One-step Stereo PIV: Theory and Performances

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Abstract The purpose of this communication is to theoretically analyze and assess the performances of a recently proposed novel Stereo PIV computation approach (Leclaire et al. 2009). The specificity of the algorithm is that it computes the three-component displacement corresponding to an interrogation window directly in one step, whereas conventional algorithms involve two steps, first interrogating separately image pairs corresponding to each camera, and then obtaining the final displacement as a result of the so-called stereo reconstruction. We first analyze theoretically the differences between the two approaches. The direct approach in fact amounts to seek the displacements pairs directly in the admissible subspace induced by the stereoscopic geometry, automatically excluding possible spurious pairs, whereas in the two-step case, peak pairs are constituted by matching the sharpest peak of each image, and then projected onto the closest pair in the subspace. We then show that under ideal imaging conditions, both approaches are theoretically equivalent, provided that the correlation peaks of each image have the same curvature. We then assess the sensitivity of both approaches to various relevant parameters, using synthetic images. The tests performed confirm that identical results are obtained for perfect imaging conditions. They also show that while the one-step approach is slightly more sensitive to peak-locking, and in a more pronounced way to the registration error, significant gains are observed in situations with low seeding, loss of particle pairs and low signal-to-noise ratio. Highest gains are achieved in particular when cameras are characterized by a different signal-to-noise ratio, which may occur in particular in forward/backward diffusion setups, whereby the method automatically weighs more importantly the data with the highest signal to noise ratio.

1. Introduction and notations

Figure 1 shows a schematic of a typical stereo PIV setup, together with the notations used in the following. Positions are denoted by upper case letters in the object plane, and lower case letters in the image planes. Bold letters are used throughout to identify vectors. Cameras are identified by a superscript \(i = 1,2\). Calibration functions \(F^i\) are supposed to be known, which yield the position of the projections of a given point \(X\) in the laser sheet on each of the cameras, i.e. \(x^i = F^i(X)\). The algorithms which will be compared in this paper use a pinhole camera model; this is an arbitrary choice as the one-step approach is independent on the choice for the calibration functions and could be used with polynomials as well.

Similarly to image mapping approaches, one hereafter defines a regular grid of points \(k = [k,l]^T\) on the object plane. Spacing within this grid is chosen equal to the smaller dimension of a back-projected pixel, so as to avoid downsampling. For each grid point \(k\), an interrogation window (IW) \(\mathcal{W}_k\) centred around this point is defined, and the Stereo PIV computation consists in determining its displacement \(\Delta X_k\) between time instants 0 and \(dt\). Note that in all the following, the subscript of a quantity will indicate at which grid point it is evaluated.

Even when based on another formulation than the present image mapping (e.g., vector warping or Soloff’s method), which may lead to practical differences in the implementation, stereo PIV algorithms available to date involve the following two steps. Two-component (2C) displacements \(\Delta x^k_i\) are first sought separately on both camera images (be they dewarped or not), yielding a four-component result \(\Delta x^*_k\). Here, and similarly in the following, a star superscript will identify the value of the displacement found by an algorithm, to distinguish it from the unknown of the problem. The three-component (3C) displacement \(\Delta X^*_k\) is then found using a least-squares resolution of the over-determined system:

\[
\nabla F_k \Delta X^*_k \overset{LS}{=} \Delta x^*_k
\]
where \( \nabla F_k = \begin{pmatrix} \nabla F^1_k \\ \nabla F^2_k \end{pmatrix} \) denotes the 4x3 gradient matrix, and \( \Delta x^*_k = \begin{pmatrix} \Delta x^1_k;* \\ \Delta x^2_k;* \end{pmatrix} \) the pair of two-component (2C) displacements. In this equality, the superscript LS above the equal sign identifies the Least Squares resolution, which is usually referred to as stereoscopic reconstruction. It is known that \( \Delta X_k^* \) minimizes the residual or stereoscopic reconstruction error \( \varepsilon_k \), built as the norm of the difference between the re-projected displacement and the displacement measured on both cameras:

\[
\varepsilon_k = \| \nabla F_k \Delta X_k^* - \Delta x^*_k \|
\]

This quantity is usually considered to provide a good estimate for the quality of the stereo displacement. If the calibration is accurate, a common rule-of-thumb is to consider a vector spurious when \( \varepsilon_k \) is above 0.5 pixel (see, e.g., Raffel et al. 2007).

Fig. 1. Principle sketch of a stereo setup and notations

In contrast to this two-step approach, FOLKI-SPIV determines the 3C displacement directly in one step, using a joint cross-correlation under the constraint \( \varepsilon_k = 0 \), so that it does not include any stereoscopic reconstruction. This will be exposed in section 2, together with details on the software implementation. In the following sections, the specificity of this direct method compared to the two-step approach will be explored. To that purpose, we will use a modification of FOLKI-SPIV implementing such a two-step approach, all other things being equal. In section 3, we will perform a theoretical comparison, which will be supplemented in section 4 by Monte-Carlo simulations, investigating the sensitivity to peak-locking, registration error, out-of-plane loss of pairs and variable signal to noise ratio. Conclusive remarks will follow in section 5.

2. Algorithms

2.1. Introduction: Lucas-Kanade paradigm (2C)

Considering in this paragraph the case of 2C PIV with a normal viewing direction, the Lucas-Kanade (LK) method (see, e.g. Baker & Matthews 2004), proposes to seek the displacement of the interrogation window \( \mathcal{W}_k \) by minimizing the sum of squared differences (SSD)

\[
SSD(\Delta x_k) = \sum_{m \in \mathcal{W}_k} (I(x_m, 0) - I(x_m - \Delta x_k, dt))^2
\]

where \( I(x, t) \) is the grey level at location \( x \) in image recorded at time \( t \). Expanding this expression leads to

\[
SSD(\Delta x_k) = \sum_{m \in \mathcal{W}_k} \left( I(x_m, 0)^2 + I(x_m - \Delta x_k, dt)^2 - 2I(x_m, 0)I(x_m - \Delta x_k, dt) \right)
\]
In practice, to cope with difficulties commonly encountered in PIV images, such as local illumination variations, linked for instance to light reflections on an obstacle, or to optical defects in the light sheet optics, a localized intensity equalization has to be used. We use a mean and standard deviation equalization (Champagnat et al. 2011), i.e. we rescale locally the intensity so that it has a zero mean value and a unit standard deviation inside $W_k$. Consequently, the first two terms in (5) do not depend on the displacement any more, and the problem becomes equivalent to maximizing the direct cross-correlation

$$CC(\Delta x_k) = \sum_{m \in W_k} I(x_m, 0)I(x_m - \Delta x_k, dt)$$

(5)

which is the objective commonly used in a large number of PIV algorithms. The reader should thus bear in mind that minimizing the SSD thus amounts to find a correlation trough instead of a peak. However, for clarity, we will hereafter use the familiar term "peak". More details on this algorithm and its performances can be found in Champagnat et al. 2011. In this article, it is shown in particular that a precision similar to state-of-the-art FFT-based cross-correlation is reached, with an increased robustness to noise.

2.2. Two-step Stereo PIV

Within this LK paradigm and still using image mapping, a two-step Stereo PIV algorithm as described in section 1 can be easily formulated, which we will use for comparison with the state-of-the-art. In this algorithm, the three-component displacement $\Delta X_k^*$ of the IW $W_k$ is be obtained by

1. For each camera i, finding the two-component displacement $\Delta x_k^{i*}$ by minimization of

$$SSD(\Delta x_k^i) = \sum_{m \in W_k} \left( I^i(x_m^i, 0) - I(x_m^i - \Delta x_k^i, dt) \right)^2$$

(6)

2. Performing the least squares inversion of system (1), yielding $\Delta X_k^*$. In that process, the stereo reconstruction error $\varepsilon_k$ (2) is determined.

2.3. One-step Stereo PIV: FOLKI-SPIV

In the FOLKI-SPIV algorithm (Leclaire et al. 2009), compared to the approach described in section 1, computation of the stereo displacement $\Delta X_k^*$ is done directly in one step, this displacement being sought as the minimizer of

$$SSD(\Delta X_k) = \sum_{i=1}^{2} \sum_{m \in W_k} \left( I^i(x_m^i, 0) - I^i(x_m^i - \nabla F_k \Delta X_k, dt) \right)^2$$

(7)

Solving this minimization problem is thus the extension of the Lucas-Kanade framework to the context of Stereo PIV. In the objective function (8), the first sum is performed over the two cameras $i = 1,2$. The 3C displacement is sought for directly, and appears via its projection on each camera in the second sum. Using the local mean and standard deviation equalization described in section 2.1, only correlation terms remain; minimizing (8) thus consists in finding the displacement $\Delta X_k^*$ which jointly correlates the projected displacement on the two camera images. Note that since the images are only interrogated at displacement values which are projections of a 3C displacement, this amounts to perform this joint correlation under the constraint that $\varepsilon_k = 0$.

2.4. Practical implementation

Both FOLKI-SPIV and the two-step approach introduced above, which will be used for the comparative tests, rely on the same implementation and differ only in their global objectives. We give here a brief overview of this implementation, and refer the reader interested by more details to Leclaire et al. 2009 and Champagnat et al. 2011.

Taking the example of FOLKI-SPIV for clarity, criterion (8) is minimized iteratively. At each iteration,
supposing that an estimate $\Delta X_{k,0}$ is available, the increment $\Delta X_k^*$ to be sought can thus be assumed to be small. It is calculated using Gauss-Newton (GN) iterations, by performing a Taylor expansion of (8), as classically done in the Lucas-Kanade framework. After some algebra, this increment is finally obtained as the solution of a 3x3 linear system (Leclaire et al. 2009). Note that this minimum search differs with respect to most PIV approaches, in which a peak interpolation or fitting is used.

In order to be able to measure displacements of any magnitude in the PIV images, we use a multi-resolution framework, by constructing a Gaussian image pyramid. From an image at a given pyramid level $j$, the image at level $j+1$ is obtained by low-pass filtering the image $j$ and retaining one pixel every 2x2 pixels. Thus image $j+1$ is four times smaller than image $j$, and displacements are divided by two. For a given displacement field with maximum displacement $d_{\text{max}}$, the number $J$ of levels to consider for the pyramid should be such that at the top level $J-1$ (raw images being level $j=0$), the maximum displacement, $d_{\text{max}}/2^{J-1}$, should be less than 2-3 pixels. In that way, local minima can be avoided and convergence of the GN iterations is guaranteed at any level $j$.

During the GN iterations, interpolation has to be performed in the raw images, at positions corresponding to the projections on the camera images of the grid points defined on the laser sheet plane, or at projections on these images of subgrid positions in the laser sheet plane. This can be performed either with bilinear or with third-order B-splines. In all the following, the B-spline interpolator will be chosen.

Objective function (8) is a non-symmetric objective, exposed in section 2.3 for simplicity. FOLKI-SPIV may alternatively use the symmetric counterpart of this function, in which half of the displacement is sought on both images, thereby leading to a second order accurate estimation, as is now widely done in PIV algorithms. This option will be taken in the following.

3. Theory

3.1. Stereoscopic displacement search and of the $\epsilon_k = 0$ constraint

As mentioned in section 1, separate interrogation of $i = 1, 2$ camera image pairs yields the two 2C displacements $\Delta x_k^i$, $i = 1, 2$, i.e. four data. However all possible displacements in the four-dimensional space spanned by the camera axes are not possible. Indeed, assuming that calibration is perfect, and that the intensity can be sampled with high precision in the image planes (setting aside the discrete character of $I$ introduced by the camera sensor), one must have

$$\nabla F_k \Delta X_k^* = \Delta x_k^*$$

This means that $\Delta x_k^i$ must lie in the image space of $\nabla F_k$, which is a subspace of dimension 3 of the global 4-dimensional space of all possible displacement pairs. In a possibly non-perfect case, finding the 3C displacement $\Delta X_k^*$ using a least-squares inversion in fact boils down to project the measured pair of 2C displacements $\Delta x_k^*$ on the closest displacement pair which lies in the image space of $\nabla F_k$. The stereoscopic reconstruction error $\epsilon_k$ measures the distance between both displacement pairs.

Objective (7) thus amounts to restricting the search of the displacement pairs on the camera images to that which are members of the image subspace of $\nabla F_k$. In other words, such a formulation may help to eliminate directly spurious displacement peaks, if the displacement pair does not lie in this subspace. The usefulness of this choice will be demonstrated in section 4.

On a more practical point of view, the image subspace of $\nabla F_k$ may be characterised quite easily. Indeed, displacement pairs belong to this subspace if and only if they are orthogonal to the null space of $\nabla F_k$, which is of dimension 1. Equivalently, there must exist a 4-dimensional vector $\mu_k$ such that for a given 2C displacement pair $\Delta x_k^*$, one has

$$\Delta x_k^* \in \text{Im} (\nabla F_k) \iff \mu_k \cdot \Delta x_k^* = 0$$

This is equivalent to saying that there must exist one relationship between the four components of the 2C displacement pair. As the gradient matrix depends on the pixel position in the laser sheet grid, $\mu_k$ also varies with $k$. In the setup considered in section 4 however (see figure 3 below), its variation along the field of view is of small magnitude. At the centre of the object plane, which projects in the centres of both images, one finds $\mu_k = (0, 1, 0, -1)$, so that $\Delta y^1 = \Delta y^2$. 
The stereoscopic geometry logically imposes that the displacements along \( y^1 \) and \( y^2 \) must be equal, as this direction is common to both cameras and parallel to the object Y direction. At the edges of the object plane, \( \mu_k \) has still nearly opposite \( \Delta y^1 \) and \( \Delta y^2 \) components, and small \( \Delta x^1 \) and \( \Delta x^2 \) components (the ratio between both groups is less than 5%) so that belonging to the image space \( \mathcal{V}_k \) of leads to \( \Delta y^1 = \Delta y^2 \).

Thus, still anticipating on the examples of section 4, and reasoning at a pixel \( k \) close to the centre of the object plane, one can understand the respective principles of the two-step method and of FOLKI-SPIV as follows:

- Two-step method: separate cross-correlation of the images leading to a 2C displacement pair \( \Delta x_k^* \), then projection of this pair on the closest pair such that \( \Delta y^1 = \Delta y^2 \).

- FOLKI-SPIV: cross-correlation of the image pairs in a subspace where the only peaks present satisfy \( \Delta y^1 = \Delta y^2 \).

### 3.2. Proof of equivalence of one and two-step methods for ideal conditions

Let us now consider the iterative 3C displacement search and suppose that the projected displacement pairs of the current 3C estimate at pixel \( k \), for the IW \( \mathcal{V}_k \), are close to correlation peaks in the image planes. The sum of squared differences for a given camera \( i \) (7) can thus be locally approximated locally by a parabola centered around it:

\[
SSD_i(\Delta x_k^i) = \alpha_k^i \| \nabla F_k^i \Delta X_k - \Delta x_k^{i,*} \|^2
\]

In this expression, \( \alpha_k^i \) characterizes the curvature of the correlation peak (see Figure 2), and displacement \( \Delta x_k^{i,*} \) is the displacement found by interrogating separately image \( i \), i.e. that found by the two-step method.

Using the same expression of the individual SSDs, one may also reformulate FOLKI-SPIV’s objective (8), and obtain:

\[
SSD(\Delta X_k) = \sum_{i=1}^2 \alpha_k^i \| \nabla F_k^i \Delta X_k - \Delta x_k^{i,*} \|^2
\]

The displacement found by FOLKI-SPIV is thus

\[
\Delta X_k^* = \min_{\Delta X_k} \left( \alpha_k^1 \| \nabla F_k^1 \Delta X_k - \Delta x_k^{1,*} \|^2 + \alpha_k^2 \| \nabla F_k^2 \Delta X_k - \Delta x_k^{2,*} \|^2 \right)
\]  

or, since none of parameters \( \alpha_k^i \) is vanishing,

\[
\Delta X_k^* = \min_{\Delta X_k} \left( \| \nabla F_k^1 \Delta X_k - \Delta x_k^{1,*} \|^2 + \beta_k \| \nabla F_k^2 \Delta X_k - \Delta x_k^{2,*} \|^2 \right)
\]

Here, one has introduced the relative curvature of the correlation peaks of the two cameras, \( \beta_k = \alpha_k^1 / \alpha_k^2 \).

If the peak seen by the two cameras have the same curvature, i.e. \( \beta_k = 1 \), then

\[
\Delta X_k^* = \min_{\Delta X_k} \left( \| \nabla F_k^1 \Delta X_k - \Delta x_k^{1,*} \|^2 + \| \nabla F_k^2 \Delta X_k - \Delta x_k^{2,*} \|^2 \right) = \min_{\Delta X_k} \| \nabla F_k \Delta X_k - \Delta x^* \|^2
\]

**Fig. 2. Quadratic approximation of the correlation peak neighborhood**
This shows that, under the present hypotheses, the 3C displacement $\Delta X_k^*$ found by FOLKI-SPIV is the least-squares solution of the system, and thus that FOLKI-SPIV and the two-step method are strictly equivalent.

From (12), it can be seen that if the peaks do not have the same curvature, then FOLKI-SPIV will provide a weighted solution, in which the larger weight will be put on the sharper peak, whereas the two-step method will still be characterized by equal weights. Such a situation where cameras viewing the same scene exhibit correlation peaks of unequal sharpness may be encountered in practice, in particular in the case of different signal to noise ratios. This will be investigated further in section 4.4.

4. Compared performance

Synthetic particle images are generated using an in-house generator, reproducing the experimental setup sketched in Figure 3. A 1 mm thick laser sheet with a Gaussian intensity profile is considered, located in the OXY object plane, where O is the origin of the world frame of reference. Two cameras view the illuminated flow, their optical axes being inclined with angles of $-45^\circ$ and $45^\circ$ relative to the OZ axis. $C_1$ and $C_2$ identify their optical centres, and $O'_1$ and $O'_2$ the centres of the cameras frame of reference. For simplicity, no Scheimpflug adapter is included. The cameras are placed at an equal distance of the laser sheet, i.e. $OC_1 = OC_2 = 1000$ mm, and have an equal focal length $C_1O'_1 = C_2O'_2 = 100$ mm. They are calibrated using a pinhole model, and, unless otherwise specified, a perfect calibration is assumed. The pixel physical dimension is taken equal to 10 µm.

Particle positions are randomly generated with a uniform distribution in space. Unless otherwise specified, the seeding density is set to $N_{ppp} = 0.1$ and the particle images have a gaussian shape with an $e^{-2}$ diameter $d_\tau = 2.4$ px. The particle physical diameters obey a gaussian distribution. Images are digitized on an 8-bit range and include a gaussian noise. Unless otherwise specified, the maximum noise intensity represents roughly 5% of the maximum particle intensity.

All images are processed with both FOLKI-SPIV and the two-step approach derived from FOLKI-SPIV’s implementation. As we want to assess directly the differences implied by the objective functionals, we will not use the multi-resolution (J = 0 image pyramid levels) and thus restrict to displacements with a maximum value of 2-3 pixels. Every set of images is processed using $N = 10$ Gauss-Newton iterations, using a B-spline interpolator and the symmetric approach described in section 2.4. The fictitious pixel grid on the laser sheet plane is chosen so as to avoid downsampling, i.e. the pixel size is chosen as the vertical side of a back-projected camera pixel. In all the examples of this section, an interrogation window size $IW = 11$ pixels is chosen.

4.1. Sensitivity to peak-locking

A first and fundamental difference between actual PIV images and the idealized situation considered in section 3 is the pixel sampling of the signal by the sensors, which may lead to peak-locking if the particle image size is too small. We investigate here the sensitivity of both approaches to this bias by considering two shearing displacements, respectively in the X and Y directions: (I) $\Delta X(X,Y) = aY$, (II) $\Delta Y(X,Y) = bX$, with constants a and b chosen such that the maximum displacement is slightly larger than 1 px on the camera images. For each case, 100 image sets are generated.

The two algorithms display a slightly different behaviour with respect to peak locking, but this does not
show in the RMS error norm $E_{\text{RMS}}$ (see Figure 4). Indeed, as the $e^{-2}$ diameter $d_e$ of the particle images decreases, $E_{\text{RMS}}$ first increases slowly, and then experiences a more dramatic increase for $d_e < 1.5$ px. Examination of the displacement histograms show no difference for case (I) between the two algorithms, whatever the particle diameter. For case (II), histograms are identical for $d_e > 1.5$ px, and slight differences appear below this value, as the rms error significantly grows. As seen in the right hand side of Figure 6, FOLKI-SPIV seems slightly more sensitive to the peak locking bias on this component, though this is not associated with a larger RMS error.

Note that in the favorable cases of this test, i.e. for the larger particle diameters, nearly identical velocity fields are found for both algorithms for each image set. This is in agreement with the theoretical model of section 3.2, and confirms the fact that the algorithms are equivalent for perfect imaging conditions. In the strong peak locking case however, this is more difficult. Indeed, due to the large enough seeding density, correlation peaks should have a high detection probability, and thus the present results are indeed only peak bias. Due to the small particle image diameter, it is possible to imagine situations where sampling leads to a sharper peak for an image than for the other. Then, equation (13) shows that FOLKI-SPIV should indeed enhance peak-locking, as it would privilege the sharper peak of both. However, this seems not to be the only explanation, as there is a difference in the histogram for the Y displacement but not for the X displacement, while there is no reason that different peak curvatures would be observed only in the case of a Y displacement. This shows that the simple quadratic model of section 3.2 cannot be extended directly to this strongly undersampled case.

![Figure 4](image)

**Fig. 4.** RMS Error for both algorithms as a function of the particle image diameter $d_e = 4\sigma$, horizontal shear I (left) and vertical shear II (right)

### 4.2. Sensitivity to registration error

Whereas it is often possible in experimental situations to obtain a good quality stereoscopic calibration, it is much more difficult to accurately determine the actual laser sheet position compared to the calibration body used. In case of mispositioning, the IWs projected on the cameras from an IW defined in the laser sheet plane do not contain the same particles, which may lead to significant errors especially in the case of important flow gradients (Wieneke 2005). Robustness to the registration error is thus an important quality for a Stereo PIV algorithm.

In the presently considered setup, the worst case displacement field in the case of a Z offset or a rotation around the Y axis of the laser sheet is a displacement with gradients in the X direction. We thus consider a sinusoidal vertical shearing motion, $\Delta Y(X,Y) = \sin(2\pi X/\lambda)$, with $\lambda$ taken equal to 15 times the IW diameter. The RMS errors obtained by displacing the laser sheet are shown in Figure 7. The maximum displacements of the laser sheet considered are of the order of 20 px (translation along Z) and 45 px (rotation around $R_Y$). In both figures, a zone close to the exact position is sampled more closely: this region corresponds to the typical residual maximum displacement obtained after a self-calibration procedure.
Fig. 5. Displacement pdfs for $d_x = 4$ px (left) and $d_x = 0.8$ px (right), horizontal shearing motion.

Fig. 6. Displacement pdfs for $d_x = 4$ px (left) and $d_x = 0.8$ px (right), vertical shearing motion.

Fig. 7. Sensitivity of both algorithms to the registration error: case of a Z-translation of the laser sheet (left), and of a rotation around the Y-axis (right), vertical shearing motion.
For both displacements, FOLKI-SPIV appears more sensitive to the registration error, with a systematically higher RMS error. This could be expected, since, as explained in section 3.1, this algorithm searches displacement pairs which belong to the image subspace of $\nabla \mathbf{F}_k$, excluding naturally other possible pairs which are not elements of this subspace. If the actual calibration functions are not known precisely, then this may lead to situations where the right displacement pairs will be discarded, and a spurious displacement detected. For possible residual values of the laser sheet mispositioning, differences are more reasonable, though still observed, with a maximum of 0.01 px.

4.3. Sensitivity to seeding density and out-of-plane loss of pairs

As shown by Keane and Adrian (1992), the detection probability of the displacement correlation peak in 2C PIV is proportional to $N_{\text{IF}} F_\text{IF} F_\text{IO}$, where $N_{\text{IF}} = N_{\text{ppp}} x \text{WS}$, with WS the size of the interrogation window, $F_\text{IF}$ the proportion of pairs lost during the two time intervals due to in-plane motion, and $F_\text{IO}$ its counterpart for out-of-plane motion. For iterative algorithms such as here, $F_\text{IF}$ can be considered equal to one.

As mentioned in the introduction of section 4, the absence of multi-resolution restricts the simulations to displacements smaller than about 2 px. As adding an out-of-plane component leading to a significant $F_\text{IO}$ may lead to reaching the limit of small displacements, thus blurring the purpose of the test, we account for this loss of pairs simply by randomly removing the required fraction of particles between the two illuminations, and replacing them by the same number of randomly located new particles.

Figure shows the result of this detection probability study, done by varying $N_{\text{ppp}}$ for $F_\text{IO} = 0.5$, 0.8 and 1, still at a fixed window size of 11x11 px. Two uniform displacement fields, $\Delta X = 2$ px and $\Delta Y = 2$ px, are considered. As the algorithms presented herein use Gauss-Newton iterations to locate the correlation peak, it is not possible to define a peak detection probability, as no exhaustive search of the correlation map is performed. We thus represent the fraction $P(E < 0.5 \text{ px})$ of displacements whose error norm $E$ is smaller than 0.5 px as an indicator of this probability.

The same trend is observed for the two displacements, with more pronounced differences between one-step and two-step Stereo PIV. $P(E < 0.5 \text{ px})$ logically decreases with $N_{\text{IF}}$ for all values of $F_\text{IO}$, confirming the increasing number of spurious displacements detected. Values at different $F_\text{IO}$ do not collapse on a single curve, as this measure also includes a proportion of bias error. For all test cases, FOLKI-SPIV yields a more efficient displacement detection, confirming the analysis of section 3.1. The highest gains are achieved at $N_{\text{IF}} = 1.2$, $F_\text{IO} = 1$ (16%), and $N_{\text{IF}} = 6$, $F_\text{IO} = 0.5$ (12%).

![Fig. 8. Proportion of vectors with an error norm less than 0.5 px for various values of $N_{\text{IF}} F_\text{IO}$. Constant displacement field $\Delta X = 2$ px (left), $\Delta Y = 2$ px (left)]
4.4. Influence of the signal to noise ratio

Still considering the constant displacement introduced in section 4.3, we investigate the influence of the signal to noise ratio (SNR) on the measurement quality, as this parameter is often crucial in the experiments. In a first test case (Figure 9, left), we vary jointly at the same level the SNR of both cameras, and in a second, we decrease only the SNR of one of the two cameras. This latter test seeks to mimic experimental situations where optical accesses are difficult, so that one camera is placed in backward diffusion and the other in forward diffusion, thus receiving a much lower light intensity. For both test cases, we consider a low (N_{ppp} = 0.02) and a high seeding density (N_{ppp} = 0.1). Note that the x axis of the figures is the inverse of the SNR.

The measurement accuracy logically decreases as the SNR of both cameras decreases jointly. There also, FOLKI-SPIV yields better results, for both seeding densities. Gains are highest for low values of the SNR at N_{ppp} = 0.1, with a difference of roughly 0.12 in P(E < 0.5 px).

When cameras have a different SNR, the gains brought by FOLKI-SPIV may be very significant, as shown by Figure 9 left. For both seeding densities, while a decrease in the SNR of one of the cameras leads to a strong decrease in P(E < 0.5 px), important values are maintained with the one-step approach, with a much slower decrease. At the lowest SNR, probabilities differ by 0.42 (N_{ppp} = 0.02) and 0.32 (N_{ppp} = 0.1). These gains stem directly from the fact that during the displacement search, FOLKI-SPIV will put more weight on the camera with the highest SNR, and less on that with the lowest SNR, as demonstrated in section 3.2 (equation 13). Indeed, a low SNR leads to a lower peak, which has thus a lower curvature since it has the same width, as can be observed on the correlation maps when performing a 2C interrogation. Besides, this important gain may also be linked to the subspace search of FOLKI-SPIV. Indeed, the two-step method will first interrogate separately the two image pairs, yielding an accurate displacement for the high SNR camera, and a spurious displacement for the low SNR one. If this accurate/spurious pair does not match the constraint ΔY1 = ΔY2, the stereoscopic reconstruction step will project it to the closest pair admissible, which has no specific reason to be the actual displacement since is unlikely to be the actual displacement since ΔY1 = ΔY2 is not a firm constraint which is verified by a large range of displacement. In contrast, this accurate/spurious pair will not be seen by FOLKI-SPIV during the SSD minimization, as it is performed directly in the subspace of admissible displacements.

Fig. 9. Proportion of vectors with an error norm less than 0.5 px vs. signal to noise ratio (SNR). Constant displacement field ΔY = 2 px. Left: joint variation of the SNR of both cameras, right: variation of the SNR of one camera only
6. Conclusion

The one-step Stereo PIV algorithm recently proposed by Leclaire et al. (2009) has been analyzed theoretically and thanks to Monte-Carlo tests, and compared with a two-step approach based on the same implementation, mimicking the present state-of-the-art. It has been first recalled that one-step Stereo PIV searches the pair of 2C displacements directly in the subspace of admissible displacements determined by the stereoscopic geometry. Then, a simple model has shown that under perfect imaging conditions, one-step and two-step methods are equivalent. From this model, differences may occur when associated peaks in the images have different curvatures, which may be the case in particular when the camera images have a different signal to noise ratio. In such a case, FOLKI-SPIV and its one-step approach weigh more importantly the high SNR data. Synthetic tests have shown that this technique should be used with care in a context where the calibration is difficult, in particular when the registration error is difficult to correct, as a larger sensitivity is observed. A slightly larger sensitivity to peak-locking has also been reported, though not associated with a higher RMS error and thus leading to no difference in the behaviour of this error with the particle image size. In contrast, the one-step method has proved more robust to difficult conditions which are frequently encountered in the experiments, such as low seeding density and loss of pairs. In the case of cameras with different SNR, such as in a forward/backward setup, very important gains in accuracy have been achieved, as predicted by the theory.

To fully assess this method, tests should be extended to non-elementary situations, where these effects are mixed together, and to situations more relevant to fluid flows, including experimental validations. Also, further work will pursue the one/two step comparison, by investigating the relative merits of their associated quality criteria, used for spurious vector rejection. For the two-step method, this criterion is the stereoscopic reconstruction error, which is not relevant in the one-step method. In this latter context, the criterion is directly the objective to be minimized. Future tests will thus also aim at determining whether one of the methods may lead to a more truthful spurious vector rejection than the other, which is an important part of PIV processing.

References