positioned in-flow for measuring the fluid dynamic phenomenon, by means of an LDA (Garcia-Pedroche and Bennett, 2011). The main disadvantage of the LDA technique, a part the fact that it needs seeding the flow, is that the data are irregularly sampled and thus a significant processing is required.

The characterization of aero-acoustic sources can be otherwise estimated from the density oscillation measured by laser interferometry, a technique used in the past for flow visualization, e.g. Schlieren method (Kleine et al., 2006). The density fluctuation measured by laser interferometry can be directly related to the aero-acoustic pressure so to derive the effect directly from the cause. It is well known that the signals acquired by interferometric techniques are line integrals over the laser beam optical path, therefore reconstruction algorithms should be used to calculate local density variation distribution.

The use of optical and interferometric techniques for the visualization of flow fields was described by Zipser and Franke (2002), but only 2D or rotationally symmetric distributions have been treated, to which inverse Abel transform has been applied. A tomography-based algorithm has been implemented for this purpose and applied to signals from multi-directional observations of the flow field (Castellini and Martarelli, 2006). In fact when symmetry property does not stand tomographic reconstruction is compulsory. That paper presented a method for post-processing the acquired data from the interferometer in order to obtain quantitative evaluation of pressure fluctuation. The technique was called Tomographic Laser Interferometry (TLI). A very interesting example of possibilities offered by this approach is shown by Tatar et al. (2007) for the reconstruction of acoustic fields produced by an array ultrasound probes.

In this paper we propose the application of the Tomographic interferometry for the complete aero-acoustic characterization of an air jet, integrating the pressure fluctuation analysis in 3D with a far field acoustic measurement performed with an omni-direction microphone. In this way, different components of the phenomenon, such as the vortex shedding, the wave packets originated in the jet core region by the vortex instability and the acoustic propagation can be separated. The proposed procedure for the aero-acoustic characterization of cold jets can be summarized in the following steps:

- Laser interferometric measurement of the optical path fluctuation across the jet over a dense grid of points and from different directions of view in conjunction with the measurement of an acoustic reference signal;
- Coherence function calculation at each acquisition point and separation of acoustically coherent and non-coherent (aero-dynamic) components;
- Calculation of the volumetric distribution of optical path fluctuation at each position \((x,y,z)\) by tomographic reconstruction;
- Derivation of the fluid density and relative acoustic pressure oscillation at each voxel of the measurement volume and representation of each voxel as a monopole sound source.

The method will be named in the following as Coherent Tomographic Laser Interferometry (CTLI).

2. Measurement procedure theoretical basis

2.1. Flow density oscillation estimation

The measurement of the density fluctuation about the DC density at standard conditions, \(\rho_0\), can be measured by optical interferometers exploiting the sensitivity of a coherent light, such as laser, to
variation of optical path travelled by the beam.

In this work a conventional Polytec Laser Doppler Vibrometer (LDV), based on the Mach-Zehnder architecture, has been employed. It gives, as output, the frequency shift ($\Delta f$) proportional to the variation of the optical path ($dz/dt$) sensed by the laser beam when pointed towards a moving surface. Being the conventional LDV output a vibration velocity, in this paper the measured signal will be called for simplicity, pseudo-velocity, $v = dz/dt$, related to the Doppler shift frequency with the following relationship:

$$\Delta f = \frac{2v}{\lambda_{st}}$$

That pseudo-velocity is referred to the laser wavelength measured at standard conditions, $\lambda_{st}$, that is not exact because, in the case when the laser light goes across a turbulent flow, the light wavelength changes with the density fluctuations and the actual laser wavelength ($\lambda$) must be considered. Therefore the true measurement output is the so called measured pseudo-velocity, $v_{\text{meas}}$:

$$v_{\text{meas}}(x,y,t) = v \frac{\lambda}{\lambda_{st}} = \frac{dz}{dt} \frac{\lambda}{\lambda_{st}} = \left(Z \frac{dn}{dt} + n \frac{dZ}{dt}\right) \frac{\lambda}{\lambda_{st}}$$

In equation (2) the optical path, $z$, is composed of two components:
- the geometrical one, $Z$, whose changes are due to displacements of the surface where the laser beam impinges, and is the basis of common vibrometers principle of operation,
- the proper optical path linked to refraction index, $n(x,y,t)$, which undergoes to spatial and temporal variation produced by the jet turbulence within the measurement volume, the light blue region, shown in FIGURE 1.

When moving object are measured in steady air, the refraction index variation is null and only the second term appears in equation (2), i.e. the interferometer input is the displacement of the object surface. This is the conventional use of the laser Doppler vibrometer. On the other hand, if the laser beam crosses a turbulent region and impinges on a target kept steady, as shown in Error! Reference source not found., the only variable is the refraction index which fluctuates inside the measuring volume.

Being $\ln = \lambda_{st}n_{st}$ (Edlén, 1966) where $n_{st}$ is the index of refraction at standard conditions, equation (2) became:

$$v_{\text{meas}}(x,y,t) = Z \frac{n_{st}}{n(x,y,t)} \frac{dn(x,y,t)}{dt}$$

In this equation the dependence of the refraction index to the $z$ direction is not considered because the information carried by the laser beam is the integral of $n$ along its travelled path.
However the vibrometer output \( v_{\text{meas}} \) can be used to estimate the reflection index variation within the jet turbulent region which is related to the air density fluctuation (\( \rho \)). Conventionally, as in Schlieren based interferometry (Merzkirch, 1974), the relation between \( n \) and \( \rho \) is given by the Gladstone–Dale relation, developed for fluidic mixtures of several pure compounds. This relation is generally satisfactory. In this paper, instead, a more accurate model linking \( n \) and \( \rho \) has been used i.e. the Lorentz-Lorenz equation (Lorentz, 1880), (Lorenz, 1880), being the increase of computational effort quite negligible:

\[
\rho(x, y, t) = \frac{W}{A} \frac{n^2(x, y, t) - 1}{n^2(x, y, t) + 2}
\]  

(4)

\( W \) being the air molecular weight, i.e. 0.02896 kg/mol, and \( A \) the air molar refractivity, i.e. \( 4.7 \times 10^{-6} \) m\(^3\)/mol (Born, 1959). Calling \( K = A/W \) the index of refraction can be expressed as:

\[
n(x, y, t) = \left[ \frac{2K\rho(x, y, t) + 1}{1 - K\rho(x, y, t)} \right]^{1/2}
\]  

(5)

and inserting equation (5) in equation (3) the vibrometer output pseudo-velocity can be written as following:

\[
v_{\text{meas}}(x, y, t) = Z \frac{n_{st}}{n(x, y, t)} \frac{1}{2n(x, y, t)[1 - K\rho(x, y, t)]^2} \frac{d\rho(x, y, t)}{dt} = \frac{3ZK}{2n_{st}} \frac{1}{[1 - K\rho(x, y, t)][1 + 2K\rho(x, y, t)]} \frac{d\rho(x, y, t)}{dt}
\]  

(6)

This first order ordinary differential equation can be integrated between two time instants:

- \( t_1 = 0 \) where the jet is off and \( \rho_1 = \rho_0 \) (the undisturbed air density) but, in practice, the initial air density has been taken equal to zero because the aim of this measurement is to determine density oscillations about the air density at rest (the DC component),
- \( t_2 = t \) where \( \rho_2 = \rho \),

thus yielding to:

\[
\int_0^t v_{\text{meas}}(x, y, t) dt = \frac{3ZK}{2n_{st}} \int_0^\rho \frac{1}{[1 - K\rho(x, y, t)][1 + 2K\rho(x, y, t)]} d\rho
\]  

(7)

whose integral is the so called pseudo-displacement, \( s_{\text{meas}} \).

The air density oscillation can be recovered, thus, from the vibrometer output pseudo-displacement \( s_{\text{meas}} \) and the known constants \( K, Z \) (the measurement volume transversal dimension along the laser line of sight) and \( n_{st} = 1.0003 \):

\[
\rho(x, y, t) = \frac{1}{K} \frac{e^{-Z s_{\text{meas}}(x, y, t)}}{2 + e^{-Z s_{\text{meas}}(x, y, t)}}
\]  

(8)

### 2.2. Sound pressure calculation

To complete the aero-acoustic description of the flow field, the sound produced by the air density fluctuation in the surroundings of the free jet should be predicted. According to the Lighthill theory and
considering the linearized case, the pressure \( p \) and density fluctuations \( \rho \) being small compared to the undisturbed quantities, the relationship between pressure and density fluctuation depends only to \( c_\infty \) (the sound speed in the undisturbed flow, 340 m/s):

\[
p(x,y,t) = c_\infty^2 \rho(x,y,t)
\]

\( (9) \)

The air density fluctuation will generate a pressure variation consisting on the superimposition of the sound field produced by the vortexes occurrences in the near field and the acoustic wave’s propagation in the far field. It can be objected that density and, thus, refraction index fluctuations can be caused also by temperature oscillations. However the temperature contribution can be considered less significant with respect to the pressure one (Castellini and Martarelli, 2006).

2.3. **Tomographic reconstruction of the 3D sound pressure**

The output of the previous calculation \( p(x,y,t) \) is the pressure fluctuation along the optical path of the laser beam (z-direction in Figure 2), that comes across the measurement volume where the air pressure oscillation causes the variation of the refractive index and therefore of the air density. The pressure datum must be further transformed on the basis of CT principles, in order to reconstruct the 3D acoustic pressure field. If a sufficient number of projections at different angles of view (\( \Theta \) in

FIGURE 2, a), i.e. a certain number of plane grids, are acquired, the 3D density distribution can be reconstructed, as addressed and discussed by Radon (1917). The evolution of this study opened, several years after, the way of modern Computer Aided Tomography. The theoretical approach of the Radon problem is out of the scope of this paper and only its practical implementation will be treated in this paper.

For each projection or angle of view \( \Theta \), a points-grid has been acquired. By applying the inverse Radon transform to the polar distribution, the Cartesian 3D pressure field can be reconstructed, in
The air pressure fluctuation calculated from the interferometric data in the frequency domain can be represented as a 4D polar matrix, whose dimensions are the $x$ and $y$ coordinates (see FIGURE 1) over the measurement grid, the angles of view ($\Theta$) and the spectral lines. The pressure oscillation measured at each point and for every angle $\Theta$ corresponds to its integration over the line-of-sight of the laser beam within the measuring volume. By applying the tomography algorithm to that matrix, the local pressure variation can be recovered on an arbitrary 3D volume grid of points, called “voxels”. The output of the CT process is, therefore, a 4D Cartesian matrix, whose dimensions are the $x,y,z$ coordinates (see sketch in FIGURE 2) and the spectral lines.

### 2.4. Pressure acoustic and aerodynamic component separation

The pressure measured with the TLI is the superimposition of the aerodynamic pressure whose oscillation creates the turbulence and the actual acoustic pressure which propagates into the far field. The separation of those components is not trivial; nevertheless it can be performed by exploiting an acoustic sensor as a microphone being sensitive to only the propagating component. By calculating the coherence function between the overall pressure measured by the TLI and the acoustic pressure measured by the microphone, the overall pressure component coherent with the far field propagation can
be estimated this being the actual acoustic component. The non-coherent component, i.e. the complement of the coherent one, will be instead the aerodynamic pressure. Of course the un-coherent component includes also the noise, it being not coherent for definition.

The coherence function $\gamma$ is given by the following equation:

$$\gamma = \frac{|S_{pp}|^2}{S_{pp}S_{pp}}$$  \hspace{1cm} (10)

where $S_{pp}$ and $S_{pp}$ are the pressure auto-spectrum measured by the microphone and the TLI respectively and $S_{pP}$ is the cross-spectrum between the pressure measured by the microphone and the TLI.

The coherent and non-coherent components, i.e. the acoustic ($FFT_{acoustic}$) and aerodynamic ($FFT_{aerodynamic}$) portions of the overall pressure ($FFT_P$) can be calculated as:

$$FFT_{acoustic} = \gamma FFT_P$$
$$FFT_{aerodynamic} = (1-\gamma) FFT_P$$  \hspace{1cm} (11)

3. Experimental set-up

The noise produced by an ideal jet, in the absence of shock waves, is due to the turbulence generated by the mixing of the high pressure flow and the surrounding fluid, at ambient conditions.

The jet produced by a nozzle with exit diameter of 0.011 m and conicity of 6.6 deg has been observed, see FIGURE 3.

The nozzle was mounted downstream of a pipe 0.5 m long and of 0.019 m diameter.

The jet produced by the nozzle has been driven by a pressure set in the compressor of about 7 bar, however the pressure upstream the jet nozzle has been measured by a pressure sensor this allowing to calculate a pressure ratio of 1.205 ($M=0.52$) and producing an exit velocity of the flow of about 179.5 m/s. The corresponding Reynolds number was 132’000.

The aero-acoustic of such kind of system will be characterized by several phenomena:

- cavity resonances of the pipe, spaced of about 268.5 Hz, it being the fundamental acoustic cavity frequency,
- vortex street frequency occurring whether the vortexes are triggered.

The vortex street frequency occurs because downstream the nozzle lips there is the detachment of annular Von Karman vortexes that, under certain pressure and velocity field, produce an audible whistle. The vortex street frequency ($f_v$) can be calculated from the aerodynamic velocity of the flow ($u$), its Strouhal number ($N_{ST}$) and the nozzle diameter ($D$):

$$f_v = \frac{N_{ST}u}{D}$$  \hspace{1cm} (12)

For the condition of pressure ratio 1.104, the flow velocity is 179.5 m/s and the Strouhal number (for Reynolds number between 2x10^4 and 2x10^5) is about 0.18. The vortex street frequency is therefore about 2937Hz.

The CTLI test has been conducted by using a LDV (Laser Doppler Vibrometer) Polytec PSV200 vibrometer, measuring the density fluctuation across the flow and a B&K microphone type 4593 placed in the near field, slightly downwards the jet exit in order to not being invested by the flow. The microphone has been used as reference for identifying the part of the density fluctuation coherent with the acoustic pressure measured by the microphone; only this component is related to the acoustic
radiation of the flow, the rest is the aerodynamic component that won’t be radiated into the far field. The measurement set-up is shown in Figure 6. The measurement was done with 20 kHz of frequency bandwidth, 80 ms of acquisition time and 32 averages at each point, for statistical validity. The number of measurement points over the grid, sketched in FIGURE 3, was 1593. The flow field was measured from 360 points of view of the LDV, by rotating the nozzle (see FIGURE 4), thus giving 360 projections with an angular resolution of 0.5 degree.

5. Analysis of results

The raw output from the vibrometer is a pseudo-velocity related to the optical path variation because of the density fluctuation. These data, processed in frequency domain, have been averaged over the complete set of measurement positions to give the averaged spectrum shown in FIGURE 5, bottom plot. In the top plot of that figure is also reported the reference microphone spectrum. In both the spectra different phenomena occurring at different frequencies are noteworthy. The most evident is the one arising at 2937Hz which is the nozzle vortex street frequency ($f_v$). Very sharp peaks also occur at higher harmonics of $f_v$ which should be originated from acoustic excitation, and not from the velocity fluctuations. The second phenomenon is linked to the small peaks arising from the spectra pedestal which are the cavity resonances of the pipe holding the nozzle. Those frequencies start from 268.5Hz and continue up to the high order harmonics (up to at least 8kHz).

The pseudo-velocity distributions are given in terms of instant value it including amplitude (|$FFT_{p-v}$|) and phase ($\angle FFT_{p-v}$) of the pseudo-velocity signal ($FFT_{p-v}$) as following:

$$|FFT_{p-v}| \cos \angle FFT_{p-v}$$  \hspace{1cm} (13)

Those distributions are reported in FIGURE 6 for three different frequencies: the vortex street frequency and its third harmonic and the pipe cavity resonance at 4837.5Hz. These frequencies appear to be the most interesting to give, in this paper, the synthesis of the observed behavior of the jet.

The observed distributions present large-scale orderly structures of the turbulence, the so-called wave packets, typically due to the integral of the annular vortexes appearing at the nozzle exit. Their spatial separation is of about 25mm corresponding to the ratio of the vortex speed and its frequency ($f_v$). The structured turbulence is the source of the pressure fluctuation which produces the sound called ‘bird tone’ (Goldstein, 1976). This phenomenon has been observed also by Schram (2003) from the fluid-dynamic point of view, by using PIV. In the high frequency distribution (8837Hz) a second phenomenon
is evident, that is the acoustic wave propagation effect.

![Microphone spectrum - Acoustic pressure [dB ref 20μPa]](image)

**FIGURE 5** Acoustic pressure (top) and TLI pseudo-velocity (bottom) spectra.

![LDV pseudo-velocity [m/s]](image)

The coherence between pseudo-velocity and acoustic pressure signals is plotted in FIGURE 7 for the three aforementioned frequencies. Observing the maps at the vortex street frequency and its harmonics, it is clear the coherence is maximum everywhere except where pure fluid-dynamic phenomena occur, i.e. in the vortexes location. When a mere acoustic phenomenon occurs, as at the cavity resonance of 4837Hz, the coherence is high only close to the nozzle exit where the source of the sound is located.

![Acoustic pipe cavity resonance](image)

**FIGURE 6** Pseudo-velocity instant value distributions (± 12.5 Hz RMS values).
FIGURE 7 Coherence distributions (± 12.5 Hz RMS values).

FIGURE 8 Overall pressure, fluid-dynamic and acoustic components distributions at the vortex street frequency (± 12.5 Hz RMS values).

The coherence maps will be used next as masks for separating the fluid-dynamic (non-coherent...
component) from the acoustic (coherent component) contribution of the pseudo-velocity.

By applying equations (8) and (9), density and pressure oscillation can be estimated from the pseudo-velocity. The pressure distributions at the three considered frequencies are reported in the left plots of FIGURE 8, FIGURE 9 and FIGURE 10 in terms of instant value (top) and amplitude (bottom).

By applying the same equations to the coherent and non-coherent pseudo-velocity the fluid-dynamic (non-coherent contribution) and the acoustic (coherent contribution) portion of the overall pressure can be estimated. Those components are reported together with the overall pressure respectively in the central and right plots of FIGURE 8, FIGURE 9 and FIGURE 10 in terms of instant value (top) and amplitude (bottom). It can be noticed that the acoustic component is located in the core region, while the aerodynamic one in the mixing region, that starts at about 5 diameters from the nozzle exit (Merzkirch, 1974) Error! Reference source not found..

<table>
<thead>
<tr>
<th>Overall pressure instant value</th>
<th>Fluid-dynamic (non-coherent)</th>
<th>Acoustic (coherent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4837.5 Hz</td>
<td>4837.5 Hz</td>
<td>4837.5 Hz</td>
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FIGURE 9 Overall pressure, fluid-dynamic and acoustic components distributions at the pipe cavity resonance (+12.5 Hz RMS values).
6. Conclusions

In this work an interferometric technique joined to a tomographic reconstruction algorithm has been applied for the complete aero-acoustic characterization of a cold jet. The procedure consents to indirectly measure the air density variation within the measurement volume downstream the jet from the air refraction index and subsequently to calculate the acoustic pressure generated by the jet itself. The main originality of the work presented in this paper is the exploitation of a correlation approach between the acoustic pressure measured with the TLI and the far field pressure measured by a microphone. The TLI, in fact, is sensitive to both the aero-dynamic and the actual acoustic component of the overall pressure fluctuation. By calculating the coherence between TLI and microphone signals the two components have been isolated:
- the non-coherent portion, being the aero-dynamic phenomenon or the turbulence producing the air density fluctuation, i.e. the cause of the sound generation,
- the coherent portion, being the acoustic pressure propagating into the far field, i.e. the effect of the turbulence.
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