Applications of Density Tagging Velocimetry

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Recently, a novel optical technique for point-wise measurement of flow velocity called Density Tagging Velocimetry (DTV) was proposed by Raffel et al (2012). This method is based on the detection and subsequent tracking of a local density variation deliberately inserted in the flow. The experimental implementation comprising tagging, detection, and velocity evaluation reverts to and combines principles of well–known optical measurement techniques. DTV is particularly suited for measuring flow velocities in regions where the use of tracer particles is difficult or undesired. The applicability of this new technique is illustrated here by wind tunnel measurements of an unsteady wake flow and temperature measurements in a convection chamber. The recording and evaluation schemes used are described and presented together with an accuracy analysis.

1 Measurement principle

The DTV method is an optical technique for pointwise velocity measurements based on the detection and tracking of a local refractive–index or density variation which is induced in the flow. The local density variation, termed density tag, acts as a tracer particle which is transported by the fluid flow. The velocity information is obtained by determining the position of the density tag at two subsequent instances in time based on the Background Oriented Schlieren (BOS) technique, also referred to as Background Oriented Optical Tomography (BOOT) or Synthetic Schlieren technique.

![Fig. 1 Schematic diagram of the DTV measurement principle.](image)
Fig. 2 Timing sequence of a DTV measurement

(Raffel et al, 2000; Richard and Raffel, 2001; Dalziel et al, 2000). The BOS technique used in order to detect the externally induced density variation is based on the deflection of light due to refractive index changes related to density gradients in the investigated flow. A background speckle pattern with a high spatial frequency and high contrast is recorded with and without the density variation influencing the light path. Dividing the Lagrangian displacement of the density tag by the time interval between the two recordings yields a direct estimate for the local flow velocity vector. A standard one detector DTV set-up, schematically depicted in figure 1, allows for a 1D2C velocity measurement, i.e. at one spatial point two components of the velocity vector are determined. The third velocity component can be obtained utilising a two camera system set-up. Mean velocity fields and related RMS-values can be obtained by inserting the tag at various positions within the measurement volume.

In the present study we conducted DTV measurements in a uniform parallel flow and in an unsteady bluff body wake. Two different velocity evaluation schemes were applied. The results were analysed with regards to measurement accuracy for steady flow cases and the feasibility of DTV measurements in unsteady flows. Furthermore, a procedure for temperature measurements by means of the DTV principle and its first application is described.

2 DTV wind tunnel applications

In order to investigate the reliability and accuracy of the density tagging velocimetry in general and the various evaluation methods in particular, DTV was applied in a parallel uniform flow and in the unsteady wake of a bluff body. The measurements were performed in a low-speed closed-loop wind tunnel with an open jet test section at the German Aerospace Center (DLR) in Göttingen.

2.1 Experimental Set-up

In the first series of measurements the velocity of the free stream flow of the wind tunnel was measured by means of both via Particle Image Velocimetry (PIV) and Density Tagging Velocimetry (DTV). The free stream velocity $U_\infty$ of the wind tunnel was stepwisely increased. For discrete values of the velocity PIV and DTV were applied successively.

In a second series, a flat iron bar with height $h = 30\text{mm}$, width $w = 5\text{mm}$, and length $l = 1200\text{mm}$ was mounted horizontally in the middle of the wind tunnel’s test section which has a
cross section of 1.05 m × 0.7 m. With the mean flow directed horizontally, the velocity along a vertical line in the symmetry plane downstream the the obstacle was measured by means of DTV. Reference velocity field data was acquired in the vertical symmetry plane directly downstream of the obstacle with a standard one camera planar PIV system. The PIV data was evaluated according to standard correlation procedures (Raffel et al, 2007).

For DTV we used laser induced plasma tagging. The plasma tag was generated utilising a pulsed frequency–doubled Nd-YAG laser operating at a repetition rate of 10 Hz with a pulse energy of 150 mJ. By focussing the laser beam, an ionisation spark was generated at a defined location within the region of interest. The plasma tag was traversed across the flow and multiple measurement were conducted at each location to obtain average velocity profiles. The density variation was detected utilising a standard BOS setup consisting of CCD camera with double–shutter mode and a retro–reflective background pattern as described in (Heineck et al, 2010). To obtain velocity information, two subsequent snapshots, i.e. recordings with a very short exposure time, of the density tag were required. Since the camera exposure time could not be set arbitrarily short, we applied pulsed illumination of the background pattern to obtain instantaneous recordings of the density variation, comparable to the double–pulsed laser illumination commonly used for double–
frame PIV. In order to avoid that the very bright ionisation spark hampers the visualisation of the density variation, the camera exposure for the first recording was started shortly after the plasma was generated. The timing of the fundamental events for the DTV is summarised in figure 2. For the present application, the interframing time between the DTV recordings was $\Delta t_{\text{DTV}} = 40\mu\text{m}$. and the time delay between the plasma ionisation and the first recording was $\Delta t_{\text{ion}} = 15\mu\text{m}$.

2.2 Velocity evaluation

In order to evaluate the velocity from the subsequent BOS recordings of the plasma tag two alternative possibilities are proposed. It concerns a first approach based on the speckle displacement field by a local cross–correlation and a second procedure where the correlation step is omitted and the position of the spot is determined directly from the recorded intensity images. Prior to evaluation, the mean displacement over the region of interest was subtracted to account for vibrations.

In the first case, the displacement of the background pattern with respect to the reference image is evaluated by means of standard correlation methods. Examples of images of the plasma tag in uniform parallel flow at two instants of time are presented in figure 3 (left).

In the second case, the reference image is subtracted from the displaced image, yielding high values in the region of the density variation, due to the local displacement of the background pattern, and low values elsewhere. The intensity centroid of the subtraction image indicates the position of the density tag with an accuracy comparable to the one of the correlation–based method, even when considering small displacements, cf. Optical Flow evaluation by Horn and Schunck (1981). Prior to computing centroids, the intensity distributions were smoothed with a dynamic mean filter to increase the signal–to–noise ratio. A two–dimensional Gaussian bell curve was then fitted to the smoothed data, which allows for a fully automatic determination of the centre position for all individual recordings. The results of the reference image substraction, filtering and subsequently determined displacement estimation are shown in figure 3 (right). The image displacement of the centroid $\Delta s = (\Delta s_x, \Delta s_y, \tau)$ is directly related to the real spatial displacement $\Delta x(x, y, \tau)$ of the plasma tag through

$$\Delta x(x, y, \tau) = \frac{\Delta s(x, y, \tau)}{M}$$

with $\tau$ the small time delay between two exposures and $M$ the optical magnification factor. The
latter is given by

$$\frac{1}{M} = A \left( \frac{L_2}{f} - \frac{L_2}{L_1 + L_2} \right) .$$

(2)

where $f$ is the focal length, $L_1$ the distance between the background and the hotspot, $L_2$ the distance between the hotspot and the lens of the camera and $A = 6.45 \text{px/mm}$ the pixel size of the camera, which was $A = 6.45 \text{px/mm}$ in the present case. Assuming that the time delay $\tau$ is sufficiently short, the translation can be considered approximately linear and the local velocity $u$ corresponds to

$$u(x, y, t_0) = \frac{\Delta x(x, y, \tau)}{\tau} = \frac{\Delta s(x, y, \tau)}{M \tau}$$

(3)

with $t_0$ being the first image acquisition time (figure 2). Omitting the correlation step also yields an important reduction in processing time. This is extremely interesting in view of real–time measurements and implementation in closed loop controlling circuits.

### 2.3 Comparison of DTV and PIV Results

The result is depicted in figure 4 together with the corresponding linear regressions. The slopes of the linear regression curves reach the ideal value of 1 with good accuracy (table 1). In the second series of experiments, the velocity in the wake of the bluff body was measured. The mean free stream flow velocity was $u_\infty \approx 25 \text{m/s}$ during both the PIV and the DTV measurements. The temporal average of the horizontal velocity component $u$ measured by means of PIV behind the bluff body is depicted in figure 5 (left). The average was calculated from 1000 measurements. The resulting velocity profile along the vertical line 23 mm behind the obstacle’s edge was taken from the

<table>
<thead>
<tr>
<th>Method</th>
<th>$b = \frac{u}{u_\infty}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIV</td>
<td>$1.005 \pm 0.002$</td>
</tr>
<tr>
<td>DTV</td>
<td>$0.99 \pm 0.02$</td>
</tr>
</tbody>
</table>

Table 1 Slopes of the linear regressions for PIV and DTV results with respect to the wind tunnel speeds
PIV measurement and compared to the DTV results in figure 5 (right). The vertical mean stream-wise velocity profiles at a position 30mm downstream of the obstacle as also measured by means of DTV and PIV and are depicted in figure 6 together with indications of the standard deviations. The PIV result, averaged over 100 instantaneous vector fields, clearly shows the uniform profile characteristic of a turbulent wake flow. Even in such unsteady flow conditions, where the hot air spot has been distorted by strong velocity gradients, the DTV and PIV results reach an acceptable agreement. However, the standard deviation, indicated by the errorbars, is visibly larger for DTV than for PIV. As can be seen in (table 2), the variation increases towards the center region behind the obstacle where the smallest velocities have been measured.

3 Temperature of a gas flow

In addition to the velocity measurement, the plasma tagging procedure allows for a simultaneous estimation of the temperature of the gas flow around the point of ignition. Since the energy deposition by the laser happens both rapidly and locally, a singularity in the density of the flow field forms. The density decreased rapidly due to thermal expansion. This discontinuity is by no means stable and dissipates in a strongly nonlinear way. A shock front is generated, propagating the discontinuity into the surrounding.

In order to illustrate the evolution of the unstable excitation we consider an isotropic medium with given temperature $T$, pressure $p_0$ and density $\rho$. After the generation of the plasma spark at $t = 0$ at an ignition point $\vec{x}_m$, a trapezoidally shaped pressure profile is generated (as shown in figure 7). After the ignition, a discontinuity in the density field dissipates by forming a sharp shock front of velocity $u_s$ and setting the air behind the shock front into motion with velocity $u_l$ due to the Rankine-Hugoniot-Conditions (applicable to discontinuous shock fronts). Until the laser is shut off after a time $\tau$, the medium absorbs the laser energy, forming another sound wave in the moving air behind the shock front. The total velocity of this secondary wave is $v = u_l + c_s$.

<table>
<thead>
<tr>
<th>Method</th>
<th>$u_{\text{min}} \text{[m/s]} (y &lt; 20\text{mm})$</th>
<th>$u_{\text{max}} \text{[m/s]} (y &gt; 30\text{mm})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIV</td>
<td>$-3.79 \pm 0.39$</td>
<td>$25.77 \pm 0.42$</td>
</tr>
<tr>
<td>DTV</td>
<td>$-4.79 \pm 1.33$</td>
<td>$25.22 \pm 0.70$</td>
</tr>
</tbody>
</table>

Table 2 Measured minimum and maximum velocities as measured by PIV and DTV
where $c_s$ denotes the speed of sound in the laboratory frame. Because the secondary wave is faster than the shock front, they coalesce leading to an increase in amplitude and pressure (rising edge of figure 7). After the laser is shut off, no new energy is added to the main wave front, which causes a plateau in the pressure distribution. Subsequently rarefaction waves are generated that deteriorate the original wave front when their coalescence leads to complete relaxation. The result of this mechanism is a regular sound wave front moving with the speed of sound $c_s$ in the laboratory frame. This model, similarly applied by Rui et al (2009) gives an approximation for the time scales of the above mentioned processes. According to their experiments with focussed lasers on aluminum targets, the rarefaction waves catches up with the sound wave after $\approx 1\mu s$. At a distance of about 5mm, the complex wave structures fully disappear. Because the time scale is short, heat transfer can be neglected here, even though a lot of heat is generated by the plasma spot. The spherically symmetric wave front in the far field region can be visualized by the BOS technique. More complex wave forms cannot be resolved by BOS methods, because of the strong laser flash and the afterglow of the laser ignition covers the entire near field region for typically a few microseconds.

The far field sound wave has a classical $n$-wave shape and is displayed in figure 8. Its propagation speed is given by the speed of sound, which, for an ideal gas, is given by

$$c_s = \sqrt{\frac{\gamma RT}{M}} \approx 20.063 \text{ m/s} \cdot \sqrt{T [\text{K}]}$$

This relation between temperature and speed of sound can be used to estimate the temperature around the point of plasma ignition, neglecting heat transfer and disruptive effects of the shock wave to the flow field, such as nonlocal density or pressure changes.

To determine the speed of the sound wave and thus estimate the temperature, we used algorithms that worked as a kind of noise filter. First, the origin of the spherical sound wave was evaluated. By fitting a two dimensional Gaussian distribution to the magnitude function of the BOS data, one can determine the center at a subpixel level. A number of other techniques to evaluate the position of the density tag are described in previous sections. Second, to determine the distance travelled by the sound wave, the radius of the circularly symmetric sound wave was measured.

The position vector measured from the previously determined center of the spherical wave is

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1. Assuming the propagation media to be air with the adiabatic constant $\kappa \approx 1.402$, the molar mass $M \approx 0.02896\text{ kg/mol}$ and the ideal gas constant $R$. 

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denoted by \( \vec{r}(x_i, y_j) \) and the displacement data at the given pixel by \( \vec{z}(x_i, y_j) \). By integrating the projection of the displacement vector onto the location vector from the center to the end of the picture in a straight line, we obtain the maximum contribution right on top of the wave front, where the displacement vectors point towards or away from the center. The noise vectors point in evenly distributed directions and thus contribute statistically to the integral. Furthermore, the length of the noise vectors was often seen to be smaller than the signal vectors.

\[
\begin{align*}
 r_{\text{proj}}(\vec{d}) & = \int_{\vec{x}_m}^{\vec{x}_m+\vec{d}} \frac{\vec{r}(\vec{x})}{||\vec{r}(\vec{x})||} \cdot \vec{z}(\vec{x}) \, d\vec{x} \\
(5)
\end{align*}
\]

This way the noise is naturally filtered by the projection. To further increase the accuracy and to take all of the data into account, the determined maximum is averaged over all rays originating from the center, generating \( r_{\text{proj}}(|\vec{d}|) \). Determining the local and global extreme of that function yields \( d_{\text{max}} \), which corresponds to the distance the soundwave has travelled. The evaluation yields a mean shock wave distance from the center point for each flow picture. The reconstruction used here does not work ad-hoc for flow fields. The experiment was performed at rest, which makes sure that the identified density tag is also the center of the shock wave. Here we used a different projection algorithm based on stereographic projections that does no longer use the center point as the origin but a point on the circular shock wave. From this data the radius can be extracted similarly. By performing a linear regression of the radius versus time based on the data taken with varying delay times, the propagation speed and temperature were determined. The linear regression fits the data very well and that all non-linear effects have relaxed after a few microseconds (9). The measurement error is determined by evaluating several hundred radii from different pictures. Since the error scales with \( \sqrt{N}^{-1} \), \( N \) being the number of measurements, the data point at 55µs with a relatively high error is due to the afterglow of the ignition in most pictures making an evaluation impossible.

This technique has been conducted for four different temperatures and compared to a reference measurement via a resistance thermometer. The results are summarized in table 3. Even though...
Fig. 9 Exemplary $r-t$-diagram of the sound wave with linear regression at $T = 306.45 \, \text{K}$

<table>
<thead>
<tr>
<th>$c_s , [\text{m/s}]$</th>
<th>$T_m , [\text{K}]$</th>
<th>$T_r , [\text{K}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>347(1)</td>
<td>300(2)</td>
<td>296</td>
</tr>
<tr>
<td>344(1)</td>
<td>294(2)</td>
<td>301</td>
</tr>
<tr>
<td>351(3)</td>
<td>306(4)</td>
<td>306</td>
</tr>
<tr>
<td>353(4)</td>
<td>309(4)</td>
<td>311</td>
</tr>
</tbody>
</table>

Table 3 Measured temperatures $T_m$ and propagation speeds $c_s$ for different reference measurements $T_r$

the temperature measurements do no fully represent the reference, the measurements yield very promising results. It is expected that further refinement of the evaluation techniques and measurements conditions can yield even more precise results.

4 Conclusion and Future Work

Applications of the optical point–wise velocity measurement technique DTV were presented. DTV proved to be a simple and robust technique allowing for pointwise measurement of velocities where tracer particles are not practical or welcome. The experimental implementation was divided in three stages, tagging, detection, and evaluation. The most promising tagging method termed plasma tagging was based on laser induced ionisation. To detect the density variation we reverted to the BOS method. Two different velocity evaluation schemes were proposed to track the plasma tag in subsequent BOS recordings. The first approach is based on the speckle displacement field by a local cross–correlation. In the second procedure, the correlation step is omitted and the position of the spot is determined directly from the recorded intensity images yielding an important reduction in processing time. The temperature measurements showed acceptable results and so did the velocity measurements in the well calibrated undisturbed mean flow of our wind tunnel. The comparison between DTV and PIV results for an unsteady bluff body wake flow were promising in terms of the applicability of the DTV method for measurements in unsteady flow regions. It is
deemed to be the strength of DTV that every laboratory equipped with a PIV laser and software can perform DTV.

References