Estimation of Particle Sample Bias in Shear Layers using Velocity-Data Rate Correlation Coefficient

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Abstract This paper reports on the location-specific changes in velocity bias in a free shear layer of a round supersonic jet using the velocity-data rate correlation coefficient as a post-facto measure, building on earlier work by Meyers et al. (1990). While such a study can enhance our understanding of spreading and mixing in particle-laden shear layers, it is further necessitated by potential applications of shear layer detection in both low and high Reynolds number flows. Velocity-data rate correlation coefficients were calculated for laser velocimetry data by alternately seeding the primary (core) flow and the secondary (entrained) flow at every measurement location. Subsonic, supersonic and partially confined supersonic jets were accordingly investigated. Subsonic jet (Mach 0.25) data, considered as a baseline measurement, demonstrated mixing activity emanating at 6 diameters downstream of the jet source and was found to have correlation values ($C \approx \pm 0.1$). Within the extent of axial measurement locations, supersonic jet (Mach 1.4) data was void of mixing activity as demonstrated by a low range of correlation values ($-0.05 \leq C \leq 0.03$). This suggested that the axial length scale of mixing between primary and secondary flow streams occurred several diameters downstream of the circular supersonic jet source. Partially confining the supersonic jet enhanced mixing activity as evidenced by the increase in correlation coefficient ($-0.08 \leq C \leq 0.08$), skewness towards the baseline measure ($C = 0.1$) and alternating trends between primary and secondary flow measurements. Considering that in mixing and shear layer growth, data dependencies build up potentially limiting conventional flow statistics from completely explaining flow behavior, the velocity-data rate correlation coefficient offers possible clarifications.

1. Introduction

Particle sampling bias in laser velocimeter measurements occurs in shear and mixing layers where the spread of the particle-laden flow is not only dependent on the mixing of the flows, but also on the spatial distribution of particles within the flow. This paper addresses the application of the velocity-data rate correlation coefficient as a measure of statistical dependence, and thus the degree of velocity bias. Measurements in a shear layer were made using a single component LDV under a hypothesis that high velocity increases the data rate while lower velocities decrease the data rate (McLaughlin and Tiederman (1973)). This implies a correlation between velocity and data rate. The study presented in this paper builds on earlier work presented by Meyers et al. (1990), wherein the correlation between velocity and data rate was determined by calculating the standard correlation coefficient between any two processes, equation (1):

$$C = \frac{\langle U - u_i \rangle \langle R - r_i \rangle}{\sigma_u \sigma_r}$$

where $U$ is the statistical mean velocity from the entire measurement ensemble, and $u_i$ is the average velocity within the $i^{th}$ period of time, considered to be independent from other times. This period of time is referred to as the Taylor temporal microscale and is the time that the flow takes to change one standard deviation (Meyers et al. (1990), Meyers (1991)). The variable $R$ in equation (1) is defined as the statistical mean data rate during the acquisition of the measurement ensemble and $r_i$ is the data rate during the $i^{th}$ flow microscale. The correlation function is normalized by the standard
deviations associated with the velocity ($\sigma_u$) and data rate ($\sigma_r$) to determine the correlation coefficient, $C$.

Since the first reports of velocity bias by McLaughlin and Tiederman (1973), researchers have investigated techniques to eliminate the bias. These correction techniques can be categorized as either post-facto correction techniques or sampling techniques. When McLaughlin and Tiederman (1973) found that the mean velocity measurements of a turbulent boundary layer (in water) were consistently higher than theory-predicted values, they developed the post-facto correction method of weighting the streamwise velocity measurements with the inverse magnitude of the streamwise velocity:

$$\bar{U} = \frac{\sum \left( \frac{1}{|u_i|} \right) u_i}{\sum \left( \frac{1}{|u_i|} \right)}$$

Hoesel and Rodi (1977), and Buchhave and George (1978) proposed the post-facto method where each velocity realization is weighted by the particle residence time (time the particle is within the measurement volume).

However, Edwards (1987) stated that individual measurements of velocity that occur at random times could be properly averaged only if they are used to reconstruct a time series possessing the same statistics as that of the flow being measured. Uniform sampling of the reconstructed velocity-time history is independent of the particle arrivals, and is thus orthogonal to the parameter being measured, in this case flow velocity. Sample and hold techniques developed by Dimotakis (1976) (backward step algorithm), and Edwards and Jensen (1983) (forward step algorithm) satisfy this realization by developing a time history where the first particle velocity is held in time until the next particle velocity where its value is then held to the next particle arrival, etc. This time history is then uniformly sampled to obtain the independent velocity data set. The accuracy of this approach however, is dependent on the fidelity with which the measured time history represents the actual velocity time history of the flow.

Another approach to obtain independent measurements was proposed by Stevenson et al. (1982) where they reasoned that if a particle was present every time the signal processor was ready to make a measurement, then true unbiased measurements could be made. This saturable detector sampling technique was implemented by disabling data acquisition for a period of time (dead-time) following each particle velocity measurement. This approach was implemented successfully in low-speed flows by Stevenson et al. (1982), Adams and Eaton (1988), and Herrin and Dutton (1992). By increasing the particle number density in the flow to the point where a high-speed burst counter would immediately acquire a new measurement following its activation after being reset, a uniform and thus unbiased sampling of the flow field would be obtained. However, the data rates required to obtain saturated detector operation were prohibitively high to be possible in normal wind tunnel applications (Meyers (1991)).

Instead of using post-facto techniques such as sample and hold, or extremely high data rates as required by the saturable detector, independent samples might be obtained by using characteristics of the flow itself to establish orthogonal sampling. Expanding on the original work by Edwards and Jensen (1983), Edwards and Meyers (1984) reasoned that any measurements obtained within
the Taylor temporal microscale of the flow were not independent, but a single measurement in one microscale was independent of a single measurement in the next microscale. Thus a method was developed where a (velocity) histogram composed of the first measurement in each microscale was normalized by a corresponding (rate) histogram of the number of measurements obtained following the first within each microscale (Edwards and Meyers (1984), Meyers (1991)). If there were no bias, the rate histogram would be flat, imposing no change on the velocity histogram when normalized. If the bias found by McLaughlin and Tiederman (1973) was present, the rate histogram would be a rising ramp toward higher velocities. Normalization of the velocity histogram by the rate histogram would remove the bias.

In 1985 a panel of nine recognized experts in laser velocimetry evaluated the various techniques to eliminate measurement biases (Edwards, 1987)). The panel recommended, or not, each technique and established particle number density regions where the recommendations were appropriate for a given technique. Beyond the individual recommendations, the panel also recommended determining whether or not sampling bias was present in a given data ensemble by using the relation given in equation (1). If no bias was present, then no correction should be used, otherwise the technique appropriate for the test conditions should be applied.

Petrie et al. (1988) and Herrin and Dutton (1993) expanded beyond previous bias research into the realm of turbulent, separated supersonic flows. Herrin and Dutton (1993) compared five velocity bias correction schemes: inverse velocity magnitude weighting, interarrival time weighting, sample and hold weighting, residence time weighting, and the velocity-data rate correlation method for high-speed separated flows. Measurements were made in the near wake region behind a circular cylinder with an approach stream Mach number of 2.5. They stated that residence time weighting was difficult to implement in practice due to the difficulties in making accurate measurements of particle residence time in the measurement volume. They also stated that velocity-data rate correlation was a mix between sampling and correction techniques and feasible for high-speed flow applications with limited wind tunnel run times.

1.1 Motivation and objectives
The motivation to investigate potential velocity bias effects arose when conventional flow statistics did not provide sufficient insight into the behavior of high-speed, jet-related shear layers. Since little or no research exists regarding bias effects in high-speed flows, guidance must be obtained from research in low-speed shear layers such as conducted by Meyers et al. (1990). This work showed that the velocity-data rate correlation indicated that bias was present only in shear regions where particle number densities between the two flows cannot be balanced. It also showed that the bias was not only affected by the particle statistics, but by the influence of signal processing characteristics on the particle statistics. Longmire and Eaton (1992) found that particles became clustered in the saddle regions downstream of the vortex rings generated by a round jet flow. Samimy and Lele (1991) stated that for convective Mach numbers less than 0.6, compressibility effects were not significant to particle motion. Edwards and Jensen (1983) found that correlation between particle number density and velocity would occur near the edge of a jet where the seeding is not the same in the entrained flow as in the jet core.

The above research indicates that location-specific changes in the velocity-data rate correlation coefficient would arise due to differences in seeding density between the two flows as they interact in the shear region before the mixing becomes homogeneous. Thus the velocity-data rate correlation was used in an attempt to obtain insight into the persistence of the shear layer in
supersonic pressure exchanging flow fields (Bulusu (2010)). The correlation relation was obtained post-facto to determine the extent of sampling bias as an indicator of the degree of mixing between the two high-speed flows. When found, no attempt was made to correct the sampling bias. Currently, a low Reynolds number application pertaining to the resolution of scale and aspect ratio of Dean- and Lyne-type vortices in arterial secondary flows is also being considered based on earlier work by Glenn et al. (2012). Meyers et al. (1990) reported velocity-bias in the shear and boundary layers surrounding an eddy generated downstream of a backward facing step. Accordingly, bias can be detected in shearing regions surrounding arterial secondary flow vortices due to a similar likelihood of statistical dependencies arising from the spatial distribution of seed particles using the velocity-data rate correlation as a measure.

2. Velocity-data rate correlation coefficient
The calculation of the velocity-data rate correlation coefficient can be approached using the method proposed by Edwards and Meyers (1984) and illustrated by Meyers (1991). The inputs to equation 1 are provided by the postprocessing of the velocity-time history of tracer particles acquired during acquisition, where either the time arrival of or inter-arrival times were measured. The following subsections describe the steps involved in determining the velocity-data rate correlation coefficient:

2.1 Filtering of outlier data
Outlier velocity data are removed from the velocity-time history by establishing velocity limits typically defined by the characteristics of the velocity distribution of the data ensemble. The time record is then adjusted to compensate for the missing data (Figure 1a).

2.2 Estimating the integral microscale time
The Taylor temporal microscale can be estimated based on the inter-arrival time \( dt \) between the particles, Figure 1b, and the following two assumptions:

1. Only statistically stationary flows are being considered, so that no distinction is made between time averaging and ensemble averaging; and,
2. The flow velocity changes minimally within a given Taylor microscale time interval.

The velocity axis is divided into ten even velocity increments, or bins, where the overall limits are set to the minimum and maximum velocities within the data ensemble. The occurrence time of the first particle is set to \( t_0 \) and the velocity bin number noted. The time of the first succeeding particle to have a velocity in a different bin is set to \( t_1 \) and the new velocity bin number noted. The time of the first succeeding particle to have a velocity in a different bin is set to \( t_2 \) and the new velocity bin number noted. This process continues until the time history for the data ensemble has ended. The Taylor temporal microscale can then be estimated using equation (3):

\[
\frac{t_\mu}{\mu} - \sum_{i=1}^{N} \left( t_i - t_{i-1} \right)
\]

where \( t_\mu \) is the average time a set of sequential particle velocities remain within a single bin. Meyers (1991) found that this average time was approximately 20-percent of the Taylor temporal microscale. The Taylor microscale was determined from hot wire measurements obtained simultaneously with the LDV measurements made along various profiles in a 50-m/s jet exiting from a fully developed turbulent pipe flow.
2.3 Calculation of velocity-data rate correlation coefficient using Edwards-Meyers approach

The variables $u_i$ and $r_i$ in equation (1) are determined by adding the estimated Taylor microscale time to the arrival time of the first particle ($t_0 + t_{\mu}$) then determining the average velocity of the particles that arrived during the time period between $t_0$ and ($t_0 + t_{\mu}$) to yield $u_1$, and dividing the number of particles that arrived during the time period by the estimated microscale time, $t_{\mu}$, to yield $r_1$, Figure 1c.

The time for the first particle to arrive following the microscale ($t_A > (t_0 + t_{\mu})$) marks the beginning of the next microscale time from $t_A$ to ($t_A + t_{\mu}$). The variables $u_2$ and $r_2$ are then computed for this microscale. This process continues until the time history has been completed. The mean and standard deviation of velocity is determined from the array of $u_i$, and the mean and standard deviation of data rate is determined from the array of $r_i$.

Using this approach, Meyers et al. (1990) reported profiles and maps of correlation coefficients during an investigation of the flow about a backward spacing step. The correlation coefficient range of ±0.2 encompassed the shear regions surrounding the large eddy downstream of the step.

Figure 1: Steps involved prior to calculating velocity-data rate correlation coefficient. a) Threshold to remove outlier data. b) Velocity-time history overlaid on horizontal velocity bins to estimate integral microscale time. c) Velocity-time history divided into vertical bins of integral microscale times.
3. Flow Field Investigation

The investigation to determine if the velocity-data rate correlation would provide insight into the shear flow surrounding a supersonic jet consisted of three parts. The first investigation was to determine if the correlation coefficients obtained from profiles of a subsonic jet had the same trends as reported by other researchers. The next case was the investigation of a Mach 1.4 free jet, followed by the investigation of the same jet with a shroud to partially confine the jet to determine the effect of the shroud on the flow.

3.1 Experimental set-up

A conventional fringe-type LDV (TSI) was configured in forward-scatter mode and placed downstream of the jet nozzle under test, Figure 2.

The measurement volume was moved axially from the nozzle exit plane downstream to stations every 12.7mm where a measure of velocity and correlation was performed. The acquired signal bursts were processed by a Dantec BSA F70 flow processor and simultaneously captured by a TDS 2024B digital oscilloscope (Figure 3). At each measurement location data were acquired for 30 seconds, or 10,000 signal bursts whichever occurred first. The digital oscilloscope was connected via USB to a PC running a MATLAB post-processing program (Bulusu and Garris (2011), Bulusu (2010)). A Laskin nozzle-based six-jet...
atomizer (TSI Model 9306A) with olive oil

as the working fluid was used to generate the particles in the manner described by Tropea et al. (2007). The generated particles could be injected into either the primary (jet) or secondary (entrained) flow, Figure 4.

3.2 Test sections and conditions

The geometry of the nozzles, shroud and locations of measurement scans are shown in Figures 5, 6 and 7 for the subsonic, supersonic and partially confined supersonic flows, respectively. The primary flow nozzle exit diameter was varied parametrically to generate both subsonic (Mach 0.25) and supersonic (Mach 1.4) jets exiting into 1 atmosphere room temperature air (300° K). The contour and area ratio of the supersonic nozzle to produce a Mach 1.4 jet was calculated using isentropic equations without applying boundary layer corrections. The supersonic nozzle was calibrated using a Pitot tube and applying the Rayleigh supersonic Pitot tube formula, which accounted for a bow shock. The Pitot tube calculations of Mach number were within ±3.45 percent of the design Mach number (Bulusu (2010), Bulusu and Garris (2011)). The inlet ports, shown in Figures 5, 6 and 7, allowed the surrounding room air to be entrained as a secondary flow surrounding the jet flow.
Figure 6: Schematic drawing and measurement locations for supersonic jet (Mach 1.4)

Figure 7: Schematic drawing and measurement locations for partially confined supersonic jet (with Mach 1.4 nozzle)
4. Results
The distribution of seeding particles in the shear layer (along the supersonic nozzle wall entrainment region) is dependent on the rate of mixing between the primary and secondary flows via eddies that feed particles into the shear layers. If the measurement volume translates from the primary jet flow into the shear region toward the secondary flow, there will be a change in relative contribution of seed particles due to the two flow streams at a given location. For example, just before reaching the shear region, the primary jet flow is present 100-percent of the time, whereas further in the shear region the contribution from each flow would reach 50:50-percent, and finally as the shear region is exited, 100-percent secondary flow. However, if the particle seeding is not exactly balanced between the two flows, the measured contributions from each of the two flows will not represent the true ratio – thus the bias. Consequently, mixing processes tend to produce bias, and thus correlation between velocity and data rate. The lines indicating measurement locations shown in Figures 5, 6 and 7 are marked with arrowheads indicating the location of the measurement volume along a potentially shearing region.

The result of subsonic jet experiment shown in Figure 8 is indicative of the presence of a shear layer beginning at approximately 6 diameters (\(d_e = 1.379 \text{ cm}\)) axial distance from the nozzle exit. Conditions downstream of \(x/d_e \approx 6\) are signified by an increase in and alternating values of velocity-data rate correlation coefficient (-0.09 \(\leq C \leq 0.1\)) between primary and secondary flows. The magnitude of \(C\) values (\(\pm 0.1\)) is in agreement with those conditions reported by Meyers (1990) in the shear layers of the backward facing step experiment.

The results obtained from the supersonic (Mach 1.4) jet shown in Figure 9 denote the presence of a shear layer...
without any significant mixing activity up to 3.2 diameters (d_e = 2.032 cm) axial length from the nozzle exit plane. Mixing between the two flow streams in the baseline subsonic case is evidenced by alternating high values of the correlation coefficient (C = ±0.1). In the supersonic case, the values of C fall within a narrow range (-0.05 ≤ C ≤ 0.03) for both primary and secondary flows. In addition, the variations in location-specific C-values are in phase and not alternating. This signifies lack of mixing between the primary and secondary flow streams within the limited axial distance traversed from the nozzle exit plane and illustrated by the narrow range and low values of the correlation coefficient. This particular result is consistent with previous studies that have shown that mixing is inhibited in supersonic mixing layers (Cottrell and Plesniak (2001), Goebel and Dutton (1991), Goebel et al. 1990). Especially in subsonic-supersonic, mixing layers with lower compressibility (convective Mach numbers M_c ≤ 0.7) linear growth in mixing layer (self-similar region of mixing) begins at greater streamwise distances as shown in experiments by Goebel and Dutton (1991) and validated by numerical results of Cottrell and Plesniak (2001). Therefore, homogenous mixing between the supersonic (primary) and subsonic (secondary) flow streams can be expected several more diameters of axial distance downstream of the circular free jet source. These regions of increased rate of mixing will lead to greater differences in the seeding density between the two flow streams and higher values of velocity-data rate correlation coefficient.

Based on the assumption that mixing enhancement occurs in jets in confined environments, a shroud was installed as shown in Figures 2 and 7. Entrainment of surrounding room air for the secondary flow stream was allowed through the same inlet port as in the previous experiments, while the primary flow supersonic jet expanded through the partially confined region of the shroud (d_{sh} = 4.572 cm). Measurements were made from the exit plane of the shroud offset by a distance (L = 10.853 cm) from the nozzle, axially outward as shown in Figure 7. The result of partially confining the jet and the entrained secondary flow are presented in Figure 10. The location-specific correlation coefficients for the primary and secondary flow streams exhibited an alternating tendency up to 3.44 diameters (d_e = 2.032 cm) downstream of the shroud exit. In addition, the range of correlation coefficients, -0.08 ≤ C ≤ 0.08 skewing towards C ≈ 0.1 (baseline measure shown in Figure 8.) suggests an interaction between the primary and secondary flow streams. The correlation values do not confirm the strong mixing tendencies evidenced by high correlation values in the

Figure 10: Velocity-data rate correlation coefficient of partially confined supersonic jet with Mach 1.4 nozzle at various measurement locations
baseline subsonic case. However, the alternating correlation values of the primary and secondary streams allude to the enhancement of mixing if the length of the shroud was increased. Furthermore, the result in Figure 10 is in contrast to the supersonic free jet case wherein, the correlation values suggested no tendencies of mixing within the range of measurements.

5. Conclusions
This paper reports on the location-specific changes in velocity bias in a free shear layer of circular jets using the velocity-data rate correlation coefficient as a post-facto measure. A standard velocity-data rate correlation was calculated using the Edwards-Meyers approach involving an estimation of the integral microscale time. The correlation estimation was performed at various locations along the shear layer of three jet configurations: subsonic, supersonic and partially confined supersonic. The subsonic jet was considered as a baseline result that is in agreement with earlier work cited, wherein the correlation values indicate mixing between primary and secondary flow streams. The supersonic free jet result illustrates that the mixing region was not reached within the limits of the traversing distance implying that the axial length scale for supersonic mixing is larger. The tendency for mixing enhancement in confined jets is demonstrated by the alternating location-specific correlation values between the primary and secondary flow streams and its propensity of correlation towards positive values. The results reported in this study suggest that velocity-data rate correlation provides a means to enhance our understanding of spreading and mixing in particle-laden shear layers and necessitates further application-specific examination. With several potential applications ranging from shear layer detection to vortical structure size estimation, where conventional flow statistics may not yield insight, the velocity-data rate correlation coefficient will clarify and elucidate flow behavior provided that the measurements are statistically stationary with sufficiently high data rates.

References


