Quad-Plane Stereoscopic PIV in Turbulent Jet

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Abstract To investigate fine scale structure of turbulent flows, quad-plane stereoscopic particle image velocimetry (QPSPIV) has been developed by utilizing the difference of laser wavelength and polarization. The system of QPSPIV consists of two Nd:YAG lasers, two dye lasers, eight high resolution CCD cameras and several optics. QPSPIV provides three-dimensional distribution of three components of velocity and nine components of velocity gradient with the same spatial resolution of general direct numerical simulation (DNS) of turbulence, $2.7\eta \times 2.7\eta \times 2.4\eta$. In QPSPIV, in-plane velocity gradient components can be calculated by a 4th-order central finite difference scheme, and out-of-plane velocity gradient components can also be calculated by a 4th-order finite difference scheme. The quality of quad-plane laser sheets is investigated carefully. Intensity profiles of quad-plane laser sheets show that the beam profile of each plane is clearly separated, and that the parallelism is enough for QPSPIV. The developed PIV system was applied to velocity measurements in a turbulent jet. Probability density functions of velocity gradients obtained by this QPSPIV system represent the intermittent nature of turbulence very well and those shapes well coincide with those obtained by DNS. The present results show that the QPSPIV system gives the three-dimensional distribution of the full velocity gradient tensor with a spatial resolution good enough to capture the fine scale structure. From these velocity gradients, various flow properties such as vorticity vectors, the second invariant of velocity gradient tensor and the energy dissipation rate are obtained faithfully. The fine scale eddy can be detected from the isosurface of the positive second invariant of the velocity gradient tensor.

1. Introduction

In our previous studies on fine scale structure of turbulence by direct numerical simulation (DNS) (Tanahashi et al., 1997, 2001, 2004a, 2007; Wang et al., 2007), the existence of universal fine scale structure (coherent fine scale structure), which is independent of the Reynolds number and types of the flow field, has been revealed. The diameter and the maximum azimuthal velocity of coherent fine scale eddies can be scaled by Kolmogorov length ($\eta$) and Kolmogorov velocity ($u_k$). Except for near wall turbulence (Tanahashi et al., 2004a), the most expected diameter and maximum azimuthal velocity are $8\eta$ and $1.2u_k$. It should be noted that the azimuthal velocity of intense fine scale eddies reaches $3 \sim 4u'_{rms}$. Coherent fine scale eddies have strong three dimensional characteristics and are advecting with velocity of the order of $u'_{rms}$ (Tanahashi et al., 1999). To investigate these fine scale structures experimentally, simultaneous measurement of three components of velocity and nine components of velocity gradient is required. Since the coherent fine scale structure in turbulence possesses strong swirling motion and determines the characteristics in fine scales, it has very important roles on the friction drag on the wall (Tanahashi et al., 2004a), turbulent heat and mass transfer (Miyauchi et al., 2002), particle dispersion in turbulence, turbulent combustion (Tanahashi et al., 2000; Nada et al., 2004), etc..

Particle image velocimetry (PIV), which gives instantaneous maps of the velocity field, is a useful, firmly established, non-intrusive measurement technique (Dudderrar and Simpkins, 1977; Keane and Adrian, 1990). Many research groups have explored PIV extensively over the past decade to apply PIV measurement to various industrial purposes as well as fundamental researches. Kähler and Kompenhans (2000) first proposed dual-plane stereoscopic PIV (DPSPIV) (they called
it multiple plane stereo PIV) which can investigate three components of velocity and nine components of velocity gradient. They separated the scattering light from parallel laser sheet by using orthogonally polarized light sheets with polarizing beam splitters. Hu et al. (2001) measured large-scale structure of a lobbed jet flow with DPSPIV by the same principle. The other way to separate the scattering light was proposed by Mullin and Dahm (2005). Instead of polarization, they used green and red light sheets with color filter. However, these PIV systems require four lasers (or two sets of double-pulsed lasers) to produce dual-plane laser sheets and four cameras to capture stereoscopic particle images on the dual planes. The cost for the additional lasers may account for a large portion of the total cost for purchasing a DPSPIV system. Therefore, the reduction of laser requirements contributes to the dissemination of DPSPIV. Tanahashi et al. (2008) developed the parallel beam forming optics, which allow polarization-based DPSPIV by adding only two extra-cameras to the conventional stereoscopic PIV system. In those DPSPIV measurements, velocity gradient in the in-plane direction can be calculated by using higher than the 4th-order central finite difference scheme, but calculation of velocity gradient in the out-of-plane direction has no choice but to apply 2nd-order central finite difference scheme. Furthermore, the separation between laser sheets is restricted by laser sheet thickness. For these reasons, higher accuracy in calculating out-of-plane velocity gradient components is needed for turbulence measurement under high Reynolds number condition.

In this study, to introduce a 4th-order finite difference scheme in calculating out-of-plane velocity gradient components, quad-plane stereoscopic PIV (QPSPIV), has been developed. From measurement results in four planes, the accuracy of the QPSPIV system is evaluated, and the QPSPIV system is applied to a measurement of fine scale structures in a turbulent jet.

2. Quad-plane stereoscopic PIV
2.1 Setup of QPSPIV

Figure 1 shows a schematic of the QPSPIV system. This system consists of two Nd:YAG lasers (Quanta-Ray, INDI-40, 532 nm, 200 mJ/pulse), two dye lasers (Lambda Physik, Scannmate2) with Rhodamine6G dye, eight high resolution CCD cameras (Roper Scientific, Megaplus II ES4020, 2048 × 2048 pixels) and several optics. QPSPIV can be accomplished by utilizing the difference of laser wavelength and polarization. The polarization of laser beams from Nd:YAG lasers are rotated
by half-wave plates to adjust the intensities of particle images at 532 nm and 560 nm to be the same. Each 532 nm laser beam from Nd:YAG lasers is divided into horizontal polarization and vertical one through the polarizing beam splitter. The beams with vertical polarization pump the dye lasers. The laser beam with each wavelength is guided by the half-wave plates and the polarizing beam splitters, and becomes double-pulsed beam. Two sets of parallel beams are formed from each wavelength beam through the parallel beam forming optics (Tanahashi et al., 2008), and four double-pulsed beams for quad-plane laser sheets which consist of different wavelength and polarization are led to parallel axes by a dichroic mirror. The four parallel beams are expanded by sheet forming optics, and illuminate tracer particles. The polarizing beam splitters and band-pass filters separate the scattered light from the particles in four laser sheets, particle images in respective measurement planes are obtained with eight CCD cameras. The lens (f# 5.6 : Nikon, Micro-Nikkor ED 200mm), which are equipped with the CCD cameras, are located with an inclination angle of 20° with respect to normal to the measurement plane to capture stereoscopic particle images. In order to focus clearly on the whole measurement region, the Scheimpflug condition is applied.

2.2 Calculation of velocities and velocity gradients

Velocity vectors are computed using 60 × 60 pixels interrogation window for 1st step and 30 × 30 pixels for second (final) step with overlap of 50% for the final interrogation window. From two-dimensional velocity fields obtained by two CCD cameras, three-component velocity vectors on a two-dimensional plane are calculated by using 2D calibration method (Prasad, 2000). In this study, spatial cutoff filter which has been developed in our previous study (Tanahashi et al., 2002) is used to eliminate high-wave-number noises introduced by the overlap of the interrogation windows. Note that the number of spurious vectors included in the raw data is very few in the present measurements, and the elimination scheme is mainly used to remove the high-wave-number noises which exceed the spatial resolution of PIV.

From three velocity components on the four measurement planes, nine velocity gradients are calculated on the three planes, which are defined at the middle of two each planes lying next to each other of the four measurement planes. The velocities on the three planes are interpolated by 3rd-order Lagrange interpolation, and in-plane velocity gradient components (∂u/∂x and ∂u/∂y) are derived by a 4th-order central finite difference scheme using the interpolated velocities. Out-of-plane velocity gradient components (∂u/∂z) are calculated by a 4th-order finite difference scheme using the velocities on the four planes defined as

\[
\frac{\partial f}{\partial z} = \sum a_i f(z_i),
\]

where

\[
a_1 = \frac{dz_2 dz_3 + dz_3 dz_4 + dz_4 dz_2}{(dz_1 - dz_2)(dz_1 - dz_3)(dz_1 - dz_4)},
\]

\[
a_2 = \frac{dz_1 dz_3 + dz_2 dz_4 + dz_4 dz_1}{(dz_2 - dz_1)(dz_2 - dz_3)(dz_2 - dz_4)},
\]

\[
a_3 = \frac{dz_1 dz_2 + dz_3 dz_4 + dz_4 dz_3}{(dz_3 - dz_1)(dz_3 - dz_2)(dz_3 - dz_4)},
\]

\[
a_4 = \frac{dz_1 dz_2 + dz_2 dz_3 + dz_3 dz_4}{(dz_4 - dz_1)(dz_4 - dz_2)(dz_4 - dz_3)},
\]

\[
dz_i = z_i - z (i = 1, 2, 3, 4).
\]
3. Experimental conditions

3.1 Experimental apparatus and conditions

Figure 2 shows the experimental apparatus of a turbulent jet (Tanahashi et al., 2003, 2004b, 2008). The air from a compressor is regulated to a constant pressure and constant flow rate by pressure regulators and a digital mass flow controller, and is seeded with droplets generated by an atomizer (TSI, Six-Jet Atomizer, Model 9306). Then, the air is led to a nozzle to form a jet. The external and internal diameters of the nozzle are 12 mm and 4 mm, respectively. To prevent spreading of particles to circumference, the test section is surrounded by an acrylic duct with a cross section of 320 mm × 320 mm.

In this study, the measurement is carried out at \( Re_D = 5316 \) and \( x/D = 60 \), \( r/D = 0 \), where \( Re_D \) is the Reynolds number based on the nozzle diameter \( D \) and mean axial velocity at the jet exit. \( x \) represents the mainstream direction and is distance from the jet exit and \( r \) denotes the radial coordinate. Preliminary measurements by a hotwire constant temperature anemometer with an X-probe (Kanomax Japan, Model0250R, tungsten, \( \phi = 5 \mu m \)) show that mean velocity \( u_m = 1.63 \) m/s, r.m.s. of velocity fluctuation \( u'_{rms} = 0.443 \) m/s, the Taylor microscale \( \lambda = 2.13 \) \( \mu m \), the Kolmogorov length \( \eta = 137 \) mm, the Kolmogorov velocity \( u_k = 0.110 \) m/s, and the Reynolds number based on the Taylor microscale \( Re_\lambda = 76.8 \). The global features of the flow including \( u_m, u'_{rms}, \) r.m.s. of axial velocity fluctuation \( v'_{rms} \) and the Reynolds stress \( \overline{u'v'} \) were presented in the previous work (Tanahashi et al., 2004b), and agreed with those obtained by Wygnanski and Fiedler (1969) and Panchapakesan and Lumley (1993). Based on these statistics, the field of view is set to 25.7 mm × 25.7 mm for 2048 × 2048 pixels, which gives the spatial resolution of 2.7\( \eta \) corresponding to the interrogation window size of 30 × 30 pixels. This spatial resolution is nearly the same with that of general DNS of turbulent flows. The time interval between successive images (\( \Delta t \)) is set to 40 \( \mu s \). In this study, DOS (dioctyl sebacate) is used as tracer particles (mean diameter is about 1 \( \mu m \)). From the experimental conditions, \( St_\eta = 1.63 \times 10^{-3} \).

3.2 Characteristics of quad-plane laser sheets

In QPSPIV, it is important to adjust the distance and parallelism between measurement planes carefully, because these parameters directly affect the accuracy of out-of-plane velocity gradient components. The thickness of laser sheets is also essential to determine the spatial resolution of
**Table 1** Characteristics of laser sheets.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>560</th>
<th>560</th>
<th>532</th>
<th>532</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength [nm]</td>
<td>560</td>
<td>560</td>
<td>532</td>
<td>532</td>
</tr>
<tr>
<td>Polarization (vertical or horizontal)</td>
<td>v</td>
<td>h</td>
<td>h</td>
<td>v</td>
</tr>
<tr>
<td>Pulse energy of Laser1 [mJ/pulse]</td>
<td>9.4</td>
<td>9.4</td>
<td>22.6</td>
<td>22.6</td>
</tr>
<tr>
<td>of Laser2 [mJ/pulse]</td>
<td>5.5</td>
<td>5.5</td>
<td>14.4</td>
<td>14.4</td>
</tr>
<tr>
<td>Laser sheet thickness of Laser1 [µm]</td>
<td>257</td>
<td>249</td>
<td>327</td>
<td>216</td>
</tr>
<tr>
<td>of Laser2 [µm]</td>
<td>243</td>
<td>261</td>
<td>320</td>
<td>257</td>
</tr>
<tr>
<td>Separation [µm]</td>
<td>349</td>
<td>676</td>
<td>310</td>
<td>257</td>
</tr>
<tr>
<td>Parallelism [mrad]</td>
<td>&lt; 1.96</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3 Intensity profile of the quad-plane laser sheets in perspective direction at the center of the measurement region.

Fig. 4 Distributions of the laser intensity peak in x-z cross section (a) and y-z cross section (b).

Velocity in the z direction. Mullin and Dahm (2005) measured these values by traversing a knife edge and detecting the transmitted light onto a photodiode. Ganapathisubramani et al. (2005) adopted the “burn method” to estimate them easily and roughly. In the present study, these values are measured by a beam profiler (Ophir, FX-50).

Figure 3 shows one example of the measured intensity profiles of the quad-plane laser sheets for each laser with Gaussian fits. The laser sheets for four lasers with the same wavelength and polarization coincide very well and have almost the same intensity. From the Gaussian fits of these profiles, the centers of the laser sheets are determined. The locations are plotted in Fig.4. The coordinate parallel to the light path is denoted as y. This figure also confirms the good agreement of the path of the four laser sheets. Since the distances between two laser sheets can be set arbitrarily within about 700 µm by translating the second polarizing beam splitter in the parallel beam forming optics, the laser sheets are aligned appropriately to prevent them from overlapping with each other. Parallelism between the laser sheets is examined from the distribution of the center points in x and y directions. As a result, the divergence angles between two planes are less than 1.96 mrad, which is enough for QPSPIV. The present laser sheet thicknesses and separations in detail are shown in
They performed “single-plane image assessment”, “coincident-plane image assessment” and “separated-plane image assessment” to ensure the accuracy of velocity gradients. In the separated-plane image assessment, the divergence error $\partial u_i/\partial x_i$ for incompressible fluid was reported to

Table.1. To evaluate the error due to the scattering from the other planes, polarized light intensities from single polarized laser beam exposure is measured. The error due to the scattering on the other planes is less than 3.5%.

4. Performance of QPSPIV
4.1 Nine velocity gradients and turbulence statistics

Probability density functions of longitudinal and lateral derivatives of velocity components at the center of the measurement planes are shown in Fig.5. The derivatives are normalized by their r.m.s. values. The results obtained from the QPSPIV are compared with those from DNS of homogeneous isotropic turbulence with $Re_d = 60.1$ and 97.1. The longitudinal and lateral derivatives obtained by the QPSPIV agree very well with those by DNS. The shape of probability density functions, which is represented by skewness and flatness factors in general, is nearly the same with that obtained by DNS. These results show that the present QPSPIV system gives all velocity gradients with enough accuracy. Figure 6 shows probability density functions of longitudinal and lateral derivatives of velocity components in the out-of-plane direction. This figure shows that accuracy of velocity gradients in the out-of-plane direction which is calculated by using a 4th-order finite difference scheme is higher than by using a 2nd-order central finite difference scheme. The accuracy of all nine velocity gradients obtained by DPSPIV was assessed carefully by Mullin and Dahm (2006). They performed “single-plane image assessment”, “coincident-plane image assessment” and “separated-plane image assessment” to ensure the accuracy of velocity gradients. In the separated-plane image assessment, the divergence error $\partial u_i/\partial x_i$ for incompressible fluid was reported to
estimate the accuracy of on-diagonal component of the velocity gradient tensor, and their r.m.s. value was 0.35. The r.m.s. value for the QPSPIV is 0.43 in the center of the measurement region and 0.45 for all the data. These results suggest that the present QPSPIV system gives the gradient of the velocity with the good enough accuracy.

From the nine-component velocity gradients obtained by QPSPIV, vorticity vectors \( \omega \), the second invariant of velocity gradient tensor defined as

\[
Q = -\frac{1}{2} (S_{ij}S_{ij} - W_{ij}W_{ij})
\]  

and energy dissipation rate \( \varepsilon (= 2vS_{ij}S_{ij}) \) can be estimated directly without the assumptions of isotropy and Taylor’s hypothesis. Here \( S_{ij} \) and \( W_{ij} \) denote symmetric and antisymmetric parts of the velocity gradient tensor;

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]  

\[
W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)
\]
Fig. 8 Distributions of velocity vectors and the second invariant of velocity gradient tensor, and an isosurface of the positive second invariant.

In the present study, $\eta$ and $u_k$ obtained from the preliminary measurements by a hot-wire are estimated based on the assumption of isotropy and Taylor’s hypothesis. On the other hand, since $\varepsilon$ ($\varepsilon = 19.0 \text{ m}^2/\text{s}^3$) is derived without any assumption from the present QPSPIV, $\eta = (\nu^3/\varepsilon)^{1/4}$ and $u_k$ ($= (\varepsilon \nu)^{1/4}$) are calculated more precisely. The results obtained by the QPSPIV are $\eta = 115 \mu\text{m}$ and $u_k = 0.13 \text{ m/s}$. Note that these values are calculated using the data near the center of the measurement region.

4.2 Typical measurement results by QPSPIV

Figure 7 shows a typical distribution of fluctuating velocity vectors and vorticity ($\omega_z$) on the four planes. Here, red color represents positive vorticity and blue does negative one. The size of the visualized region is $7.4 \text{ mm} \times 7.4 \text{ mm}$, and the visualized distance between measured planes is five times the real scale for easy understanding. Since the measurement is conducted in a fully developed turbulent jet, fluctuations of velocities and vorticity are relatively high. The existence of the vortex structure which penetrates the measurement planes is observed.
Figure 8 shows the distributions of velocity vectors and the second invariant of velocity gradient tensor, and isosurface of the second invariant. The distributions are shown at the three planes where the velocity gradients are computed. Here, red color represents positive second invariant of velocity gradient tensor and blue does negative one. The visualized distance between measured planes is seven times of the real scale. Positive second invariant of velocity gradient tensor enables detection of the fine scale eddy. It is confirmable that the fine scale eddy having the axis normal to the measurement plane exists, and that the swirl motion of the fluid around the eddy occurs. From these results, the QPSPIV system enables to capture the three-dimensional fine scale structure in turbulence, and is an efficient measurement technique to reveal turbulent structure.

5. Conclusion

In this study, a QPSPIV system has been developed to investigate fine scale structure of turbulence. QPSPIV can be accomplished by utilizing the difference of laser wavelength and polarization. Intensity profile of the quad-plane laser sheet shows small enough divergence angle and parallelism for QPSPIV. The QPSPIV system was applied for turbulence measurement in a turbulent jet. The developed QPSPIV system provides three components of velocity and nine components of velocity gradient with the same spatial resolution as that of general DNS of turbulent flows. It has been shown that probability density functions of the measured nine velocity gradients agree well with those obtained from DNS. Fine scale structure in turbulence is well captured three-dimensionally by QPSPIV. The present study demonstrates that QPSPIV system is an efficient measurement technique to reveal turbulent structures.

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