Theoretical evaluation of droplet concentration limits for interferometric droplet sizing measurements

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Abstract This study reports the development of a simulation process, which involves the generation of artificial fringe pattern images that are obtained when planar droplet sizing measurements with interferometric imaging are taking place. This simulation procedure evaluates the droplet concentration limits of interferometric droplet sizing techniques and allows the design and optimisation of the optical arrangement. Spatial distributions of droplets are constructed using random number generators for their positioning and sizes, which follow a Rosin-Rammler size distribution. The fringe patterns created during the interferometric imaging of the constructed droplet distributions are then evaluated, so that the probability of overlapping between fringe patterns of different droplets can be quantified for different system parameters. Number, volume and mass concentration can be therefore estimated by employing statistical distributions for the number of droplets and their size per image. Finally, the concentration limits of interferometric imaging sizing techniques are discussed.

1. Introduction

In the field of spray droplet size and velocity distribution characterisation, many methods have been proposed. Phase Doppler Anemometry (PDA) as described by Bachalo and Houser (1984) has long been the most common and reliable choice for point measurements in sprays. However, its limitation to measurements of droplet size and velocity distribution at a small volume defined by two intersecting laser beams implies that in order to obtain the spray droplet flow field information many measurements need to be performed and only one droplet must be present in the probe volume during each measurement. With the advancement of the Charged Coupled Devices (CCD) technology, imaging measurement techniques have become a more realistic and practical approach to spray droplet characterisation. These methods lack the temporal resolution of the PDA, but compensate by providing instantaneous spatial information and the recent availability of high-speed imaging cameras will render the temporal advantage of point measurement techniques to be irrelevant.

One of the most prominent methodologies for modern imaging droplet characterisation techniques is the far-field imaging of the undulating light pattern that emerges when a spherical droplet passes through laser light. König et al. (1986) used the geometrical optics approximation analysis of Glantschnig and Chen (1981) to measure the size of the droplets created by a monodisperse droplet generator via the far-field scattered intensity pattern that each droplet generated when crossing a laser beam. In their setup, the far-field intensity is the result of the interference of the light emanated by two bright spots (van de Hulst and Wang (1991 created from the reflected and first-order refracted rays for the specific scattering angle of observation.

Glover et al. (1995 expanded their method by using a laser sheet, therefore creating a planar measurement technique capable of measuring the size and velocity distributions of droplets, known
as Interferometric Laser Imaging Droplet Sizing (ILIDS).

In ILIDS, the interfering scattered light pattern—or fringe pattern—originating from each droplet in the field of view is imaged in the CCD via an optical assembly. The shape of the aperture of the imaging optical assembly defines the outer shape of the fringe pattern on the acquired images and, since circular apertures are the most common, the resulting fringe patterns are also typically circular (Fig. 1). The frequency of the undulating light pattern that is contained in each fringe pattern provides the droplet size, after the droplets have been identified and processed via usage of specialised image processing software. The size of the droplets is obtained by the simple formula (Hesselbacher et al., Hess (1):

\[
d = \frac{2\lambda_{w}N_{f}m}{a} \left[ \cos \left( \frac{\theta}{2} \right) + \frac{m \sin \left( \frac{\theta}{2} \right)}{\sqrt{m^2 - 2m \cos \left( \frac{\theta}{2} \right) + 1}} \right]^{-1},
\]

with \( \lambda_{w} \) being the illumination wavelength, \( a \) the collecting angle, \( m \) the relative refractive index and \( N_{f} \) the number of undulations inside the resulting fringe patterns. The velocity of the droplets is obtained for each individual droplet by tracking it between consecutive images.
The sizing principle of ILIDS is also found in the Global Phase Doppler (GPD) technique proposed by Damaschke et al. (2001). GPD employs two interfering laser sheets and uses the full range of scattering angles available for measurement with the necessary geometrical optics adjustments for inclusion of higher-order refracted light so that it is valid in all the scattering field. The existence of two laser sheets results to each droplet being represented by two overlapping fringe patterns in the resulting image. This exacerbates a very important obstacle in the application of interferometric droplet sizing techniques (both ILIDS and GPD), the inevitable overlap of fringe patterns for neighbouring droplets, when imaging dense spray regions. The size of the circular fringe patterns and the random nature of droplet positioning in a spray are responsible for pattern overlapping making image processing a difficult task.

To alleviate this problem, Maeda et al. (2000) proposed a system with a rectangular slit and a pair of cylindrical lenses that compresses the circular fringe patterns in the vertical dimension, which does not contain the droplet size information. Fig. 3 illustrates the standard and compressed ILIDS optical system arrangements. Typical experimentally recorded images of these optical arrangements are shown in Fig. 2; it is apparent that with this enhancement, the overlapping of the fringe patterns is radically reduced and, as a result, the number of droplets on the image that can be measured is increased.

Kawaguchi et al. (2002) quantified the increase in the droplet validation probability for both the standard and compressed approach, for a specific fringe pattern size and four different CCD image sizes. Their approach involved the generation of images with randomly placed droplets and the calculation of the droplet validation ratio. They reported that for 1 Mpixel images with 100 droplets, the compressed images validate more than 95% of the total droplets in contrast to the less than 20% validation rate for the uncompressed images. In the 4 Mpixel case a validation ratio of almost 90% for images with 1000 droplets is reported for the images with the compressed fringe patterns against the 0% for the images with the uncompressed fringe patterns.

Damaschke et al. (2002) performed a theoretical quantification of the concentration limits of the above spray measurement techniques for specific optical systems. They considered an ideal imaging system and expressed the overlap coefficient in terms of Poisson statistics, as the ratio of the overlapping fringe pattern area to the overall fringe pattern area. They concluded that for the example systems they considered, the drop number density limit is greater for PDA systems than for the interferometric techniques. However, their results on the allowed overlapping are purely derived by using the geometrical characteristics of the systems defined and do not take into account the actual spray characteristics that is encountered in real-life measurements.

In the present contribution we describe a simulation process that incorporates both previous approaches, in addition to the characteristics of sprays with the objective to quantify the effect of pattern overlapping. The results are generic, since they are considered without a specific optical system in mind and emphasize on the images generated as part of a typical measurement. The overlapping allowed between fringe patterns is treated as a parameter in the analysis affecting the allowed concentration limits of the imaging system. The current analysis, takes into account the work of Edwards and Marx (1995a) and Edwards and Marx (1995b) on the simulation of the ideal spray and gives a qualitative insight on the effect of overlapping fringe patterns on the ability to process the resulting images in a quick and cost-effective way. As such, the design of new and the optimisation of existing interferometric sizing systems can be assisted.
2. Experiment simulation process

For the qualitative and quantitative determination of how fringe pattern overlapping affects the concentration limits of interferometric measurement techniques and the ability of the image processing software to extract meaningful information from the acquired images, we developed special software. The simulation process includes the generation and processing of artificial images of droplet fringe patterns for a region of a theoretical spray with known characteristics.

The images are artificially created with the main parameter driving the analysis being the ratio of the fringe pattern pixel area over the total image pixel area

$$\beta = \frac{\text{fringe pattern pixel image area}}{\text{total image pixel area}}$$

which is a measure of the fringe pattern size in the image. This parameter is defined by the optical arrangement of the real measurement system and is the result of the magnification the system exhibits in each dimension. When calculated accordingly for the shape of the aperture, $\beta$ allows for generic characterisation of the droplet overlapping probability and, therefore, the overall measurement capability. The fringe patterns in each image are considered to have the same size, i.e. implying that the Scheimpflug condition is met in the imaging optical configuration and no droplet depth variation is taking place in the laser sheet. In addition, the variability of the fringe pattern size that is the result of not enforcing the Scheimpflug condition can also be implemented, but this limits the generic nature of the results and requires the parameters of the optical assembly to be incorporated so that the variable magnification ratio in each pixel can be somehow defined. In the context of the present work, the size of the fringe patterns in the image is considered to be constant, consistent with Scheimpflug imaging configuration.

In contrast to the previous approaches, the morphology of the spray under investigation is taken into account. The user provides the total number of images and the mean number of droplets per image. We assume that the spray is ideal, as defined by Edwards and Marx (1995a. As such, the droplets are considered to be non-interacting, each one

![Fig. 3. The typical optical arrangement for capturing standard (left) and compressed (right) droplet fringe patterns.](image-url)
with an individual diameter and that the droplet field is “random” in nature and unaffected by events in the past or future (i.e. the spray is treated as a Markov process). The spray is considered to be homogeneous, which requires that the probability of \( n \) droplets arriving in the volume under examination follow a Poisson distribution

\[
P(n) = \frac{\lambda^n}{n!} e^{-\lambda},
\]

with \( \lambda \) the mean droplet arrival rate in the image. Due to \( \lambda \) being a constant, the spray is considered to be steady; however, an unsteady function \( \lambda(t) \) can be readily employed instead. All of the droplets are spatially positioned in each simulated image randomly, with a uniform probability distribution providing for the droplet centre coordinates in the image. This assumption is acceptable due to the generally small measurement area of interferometric sizing systems.

The droplets of this artificially imaged spray follow a typical Rosin-Rammler (Rosin and Rammler (1933 size distribution, whose probability density function is

\[
P(d) = \frac{q}{D} \left( \frac{d}{D} \right)^q \exp \left[ -\left( \frac{d}{D} \right)^q \right],
\]

with \( D \) being the mean droplet diameter and \( q \) is a parameter indicating the distribution spread. Since there is no readily available function to provide for random numbers that follow this particular distribution, the inversion principle of Devroye (1986 had to be used. The principle dictates that when random numbers following a uniform-probability distribution are being fed as input to the inverse function of a cumulative distribution function, the resulting values follow the desired distribution. In the case of the Rosin-Rammler, the cumulative distribution function is

\[
CDF_{RR}(d) = 1 - \exp \left[ -\left( \frac{d}{D} \right)^q \right]
\]

and therefore its inverse function is

\[
CDF_{RR}^{-1}(d) = D \left[ -\log(1-d) \right]_{\frac{1}{q}}.
\]

The random number generator provides the quantity \( d \) of Eq. (6) in a uniform-probability distribution manner. A sample histogram of generated droplet diameter values can be seen in Fig. 4 along with the continuous Rosin-Rammler size distribution that is being simulated.
It is apparent that random number generation is a very important step in the image creation process. Given the fact that there is no real “randomness” in computer-simulated random number, only pseudo-randomness generated by predefined number tables, there was a need for an efficient algorithm to handle the numerous random number calls. For this reason, all the random number generation processes made use of the effective Mersenne Twister algorithm of Matsumoto and Nishimura (1998 with the time of the software invocation being the random seed for the algorithm.

After the droplets are generated, they are “drawn” in the array that contains the artificial image. Each fringe pattern occupies the pixel area around its center as dictated by parameter $\beta$ of Eq. (2), with the intensity value ‘1’ being added on the pixels that surround the fringe pattern. As such, pixels that are occupied by more than one droplet have intensity values that are equal to the fringe patterns occupying them, as shown in Fig. 5. If the droplet pattern area percentage covered by values that are greater than ‘1’ is larger than the allowed overlapping percentage, then the droplet is rejected as invalid and so are its contribution in mass and volume in the illuminated measurement volume. The allowed overlapping percentage is a direct indication of the capability of the image processing software that is typically extracting the droplet information from the acquired images. As an example, Fig. 6 depicts one generated image with 116 droplets. The resulting standard ILIDS image is shown in Fig. 7 with $\beta=7.85\times10^{-3}$, while the compressed ILIDS image is given in Fig. 8 with $\beta=4\times10^{-4}$. For these two values and the size of the image set (1000x1000 pixels) both cases produce fringe patterns of the same length of 100 pixels, i.e. the size resolution is the same for both cases. The allowed overlapping percentage is set to be $v=20\%$ for both cases. The droplets whose overlapping area is not greater than 20\% are depicted as bright red. It is apparent that standard ILIDS is bound to identify a small number of droplets, even in such a relatively forgiving overlapping percentage.

Fig. 4 The Rosin-Rammler drop size distribution and the histogram of $10^6$ randomly generated diameter values with a mean value of $D=50\mu m$ and spread parameter $q=4$.

Fig. 5 The fringe patterns in the artificial images are rejected when the overlapping fringe pattern area percentage exceeds the effectiveness of the image processing software.
3. Results and discussion

Using the above algorithm for the allowed overlapping fraction applicable to the measurement system under investigation, an estimation of the number of droplets per image that can be processed by the image-processing software can be provided. In addition, by using the desired Rosin-Rammler size distribution and mean droplet arrival rate an estimation of the valid number, mass and volume concentrations per image can also be provided. Therefore, a quantification of the expected overlapping probability between neighbouring fringe patterns for a set of system parameters is achievable.

From the previous section, it is clear that the optimum number of measureable droplets on an ILIDS image is defined by two parameters, the parameter $\beta$ and the fringe pattern overlapping percentage that the image processing software allows. The value of $\beta$ is determined by the optical setup and is essentially the outcome of the combination of its focusing capabilities in conjunction to the current defocusing state. The more the optical system deviates from the focused state, the more a glare point in the focused image is transformed into a fringe pattern in the out-of-focus image and
\( \beta \) is the convenient parameter to characterise overlapping effects of the system, since it only requires knowledge of the fringe pattern's size in the images, a quantity easily measurable by the image processing software. Its inverse value, \( 1/\beta \), is an extreme estimation of the maximum number of droplets allowed in an image, if all droplets were arranged orderly and no unoccupied pixels exist. Also, due to the general ability of interferometric systems to have variable fringe pattern sizes it frees the analysis of involving the parameters of the optical assembly. For the above reason, the presented results can provide a guideline to the design and application of measurement systems with desired concentration conditions in the droplet imaging volume or quantifying the expected concentrations for a set optical system.

The second parameter, the allowed overlapping \( v \), is a measure of the image processing software capabilities and it states the maximum area in a fringe patterns that is shared with another fringe pattern that allows for the pattern to be properly identified and its size and velocity be estimated accurately. This parameter is something that has not been yet quantified due to the diversity of software that processes such images and the difference in the operating evaluation algorithm. In the present work, it is assumed that when the conservative limit of 20% area overlapping is crossed, the fringe pattern is considered non-measurable by most of the image processing software developed so far. Even so, this assumption can and probably will be challenged by the advancement of signal processing algorithms.

The nature of parameter \( \beta \) provides evidence that high concentrations can be reached, when the area a fringe pattern occupies is small, especially when there is a degree of overlapping allowed. It is apparent that high droplet number concentrations as high as the ones implied by \( 1/\beta \) are impossible to achieve especially for standard ILIDS and GPD. In the case of compressed ILIDS though, the vertical dimension of the fringe patterns is much smaller than the other making the respective \( \beta \) much smaller than the standard ILIDS or GPD cases, leading to realistically greater concentrations in the resulting images for the allowed overlapping set. Values of \( \beta \) between \( 10^{-3} \) and \( 10^{-4} \) are realistic, setting the maximum theoretical droplet number concentration values of more than 1000 droplets per image, albeit with concessions made in the number of droplets that can be evaluated.

Greater concentrations have a practical meaning in reducing the number of measurements and providing a larger flow-field, but also lead to sacrifices to the sizes of the measurable droplets as Eq. (1) indicates. Particularly, the minimum diameter can be evaluated in an ideal manner for a specific system when a minimum number of fringes is set, such as \( N_f=1 \). Similarly, the maximum diameter has the upper ideal limit that is set by the Nyquist theorem and is linked directly with \( \beta \) via the half-length of the fringe pattern as the number of undulations \( N_f \). Still, these limits are only taken from the simple sizing relation of Eq. (1), whereas only analysis of the scattering pattern from small droplets can provide the answer to the minimum measurable size. A complete quantification of the lowest measurable diameter has not yet materialised, but it surely is a highly sought quantification for the designing and optimising of measurement systems.

Considering the above, one can quantify the optimum number of droplets per image for various values of \( \beta \) and \( v \). Such a quantification is given in Fig. 9 for standard ILIDS/GPD and Fig. 10 compressed ILIDS for \( v=0\% \), i.e. no overlapping allowed by the image processing software. The images were assumed to be consisting of 1000×1000 pixels and \( \beta \) corresponds to various sizes of fringe patterns. The considered values for \( \beta \) correspond for this image size to individual droplet images of 50, 75, 100, 125, 150, 175 and 200 pixels in the horizontal dimension that contains the size information, while the vertical size of the compressed patterns is assumed to be 4 pixels—a typical size found in such systems. However, due to the nature of the analysis these results can be generalised for different image sizes and the corresponding fringe pattern sizes \( \beta \) applies to. Each point in these two graphs is the mean value of overlapping probability of 2000 images whose droplet number arrival rate \( \lambda \) is defined in the point's abscissa. Fig. 9 and Fig. 9Fig. 10 also indicate the droplet number concentration, even thought the analysis does not take it into account. This is
due to the imaging nature of interferometric measurement systems, which are drop-number dependent instead of drop mass/volume dependent. As such, a typical measurement area times the typical laser sheet depth can be set to provide indicative mass and volume concentrations; typical measurement volume sizes range from $10 \times 10 \times 1 \text{mm}^3$ to $20 \times 20 \times 15 \text{mm}^3$. As discussed in the previous section, the artificially created droplets from a water spray were allocated a random diameter that followed a Rosin-Rammler size distribution with parameters $D=50 \mu \text{m}$ and $q=4$. The above parameters create a spray with a wide drop size distribution. For that artificial spray, the liquid volume and concentration were also calculated. Fig. 11 depicts the liquid volume concentration for various fringe pattern sizes. It is clear that the measurement system has an optimal position for which it can resolve the maximum number of droplets, depending on the size of the fringe patterns and the droplet arrival rate.

It is apparent that the standard approach cannot match the concentration capability of the compressed version for the same pattern size case; the compressed fringe patterns are realistically measureable in most practical cases, whereas the standard approach struggles with images containing more than 50 droplets regardless of the fringe pattern size. Additionally, increasing the allowed overlapping does not improve substantially the number of validated droplets as Error! Reference source not found. and Error! Reference source not found. show, especially for the standard ILIDS/GPD case. It is clear though that for realistic $\beta$ values of more than $10^{-4}$, the probability of overlap is ranging between 10%–40% when 50–100 droplets are present and some sort of compromise must be considered, in conjunction with the nature of the measurement.

The maximum number of fringe patterns for the present calculations were 500, resulting in a relatively small number concentration of approximately 5 drops/mm$^3$ for the typical $10 \times 10 \times 1 \text{mm}^3$ measurement volume. By considering a larger CCD array, e.g. a 2000×2000pixel array, the $\beta$ parameter becomes four times smaller with the fringe retaining the minimum and maximum measureable diameters. However, the larger size of the array also increases the field of view and hence the measurement volume. Typical arrays like this have an approximate $15 \times 15 \times 1 \text{mm}^3$ field of view. Unless denser CCD arrays are readily available, this optical arrangement would definitely increase the droplet number concentration limit; however, it would be a value much lower than the 27.4 drop/mm$^3$ for the test system C, as reported by Damaschke et al. (2002). This is to be expected though, as that approach did not take into account the random nature of the spray under
investigation. Also, the ILIDS drop concentration is far lower compared to the number concentration of PDA that is 100 drop/mm$^3$. This result though is purely mathematical in nature derived by the existence only of a single droplet in a very small probe volume, a necessary assumption in PDA measurement. The above advantage in reality cancels out with the requirement of many more measurements that are required for the PDA to extract the same information that just a pair of ILIDS (standard or advanced) images can extract. Therefore, even though the PDA droplet concentration limit seems to be much greater for ILIDS to surpass, it is in reality quite larger in terms of the required time to complete a set of measurements. In this aspect, interferometric sizing systems demonstrate their full potential.

4. Conclusions

In the present contribution, we proposed a methodology for the quantification of the concentration limits of interferometric planar droplet sizing measurement systems. Our approach is taking into account the randomness and droplet size distribution of the spray under investigation to produce realistic evaluations of the droplet number, volume and mass concentration measurement capability of the optical system.

Although the compressed ILIDS system is a great improvement on the standard ILIDS/GPD systems, it still lags behind the concentration limits of the PDA. However, with proper design and analysis such a system can be optimised for measurements considerably, especially after taking into account the physiology of the spray in terms of droplet arrival rate. It is worth noting that when the

![Figure 11](image.png)

**Fig. 11** Liquid volume concentration of a spray following a Rosin-Rammler droplet size distribution with $Q=50\mu m$ and $q=4$ for a standard measurement volume (10x10x1mm$^3$) with compressed ILIDS.
mean diameter of the spray reduces, the droplet number density increases for the same amount of liquid mass and hence reduce the maximum number of resolvable fringe patterns. Knowledge of the optimal operation parameters for such systems can help to acquire droplet information from areas that are very dense by reducing the pattern overlapping. Such areas would not necessarily benefit from taking more measurements since the pattern overlapping of an un-optimised system would be dominating the acquired images.

The effect of the mean droplet size, size spread of the size distribution of the Rosin-Rammler distribution and number density on the maximum number density measured by ILIDS will be completed in the future.

References


