Comparison of PIV-Based Methods for Airfoil Loads Evaluation

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Abstract. We compare the accuracy achievable with different methods of calculating time-averaged airfoil loads if the surrounding velocity field is known, e.g., from Particle Image Velocimetry. These methods require integration over a control volume enclosing the body. For separated flow around an inclined flat plate at Re = 10⁴, we investigate the effect of varying the control volume. Some methods yield excellent results for both lift and drag. Preliminary results for a corresponding experiment indicate that the calculated lift coincides well with direct force measurements, whereas agreement for the drag can be considered fair. Implementation of the methods was validated using a circular cylinder flow at Re = 200.

1. Introduction

Lift and drag of an airfoil are usually obtained by direct measurements using a force balance. At low Reynolds numbers (Re), however, accurate measurements may be difficult, at least, due to the very small magnitude of the airfoil loads. Also, if flow control is applied, wires or tubes for driving actuators may introduce additional uncertainties. Finally, direct force measurements cannot deliver sectional loads. For these cases, an indirect, non-intrusive method is preferable.

Alternatively to direct force or surface pressure measurements, one may deduce the loads from the velocity field measured by Particle Image Velocimetry (PIV) or Laser Doppler Velocimetry (LDV). This includes the evaluation of surface and, in most cases, volume integrals over a control volume enclosing the body, whereby two classes of methods have been proposed.

The first class is based on the momentum equation in integral form, hence requires explicit knowledge of the pressure at a surface in the fluid. As it cannot be measured directly, it must be computed in an intermediate step, either by using the differential momentum equation (e.g. Unal et al, 1997; van Oudheusden, 2008), or by solving the pressure Poisson equation (e.g. de Kat et al, 2008; Ragni et al, 2011). This step is crucial since the pressure accounts for almost the complete lift and, for separated flows, most of the drag.

For the second class of methods, the pressure has been eliminated via integral identities. Noca et al (1997, 1999) proposed three variants: the “impulse equation”, the “momentum equation” and the “flux equation”, the latter requiring only velocity information at a surface. Wu et al (2005) offered an alternative and more compact formulation.

Many of the previous studies were supported by numerical simulations which do not suffer from experimental noise. However, all comparisons were performed at very low Reynolds number ensuring fully laminar flow. Furthermore, to the best of our knowledge, there are no studies evaluating the performance of the different classes of methods on a common set of data.

In the present study, we compare the accuracy achievable with different methods for both numerical and experimental data at a reasonably high Re = O(10⁴). We look at the flow around an inclined flat plate at post-stall angles of attack, measured by 2D2C PIV and computed by Direct Numerical
Simulation (DNS). Mainly for validation, we also include a fully laminar case: a circular cylinder flow at Re = 200.

2. Methods

**Figure 1:** Principle of force evaluation. Integrals are computed over $\Gamma$ and $\Omega$. Alternative paths (see Section 3 and figures therein) are separated by $\Delta$.

**Force evaluation.** We consider incompressible flow of a fluid of unit density. Typically applied for numerical simulations, a force results from the integration of stresses over the body’s surface $S$:

$$F = - \int_S (-pn + \tau) \, ds,$$

where $p$ is the pressure, $n$ the unit normal vector, and $\tau$ the shear stress.

Alternatively, the force can be obtained from the momentum equation in integral form:

$$F_{\text{IM}} = - \frac{d}{dt} \int_\Omega u \, dV + \int_\Gamma n \cdot (-pI - uu + T) \, ds,$$

applied to a control volume $\Omega$ (enclosing the body) which is bound by a surface $\Gamma$. Herein, $u$ denotes the velocity, $T$ the viscous stress tensor, and $I$ the identity matrix.

Noca’s ‘‘momentum equation’’ also includes a surface and a volume integral,

$$F_{\text{mom}} = - \frac{d}{dt} \int_\Omega u \, dV + \oint_\Gamma n \cdot \gamma_{\text{mom}} \, ds,$$

where

$$\gamma_{\text{mom}} = \frac{1}{2} u^2 I - uu - \frac{1}{k-1} u(x \times \omega) + \frac{1}{k-1} \omega(x \times u)$$

$$- \frac{1}{k-1} \left[ \left( x \cdot \frac{\partial u}{\partial t} \right) I - x \frac{\partial u}{\partial t} \right]$$

$$+ \frac{1}{k-1} \left[ x \cdot (\nabla \cdot T) I - x(\nabla \cdot T) \right] + T,$$

whereas the ‘‘flux equation’’ requires a surface integral, only:

$$F_{\text{flux}} = \oint n \cdot \gamma_{\text{flux}} \, ds,$$

with

$$\gamma_{\text{flux}} = \frac{1}{2} u^2 I - uu - \frac{1}{k-1} u(x \times \omega) + \frac{1}{k-1} \omega(x \times u)$$

$$- \frac{1}{k-1} \left[ \left( x \cdot \frac{\partial u}{\partial t} \right) I - x \frac{\partial u}{\partial t} + (k-1) \frac{\partial u}{\partial t} x \right]$$

$$+ \frac{1}{k-1} \left[ x \cdot (\nabla \cdot T) I - x(\nabla \cdot T) \right] + T.$$
Position and vorticity vectors are given by \( x \) and \( \omega \). The constant \( k = 2 \) if Equations (3)-(6) are evaluated on planar data (yielding a two-dimensional force), and \( k = 3 \) for volumetric data.

Noca et al (1999) showed that their third equation, the “impulse equation”, performs no better than the “momentum equation”. Hence, it is omitted from the discussion. Furthermore, all of the above formulas originally involved terms to account for non-zero wall velocity, which are neglected here for simplicity.

We also implemented the method of Wu et al (2005). However, it gave unreasonable results for all tests we conducted. Since we cannot rule out implementation errors at this point, we did not include this method here.

**Pressure reconstruction.** The differential momentum equation yields the pressure gradient,

\[
\nabla p = -\frac{\partial u}{\partial t} - (u \cdot \nabla)u + \nu \nabla^2 u, \tag{7}
\]

where \( \nu \) is the kinematic viscosity. In principle, it can be directly integrated along the surfaces \( S \) or \( \Gamma \) to give the pressure required in Equations (1) or (2), respectively.

Another option is taking the divergence of (7), which leads to a Poisson equation for the pressure:

\[
\nabla^2 p = \nabla \cdot (\nabla p) = -\nabla \cdot (u \cdot \nabla u - \nu \nabla^2 u). \tag{8}
\]

Dirichlet boundary conditions could be feasible at upstream or far field boundaries where the flow can be considered irrotational, such that the pressure is known from the Bernoulli equation or can be integrated directly using Eq. 7. However, neither approach is suitable for downstream or inner boundaries (\( S \) in Fig. 2, or any surface inside \( \Gamma \) enclosing the body), so one has to resort to Neumann conditions there. For simplicity and accuracy, we use a rectangular inner boundary.

In general, the Poisson approach seems to produce less noisy pressure results (Ragni et al, 2011), however, implementing it is also more demanding.

3. Results

We compare the following methods:

- integral momentum equation (2), using original pressure from DNS, denoted by IM DNS,
- integral momentum equation (2), with pressure reconstruction by the differential momentum equation (7), denoted by IM DIFF,
- integral momentum equation (2), with pressure reconstruction by the Poisson equation (8), denoted by IM POIS,
- Noca’s Momentum and Flux equations (3) and (5), denoted by MOM and FLUX, which do not require the pressure field explicitly.

Of course, IM DNS cannot be applied to experimental data; however, it is useful to estimate errors introduced by the pressure reconstruction.

We first look at two numerical test cases, both computed using a highly accurate spectral element Fourier method (Blackburn and Sherwin, 2004). All variables are non-dimensionalized using a reference length (diameter \( D \) or chord length \( c \)) and the free-stream velocity \( U_\infty \), and denoted by an asterisk.
3.1. Circular cylinder flow, Re=200

The first test case is unsteady, laminar flow around a circular cylinder of diameter $D$ at $Re = 200$, computed by 2-D DNS. We chose it to validate the implementation of the different methods using a case with no missing out-of-plane components or underresolved gradients. Before submitting to the force calculation, velocity data has been re-sampled to a resolution of $D/100$, similar to what would be obtainable from a rather good PIV setup. Figure 3.1 shows contours of the instantaneous velocity magnitude at $t^* = 85$, and indicates the minimum and maximum integration path $\Gamma$ (circular path, placed at $r^* = 0.51$ and 0.91) used for the following discussion.

![Figure 2: Contours of instantaneous velocity magnitude for the numerical circular cylinder case at $t^* = 85$, also indicating minimum and maximum integration path.](image)

![Figure 3: Time history of lift and drag for the numerical circular cylinder case. Radius of integration path $r^* = 0.65$.](image)

Figure 3 compares the history of lift and drag coefficients, computed using all available methods, with the reference. For this case, we chose an integration path of radius $r^* = 0.65$. In general, all methods yield excellent results almost indistinguishable from the reference curves, indicating the methods and our implementation are correct.

Fig. 4 provides a more detailed picture of the accuracy of different methods, comparing the rms of
the absolute errors
\[ \text{rms}(c_{L/D} - c_{L/D}^{\text{ref}}) \equiv \sqrt{\frac{1}{t_1 - t_0} \int_{t_0}^{t_1} (c_{L/D} - c_{L/D}^{\text{ref}})^2 \, dt} \]
for varying \( r^* \).

As expected, calculating the coefficients using the original pressure from DNS (IM DNS) performs best. For the lift (Fig. 4, left), the error is minimum if the integration path is placed near the cylinder, but outside of the boundary layer (\( r^* = 0.52 \)). It increases roughly linear with \( r^* \), a trend which holds for all other methods, too. However, they produce even larger errors if the integration path approaches the cylinder’s wall (\( r^* \to 0.5 \)). Pressure reconstruction (of any kind) roughly adds an order of magnitude to the error. Still, an accuracy of approximately \( 10^{-3} \) is possible. The second best method is IM DIFF, but all others follow rather closely. Comparing Noca’s methods, MOM is slightly better than FLUX.

Curves for the drag are similar to some extend (Fig. 4, right). IM DNS is again best, and intersecting the boundary layer produces large errors. However, no clear trend is apparent when increasing \( r \). IM DIFF reaches a minimum at \( r^* \approx 0.57 \), then increases up to \( 10^{-3} \), whereas Noca’s methods oscillate around \( 10^{-4} \) and fare second best for most \( r \). Again, MOM performs slightly better than FLUX.

3.2. Separated flow around an inclined flat plate, Re=10 000

The second test case was chosen to estimate the accuracy achievable in flows with turbulent wakes. We consider the flow around a flat plate inclined at an angle of attack \( \alpha = 13^\circ \) at Reynolds number \( \text{Re} = 10^4 \), computed by 3-D DNS. The flow separates near the leading edge, undergoes transition in the separated shear layer, intermittently re-attaches, and forms a turbulent wake.

Figure 5 (left) provides an overview of the setup and indicates the initial integration path \( \Gamma \) (dotted line). It is located rather close to the flat plate, except for its upper boundary, which was placed just outside the most turbulent flow. From this initial path, alternative paths were constructed by shifting all boundaries outwards by \( \Delta \) (see Fig. 2).

Figure 6 shows lift and drag coefficients for varying distance \( \Delta \). A negative distance \( \Delta \) indicates that only the upper boundary is moved inwards, while all other boundaries then remain at their initial location. Thereby, we show the effect of this boundary moving from free stream into the separated flow.
Figure 5: Left: Contours of turbulent kinetic energy for numerical flat plate case, also indicating initial path (dotted line) and inner boundary for pressure Poisson solver (solid line). Right: Comparison of pressure computed from the Poisson equation (thick lines) with the original pressure from DNS (thin lines).

Figure 6: Lift and drag coefficients for the numerical flat plate case.

The coefficients are calculated using the time-averaged velocity field, taking into account the Reynolds stress terms for IM DNS, IM DIFF, and IM POIS.

IM DNS nicely matches the reference, indicating that given a correct pressure, the force calculation is practically independent of the path. Direct integration of pressure (IM DIFF) leads to some deviation (10% for lift and 15% for drag) if the path intersects the separation region ($\Delta < 0$). Avoiding that region, the coefficients first converge to reference values, then tend to get less accurate as $\Delta$ further increases.

Using the Poisson equation (IM POIS) in principle yields coefficients similar to IM DIFF. The roughly linear decrease for the drag between $\Delta = 0$ and maximum $\Delta$ is caused by the boundary conditions for the Poisson equation, as can be seen from Fig. 5 (right): Note that the Poisson equation is always solved on the full domain first. Then, the force is computed using the current integration path. While the Dirichlet condition at the upstream boundary apparently prescribes a correct pressure, the Neumann condition at the inner boundary leads to less accurate pressure in its vicinity. This might be caused by insufficiently resolved gradients near the leading edge. In total, a force calculation will now
tend towards a correct value for large $\Delta$, and get less accurate as $\Delta$ decreases.

Noca’s methods MOM and FLUX fare basically identically. We find very large errors for negative $\Delta$, the lift being over- and the drag underpredicted. There is a point $\Delta \approx 0.1$ somewhat away from the flat plate where Noca’s methods give accurate results. Beyond that, however, errors in lift and drag increase and exceed 10% and 15%, respectively.

3.3. Application to 2D-PIV

**Experimental apparatus and procedure.** The main goal of this experimental campaign was to have simultaneous measurements of the forces impinged on a flat plate at different angles of attack and Reynolds number conditions, both from a highly sensitive balance and from PIV velocity fields.

The measurements were obtained in an open circuit wind tunnel with a closed test chamber of $135 \times 160 \text{mm}^2$ cross section, as sketched in Fig. 7. The flat plate model (chord $c = 44 \text{mm}$, span 136 mm, thickness 3.3 mm) is made of acrylic glass so as to not block the laser light. To minimize flow disturbance, the balance (strain gages, sensitivity 0.001 N) was placed outside of the wind tunnel and connected to the ends of the airfoil by an aluminium structure.

The flat plate was operated at an angle of attack $\alpha = 15^\circ$, and four free stream velocities ranging from 10 m/s to 45 m/s ($Re = 30000 \ldots 120000$). For each operating condition, 200 image pairs were recorded at a frequency of 7.5 Hz. Simultaneously, the balance measured lift and drag forces at a frequency of 10 Hz. Mean values and their corresponding standard deviations from both techniques were finally used for comparison. Further details on the PIV setup are shown in Tables 1 and 2.

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<th>Illumination</th>
<th>Seeding</th>
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<tr>
<td>laser type</td>
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<tr>
<td>energy per pulse</td>
<td>composition</td>
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<td>wave length</td>
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<td>laser thickness</td>
<td>particle diameter</td>
</tr>
<tr>
<td>frequency</td>
<td>fog generator</td>
</tr>
</tbody>
</table>

- laser type: Litron Nd:YAG
- energy per pulse: 120 mJ
- wave length: $\lambda = 560 \text{mm}$
- laser thickness: 2 mm
- frequency: 7.5Hz (double pulse, delay 25…8\mu s)
- fluid: Shell Ondina Oil15
- composition: refined mineral oil
- particle density: $\approx 1.20\text{kg/m}^3$
- particle diameter: 1.5\mu m (median)
- fog generator: Teknova RG:100

Table 1: Illumination and seeding characteristics
### Imaging and Acquisition Parameters

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Final Field of View</td>
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Table 2: Imaging and acquisition parameters

**Preliminary results.** Figure 8 compares force coefficients computed by IM DIFF with corresponding force balance readings. A first variation of the coefficients with the integration path is indicated by error bars. We find an excellent agreement for the lift below $U_\infty = 23 \text{ m/s}$. At the highest velocity, however, IM DIFF overpredicts the lift by $\approx 35\%$. The method appears to be less accurate for the drag, with errors ranging from $10\%$ up to $50\%$.

![Figure 8: Lift and drag for the experimental flat plate case.](image)

**4. Summary**

We compared different methods for the calculation of force coefficients from a given velocity field, applied to both numerical and experimental data. Ideally, the computed force coefficients would be invariant with $\Gamma$. However, even for DNS data, we have found some variations. For numerical data of laminar flow, all methods yielded excellent results as long as the integration path did not intersect the body’s boundary layer. For numerical data of a turbulent, separated flow, excellent time-averaged forces could be obtained by the integral momentum equation and direct pressure integration.
(IM DIFF), if the integration path had been placed outside the separation region. A numerical grid of less than 100 x 100 grid nodes, a common PIV resolution, was found sufficient for this case. Using the pressure Poisson equation (IM POIS) offers no advantage over direct pressure integration (IM DIFF) for the time averaged force coefficients. Noca’s equations MOM and FLUX give rather large errors.

Preliminary results from a corresponding experiment indicate that IM DIFF yields a good agreement with balance readings for the lift. Agreement for the drag can be considered fair. Results for other methods, more angles of attack and a detailed investigation of the effect of the integration path are currently underway.

Acknowledgments

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References


de Kat R, van Oudheusden B, Scarano F, Delft T (2008) Instantaneous planar pressure field determination around a square-section cylinder based on time resolved stereo-PIV. In: 14th International Symposium on Applications of Laser Techniques to Fluid Mechanics, Lisbon, Portugal


