Comparison of planar PIV and tomographic PIV for aeroacoustics

V. Koschatzky,¹ E.F.J. Overmars,¹ B.J. Boersma¹ and J. Westerweel¹,*

¹Laboratory for Aero & Hydrodynamics, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands
*corresponding author: j.westerweel@tudelft.nl

Abstract We compare planar PIV and tomographic PIV in a thin volume applied to the flow over a two-dimensional cavity. The results are used to compare the source terms of both Curle’s analogy and vortex sound theory. The results confirm that there is little difference between the source terms computed in two or three dimensions, owing to the fact that the flow is nearly two-dimensional. Results also point out that Curle’s analogy is extremely sensitive to the presence of missing or damaged data.

1. Introduction

In this paper we compare the application of both planar particle image velocimetry (PIV) and tomographic PIV [1,2] for the estimation of sound emission. We consider an air flow over an open cavity. Previous measurements by means of planar PIV are described elsewhere [3]. In the planar PIV measurement only two out of three velocity components could be determined, and it was assumed that the flow in the cavity is statistically homogeneous along the cavity. However, for a full estimation of the sound generation, it is required that the full deformation tensor is determined. We therefore carried out a second set of measurements by means of tomographic PIV.

The measurements provide sufficient velocity data in the span-wise direction to allow for the computation of the differential flow quantities that appear in both Curle’s analogy [4] and vortex sound theory [5]. These two approaches were used to compute the sound emission and the sound level in the far field.

We use the tomographic PIV data to determine the contribution of the missing components and derivatives in the planar PIV measurements. By using only two out of three velocity components in a single plane we determine the effect of not having the third velocity component available. In a similar manner we exclude the out-of-plane derivatives.

2. Estimation of sound source terms for planar and volumetric PIV data

A detailed description of the estimation of sound source terms from PIV data by means of Curle’s analogy [4] and vortex sound theory [5] is described by Koschatzky et al. [3,7]. Here we compare the estimation from planar PIV data, when only two out of three velocity components are available and no out-of-plane gradients, with the estimation from thin-volume tomographic PIV data, when the full deformation tensor (in a planar domain) is available. We consider the case of a flow over an open spanwise cavity. Although the mean flow in this case can be considered as (nearly) two-dimensional, the instantaneous flow, especially inside the cavity, is three-dimensional. This geometry is representative for a number of flows relevant to aeroacoustic studies, where the mean flow is (nearly) two-dimensional.

Curle’s analogy source terms

The source term for Curle’s analogy is the pressure time derivative at the cavity walls [7]. In the 2-
D computation, the pressure field is obtained by solving the Poisson equation:

\[
\frac{\partial^2 p}{\partial y_1^2} + \frac{\partial^2 p}{\partial y_2^2} = -p_0 \left[ \frac{\partial}{\partial y_1} \left( \frac{Du_1}{Dt} \right) + \frac{\partial}{\partial y_2} \left( \frac{Du_2}{Dt} \right) \right]
\]  

with Neumann and Dirichlet boundary conditions:

\[
\begin{align*}
\frac{\partial p}{\partial y_1} &= -\rho_0 \frac{Du_1}{Dt} \quad \text{along boundaries with } x = \text{constant} \\
\frac{\partial p}{\partial y_2} &= -\rho_0 \frac{Du_2}{Dt} \quad \text{along boundaries with } y = \text{constant} \\
p &= \frac{1}{2} p_0 \left[ U_0^2 - \left( u_1^2 + u_2^2 \right) \right] \quad \text{at the upper domain boundary}
\end{align*}
\]  

Here the vector \( y = (y_1, y_2, y_3) \) represents the position of the source. The velocity material derivatives are computed in a Lagrangian reference frame along path lines that remain in the measurement plane. The pressure material derivative is computed in the same way as the velocity material derivatives, and the pressure time derivative is obtained by [9]:

\[
\frac{\partial p}{\partial t} = \frac{Dp}{Dt} - \left( u_1 \frac{\partial p}{\partial y_1} + u_2 \frac{\partial p}{\partial y_2} \right)
\]  

In the three-dimensional computation, the complete three-dimensional Poisson equation is used:

\[
\frac{\partial^2 p}{\partial y_1^2} + \frac{\partial^2 p}{\partial y_2^2} + \frac{\partial^2 p}{\partial y_3^2} = -\rho_0 \left[ \frac{\partial}{\partial y_1} \left( \frac{Du_1}{Dt} \right) + \frac{\partial}{\partial y_2} \left( \frac{Du_2}{Dt} \right) + \frac{\partial}{\partial y_3} \left( \frac{Du_3}{Dt} \right) \right]
\]  

with the Neumann and Dirichlet boundary conditions:

\[
\begin{align*}
\frac{\partial p}{\partial y_1} &= -\rho_0 \frac{Du_1}{Dt} \quad \text{along boundaries with } x = \text{constant} \\
\frac{\partial p}{\partial y_2} &= -\rho_0 \frac{Du_2}{Dt} \quad \text{along boundaries with } y = \text{constant} \\
\frac{\partial p}{\partial y_3} &= -\rho_0 \frac{Du_3}{Dt} \quad \text{along boundaries with } z = \text{constant} \\
p &= \frac{1}{2} p_0 \left[ U_0^2 - \left( u_1^2 + u_2^2 + u_3^2 \right) \right] \quad \text{at the upper domain boundary}
\end{align*}
\]  

As for the two-dimensional computation the velocity material derivatives are computed in a Lagrangian reference frame. But in this case, the path-lines can develop in every direction inside the measurement volume. The pressure material derivative is computed in the same way as the velocity material derivatives, and the pressure time derivative is obtained by:

\[
\frac{\partial p}{\partial t} = \frac{Dp}{Dt} - \left( u_1 \frac{\partial p}{\partial y_1} + u_2 \frac{\partial p}{\partial y_2} + u_3 \frac{\partial p}{\partial y_3} \right)
\]  

**vortex sound theory source term**

The source term for vortex sound theory is the time derivative of the term \((\omega \times \mathbf{u}) \cdot \nabla Y_i\) in the source region [7]. Here \( \mathbf{Y} = (Y_1, Y_2, Y_3) \) is the Kirchoff vector for the surface and cavity [7]; for a detailed description and interpretation of \( \mathbf{Y} \), we refer to the work of Howe [5,8]. For the two-dimensional computation only the components \( u_1 \) and \( u_2 \) of the velocity field and the component \( \omega_3 \) of the vorticity field are used. Therefore, the term \((\omega \times \mathbf{u}) \cdot \nabla Y_i\) reduces to:

\[
(\omega \times \mathbf{u}) \cdot \nabla Y_i = -\omega_3 u_2 \frac{\partial Y_i}{\partial y_1} + \omega_3 u_1 \frac{\partial Y_i}{\partial y_2}
\]  

\[
(\omega \times \mathbf{u}) \cdot \nabla Y_i = -\omega_3 u_2 \frac{\partial Y_i}{\partial y_1} + \omega_3 u_1 \frac{\partial Y_i}{\partial y_2}
\]  

(7)
As for the time derivative in Curle’s analogy, the material derivative \((D/Dt)((\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i)\) is computed in a Lagrangian reference frame along in-plane path-lines. The time derivative \((\partial/\partial t) \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] \) is therefore calculated as:

\[
\frac{\partial}{\partial t} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] = \frac{D}{Dt} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] - \left\{ u_1 \frac{\partial}{\partial y_1} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] + u_2 \frac{\partial}{\partial y_2} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] \right\}
\]

(8)

For the three-dimensional computation all three components of the velocity and vorticity are used. Therefore, the term \((\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i\) is (given that \(\partial Y_i/\partial y_3 = 0\) by definition [7]):

\[
(\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i = (\mathbf{\omega} \times \mathbf{u}) \cdot \left[ \frac{\partial Y_1}{\partial y_1} + (\mathbf{\omega} \times \mathbf{u}) \cdot \left( \frac{\partial Y_1}{\partial y_1} + \frac{\partial Y_2}{\partial y_2} \right) \right]
\]

(9)

The material derivative \((D/Dt)((\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i)\) is computed in a Lagrangian reference frame along path-lines that can develop in any direction inside the measurement volume. The time derivative \((\partial/\partial t) \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] \) is therefore calculated as:

\[
\frac{\partial}{\partial t} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] = \frac{D}{Dt} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] + \\
\left\{ u_1 \frac{\partial}{\partial y_1} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] + u_2 \frac{\partial}{\partial y_2} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] + u_3 \frac{\partial}{\partial y_3} \left[ (\mathbf{\omega} \times \mathbf{u}) \cdot \nabla Y_i \right] \right\}
\]

(10)

3. Experimental setup

The optical configuration is shown in Figure 1. The cavity has a length of 30 mm and a depth of 15 mm, and spans the whole width (600 mm) of the test section. We used four high-speed digital cameras to record the flow in a thin volume with a thickness of 4 mm. Images were recorded at a frame rate of 6 kHz with a 70 µs time delay for each frame pair. The free-stream flow velocity was 12 m/s, giving a Reynolds number of 12\times10^3.

Measurements were conducted in the vertical open-jet wind tunnel of the Low-speed Aerodynamics Laboratories of the Aerospace Department at the Delft University of Technology. The cavity was machined in a flat plate and positioned in the test section aligned with the flow. The cavity had a length \(L\) of 30 mm, a depth of 15 mm and a width \(W\) of 600 mm, spanning the full width of the tunnel and giving a width to length ratio of \(W/L = 20\).

The PIV system consisted of four 12-bit 1024×1024-pixel cameras (Photron FastCAM A1S), used at a 448×832-pixel image format that allows an increase of the frame rate up to 12 kHz, and a dual-cavity pulsed Nd:YLF laser (Quantronix Darwin Duo). The flow was seeded by means of a stage smoke generator (Safex) that produces particles of approximately 1 µm in diameter. PIV image
pairs were acquired at a rate of 6 kHz with a time delay of 70 $\mu$s between the first and second frame. This time delay was chosen as an optimum between two opposing demands: it allows enough particle-image displacement between the two frames in the slower flow regions inside the cavity (of the order of 2-3 pixels) while keeping it within reasonable limits in the regions with a faster flow (typically 16 pixels in the free-stream area). The laser sheet thickness was estimated to be approximately 4 mm. This allowed for the reconstruction of sufficient vectors in the span-wise direction to compute differential quantities in that direction. Data were processed using commercial software (DaVis, LaVision GmbH).

The three-dimensional calibration was performed by translating a perforated target across the interrogation volume. The target was 1 mm thick and the holes were 2 mm spaced in both directions. The target was moved in steps of 1 mm from -3 mm to +3 mm with respect to the central location. Residual calibration errors were corrected with a self-calibration procedure [10]. For this purpose we used a series of recordings with a low-seeding density. The disparity map was computed by dividing the measurement volume into $5 \times 5 \times 5$ sub-volumes.

The imaged volume was reconstructed by four MART iterations [1]. Data were processed using a multi-pass algorithm [11] and window deformation [12] to accommodate the large dynamic spatial range and dynamic velocity range of the flow [2]. The interrogation windows were placed in such a way that their boundaries coincided with the walls of the cavity. The distance between the cavity walls and the location of the first vector is therefore half the size of an interrogation window. The final passes in all domains were done with cubical interrogation windows of $32 \times 32 \times 32$ pixels with a 75% overlap, giving a vector spacing of 0.47 mm throughout the entire measurement domain.

Figure 2 shows the measured turbulence intensity and the mean flow field of the cavity flow. The flow approaching the cavity has a laminar boundary layer over the upstream plane wall. The ratio of the cavity length and the boundary layer momentum thickness is $L / \theta = 104$, so that the flow over the cavity induces a “shear layer mode” cavity flow [6].
Figure 3 Instantaneous flow field. Left: iso-surfaces represent vorticity magnitude (green) and Q-criterion (red). Slices represent out-of-plane velocity contours (in m/s) and velocity vectors. Right: iso-surfaces represent source terms for vortex sound theory; the slice at the aft cavity wall the source term for Curle’s analogy.

Figure 4 Deviations of some flow quantities and of the source terms computed from planar PIV data extracted from the volumetric data relative to the fully three-dimensional tomographic PIV data. Data are normalized with the free-stream velocity $U$, the cavity length $L$, and the fluid density $\rho$. 
3. Results

We apply both Curle’s analogy and vortex sound theory to compute the source terms for the sound generation and the far field sound level. Figure 3 shows a snapshot of the two source terms obtained with the three-dimensional computation. The iso-surfaces represent positive (in red) and negative (in blue) iso-values of the vortex sound theory source term, \((\partial/\partial t) [\[\hat{\Gamma}\] \cdot \nabla \chi]\). The contour plots at the aft wall of the cavity represent the Curle’s analogy source term, \(\partial p/\partial t\). The time between successive frames is 167 µs. The images confirm the (near) two-dimensional form of the source terms in the measurement domain.

Figure 4 shows the root-mean-square statistical deviation between the two and three-dimensional computation for some flow quantities and for the source terms in the two acoustic analogies. The statistical deviation is appropriately normalized with the free-stream velocity, the cavity length, and the fluid density. Results for the divergence (Fig. 4a) and for the vortex sound theory source term (Figs. 4c,e) show there is a little difference between the 2-D and 3-D computations. The largest deviations occur in the shear layer close to the aft wall of the cavity. This is expected since the absolute values of the investigated quantities are expected to be larger in this region than elsewhere. The deviations for the pressure and its time derivative, and the source term for Curle’s analogy (Figs. 4b,d) are dominated by the low quality data region after the cavity and close to the wall. The pressure field is in fact obtained by iteratively solving a Poisson equation. Therefore, local errors propagate into the whole integration field. The deviation between the 2-D and 3-D computed pressure and time derivative would be, by itself, rather small and is therefore dominated by the spurious data. Unfortunately, a careful preprocessing of the images could not recover the signals completely. Moreover, the position of the area with damaged data did not allow for its exclusion from the computation.

We now compare the estimates of the sound emission in the far field computed with Curle’s analogy and vortex sound theory. For both analogies we compare the results obtained with the source terms computed in two and three dimensions from the three-dimensional tomographic PIV data.

![Figure 5 Results for Curle’s analogy: (a) power spectra in the far field and (b) directivity plots at the same distance. Comparison of the sound emission estimated from a high-resolution planar PIV measurement with the sound emission estimate from planar PIV data extracted from the tomographic PIV data, and with those from the full 3-D tomographic PIV data.](image-url)
measurements, as well as the results obtained from the source terms computed in two dimensions a high-resolution planar PIV measurement [3]. The estimation of the sound emission from the computed source terms is described elsewhere [7]. The results for the far-field spectra and directivity plots for the computation using Curle’s analogy are given in Figure 5, while the corresponding results for the computation using vortex sound theory are given in Figure 6.

Both approaches generate the primary sound mode that is generated by the cavity. It is noted that the sound levels obtained by Curle’s analogy shows higher levels for higher frequencies. This could be attributed to errors in the measurement data in the flow near the aft wall of the cavity. Comparison with the planar PIV data and with microphone data reveals that the theory of vortex sound produces more accurate and more reliable results.

4. Conclusions

In this paper we discuss the results obtained by time-resolved thin-volume tomographic PIV measurements. These measurements provided sufficient velocity vectors in the span-wise direction to allow for the calculation of the differential quantities in that direction and therefore for the computation of the full source terms of both Curle's analogy and vortex sound theory. We could therefore study the uncertainty in computing acoustic source terms from planar PIV data for case of a flow that exhibits span-wise coherency.

The flow statistics show that the flow is indeed two-dimensional. The difference in the computed acoustic source terms from the planar data and volumetric data from the same set of measurements confirmed that there is little difference between the two computations. The larger deviations occur in the shear layer region were the source terms are large because of the large intensities of the investigated quantities, but were the flow is nearly two-dimensional. Inside the cavity, where the flow is three-dimensional and might play a significant role, the differences are small. The statistical analysis showed that the computation of the Curle's analogy source term is heavily affected by the presence of an area of damaged data in the measurement volume. This introduces a spatially localized high-intensity broadband noise in the time series. The different formulation of the source terms for the vortex sound theory prevents that the result becomes seriously affected by this area of damaged data. Finally, we computed directivity plots and spectra of the sound emission. The results confirm that there are no significant differences between the sound estimate computed in two or three dimensions, at least for the spatial and temporal resolution we could achieve in our
experiment. The vortex sound theory sound emission estimate also matches well with the estimate obtained from planar PIV data and actual microphone data obtained previously [3]. When compared to two-dimensional measurements, volumetric PIV measurements involve a much higher cost, a higher level of complexity of the experimental setup, and long processing times of the measured image data. It is noted that the experimental data achievable through tomographic PIV are, in general, of lower quality than planar data. This is due to ambiguities in the volume reconstruction and to the lower seeding density applied in tomographic PIV. It is therefore not possible to achieve small interrogation windows, which limits the spatial resolution of the measurement. Other complications in our experimental setup were the presence of solid boundaries in the measurement field, which led to low signal to noise ratio and some localized strong reflections that significantly affected the final results. In conclusion, thin-volume tomographic PIV measurements do not seem to add relevant information to the computation of the sound emission for our experimental case, which is representative of quasi two-dimensional wall-bounded flows where the main source of sound is determined by large span-wise coherent flow structures.

References