Visco-inertial pumping by an array of oscillating plates

Ken T Kiger\textsuperscript{1,*} and Mary Larson\textsuperscript{2}

1: Department of Mechanical Engineering, University of Maryland, College Park, MD 20910 USA
2: GE Aviation, Evendale, OH USA
* correspondent author: kkiger@umd.edu

Abstract A programmable oscillating plate-array was constructed in order to study the detailed hydrodynamics of external pumping by a series of oscillating plates at Reynolds numbers on the order of 10. The array was modeled after the geometry and kinematics found in the nymphal mayfly \textit{Ephemoroptera Centroptilum}, and consisted of 5 plates, each of which could be actuated independently for stroke and pitch. Scaled tests were performed with at a Reynolds number, \( Re = \frac{f l}{\nu} = 18 \), with a single stroke kinematic pattern modeled after the living animal. In mayflies, and in many other oscillating plate systems, an antiplectic metachronal wave is used with a phase delay of approximately 90 degrees, which corresponds to a travelling wave that moves from posterior to anterior with a wavelength of approximately 4 plates. In order to better understand possible reasons for why the animal system might favor the observed phase lag, ensemble-correlation stereo PIV measurements were made to reconstruct the unsteady three-dimensional flow field at a resolution that allowed a uniform and converged estimate of the net pumped flux and the total energy dissipation within and around the vicinity of the gill array. The results indicate that the baseline case offered an optimal spot in the mass flux of fluid pumped through the array per unit energy expended, while also providing a great deal of flexibility in modifying the stroke amplitude without interference effects from adjacent gills.

1. Introduction

Small-scale chemical sensors and micro-reactors intrinsically rely on pumping and scalar mass transport to accomplish their design goals (Vitko, \textit{et al}. 2004). Traditionally, the size and flowrate through the mechanical components used for these functions have been large enough that inertial convective mechanisms can be effectively implemented, as exemplified by devices such as centrifugal fans. One challenge faced by continually shrinking packages, however, is a concurrent decrease in the operational Reynolds number and a transit towards viscous pumping mechanics where such inertial mechanisms are ineffective (Quin & Grimes, 2008). A variety of micropump mechanisms have been studied for these applications with the most prevalent concepts being reciprocating diaphragm pumps (Thomas and Bessman, 1975; van Lintel et al, 1988; Bourouina \textit{et al}, 1997), electrohydrodynamic pumps (Bart \textit{et al}, 1990), electroosmotic pumps (Jiang \textit{et al}, 2002; Wang \textit{et al} 2009), and magnetohydrodynamic pumps (Jiang and Lee, 2000). Many of these pumps exhibit effective fluid transport for particular conditions, but are limited to fluids with specific chemical properties, finite volumes of fluid or small flow rate ranges (Laser & Santiago, 2004; Woias, 2005). The development of a versatile micropump that can be effective over a wide range of pumped volumes, flow rates, and fluid properties in the range of Reynolds number less than 100 is desirable but has not yet emerged.

One approach to exploring improved pump performance across the transition from inertial to viscous dominated regimes is to observe how animals at this intermediate scale cope with similar requirements. Animals frequently use oscillating appendages for propulsion or pumping across the entire range of Reynolds number, from inertialess creeping flow (Re << 1) to inviscid potential flow (Re >> 1) (Childress, 1973). Distinct patterns of appendicular motion have been observed on either side of this intermediate transition region, dividing stroke kinematics into two main categories:
flapping and rowing. Flapping is defined as a motion where the net thrust is perpendicular to the stroke plane and is predominantly exhibited at higher Reynolds number \((Re > 100)\). In contrast, rowing is categorized as a drag-based mechanism with the net thrust being parallel to the stroke plane, and is found to be prevalent at low Reynolds numbers (Walker, 2002). Rowing necessitates use of asymmetry in the motion or geometry of the appendage, generally with distinct power and recovery strokes, while flapping can frequently allow for symmetric motions and still generate a net thrust.

**Figure 1:** Mayfly (Ephemoptera) *Centroptilum Triangulifer* (left, photo courtesy of David Funk), and kinematic angle definitions and coordinate system for stroke motion.

While the simplest geometry calls for a single pair of oscillating appendages, many animals (such as ciliated organisms, crustacea and arthropods) use appendage arrays. This introduces limb spacing and the relative phase difference between adjacent appendages as additional parameters for the design. For low Reynolds numbers, cilia have been observed beating slightly out of phase when positioned in large groups. The phase lag between adjacent cilia creates a motion referred to as a metachronal wave. Simulations of artificial cilia have been able to induce flow using an asymmetric movement and geometry (Alexeev et al, 2008). A computational study by Hussong et al. (2010) used an array of symmetric beating cilia that only exhibited asymmetry through the use of a metachronal wave. The results show that this motion is an effective transport method for flow in a channel for Reynolds numbers ranging from 1 to 2000. The size of the phase lag in between adjacent appendages is also a factor that can change the effectiveness of the coordinated motion. At a much higher Reynolds number \((Re \sim 800)\), a study on the locomotion of krill compared how using synchronized or metachronal pleopod motion affected the efficiency of propulsion. It was found that the metachronal wave is the most efficient and also produces the highest body velocities for the krill (Alben et al., 2010). The metachronal wave is exhibited on animals that function both in low and high Reynolds number ranges and may be an important contributor to effective pumping in both cases.

The current study focuses on a specific animal (the mayfly nymph *Centroptilum triangulifer*) that functions across the intermediate range of Reynolds numbers, transitioning from a viscous to inertial dominated regime. This animal uses seven pairs of external gill plates to pump water around its body in order to acquire an acceptable concentration of oxygen necessary for breathing (see Figure 1). This particular animal is interesting because previous studies on live mayflies have shown that they exhibit a distinct shift in their appendage kinematics at a Reynolds number \((Re_f = L_0^2f/\nu)\) of order 10 (Sensenig et al, 2009). Although the flow generated by the animal has been documented in prior studies, the limited range of behavior of the animal prevented a detailed understanding of why and how such a pumping mechanism might be optimized. One specific question concerning the kinematics of the array is what is the optimal phase delay of the
adjacent gills for a given set of stroke kinematics? In the mayfly *C. Triangulifer*, a phase delay of $\Delta\phi = 60$ to $90^\circ$ was observed throughout its life cycle, which corresponds to cycle wavelength to plate length of $\lambda/L = 2.3$. A similar phase shift has been noted in other animals using arrays of appendages operating in a similar Re range, such as copepods ($\Delta\phi = 64^\circ$, $\lambda/L = 3$, van Duren and Viedler, 2003), remipeds ($\Delta\phi = 30^\circ$, $\lambda/L = 3.9$, Kohlhage and Yager, 1994) and ctenophores ($\Delta\phi = 40^\circ$, $\lambda/L = 4.5$, Barlow, Sleigh and White, 1993), but no detailed explanation has been given for why this particular phasing may represent a favorable range of operation at these transitional conditions. Does this somehow represent an optimal efficiency point in terms of the cost of pumping fluid? In order to determine the influence of individual kinematic and geometric characteristics on the pumping efficiency, further experiments that allow for isolated variation of different properties are necessary. The current study focuses on the effects of the phase delay in between the gills.

2. Experimental Design and Setup

In order to perform proposed studies, it was first necessary to develop a device that could both replicate the nominal kinematics of typical mayfly gills as well as extend the range beyond what was exhibited in nature. The array of gills used for this experiment was designed using a simplified geometry compared to the live mayfly, which was controlled using programmed microservomotors. The kinematics and physical design of the gill plate array was taken from Sensenig et al (2009). Kinematic definitions from this previous study can be seen in Figure 1.

![Figure 2: Mayfly (Ephemoptera) Centroptilum Triangulifer (left, photo courtesy of David Funk), and programmable oscillating plate array (middle), and details of linkage assembly at 3 different positions in the stroke cycle (right). The programmable array mimics the mayfly geometry, but can be programmed with arbitrary kinematics for the stroke and pitch motion.](image)

For simplicity, only the most significant kinematic parameters were considered. Examining the stroke, pitch and stroke plane deviation angles revealed that the stroke range was a dominant kinematic parameter throughout the mayfly’s life cycle (Sensenig et al, 2009). For the lower Reynolds number range, the pitch appears to have comparable significance to that of the stroke angle, while for the higher Reynolds number case, the pitch is comparably small. The stroke plane deviation is the least significant factor in both cases. By retaining stroke and pitch functionality, the device has the ability to replicate simplified kinematics for both the high Reynolds number case and the low Reynolds number case.

To simplify the construction, identical gill planform shape and kinematics were used for all gills. These were based on the kinematics of gill 4 examined by Sensenig et al (2009), as this was located in the mid-point of the array. Although the living animal consistently used an intragill phase delay of $\Delta\phi = 60$ to $90^\circ$, the current study focuses on effect of varying the phase delays between the gills, which imposes limitations on the kinematics. It was necessary to reduce the range of the
stroke and the pitch to 64% of their original values to prevent collision between adjacent gills and control linkages for the conditions of \( \Delta \Phi = 180^\circ \) (asynchronous motion).

The robotic gill array achieves the desired kinematics using two micro servomotors, crank arms and sliding linkages to control each gill (See Figure 2). The linkage is designed such that correlated motion of both servos changes the stroke of the gill, while a differential movement provides the pitch variation. To control the stroke of the gills, the servomotors are arranged such that the motor rotation axis is perpendicular to the stroke plane of the gill. A steel extension arm is attached to each servomotor and connected to a thin metal push-rod that is embedded in the gill, passes through the stroke extension arm, and terminates after an s-bend within a slot of the pitch extension arm. When the servo arm moves, the extension arm rotates, which then moves the gill push-rod the same angular displacement as the servo arm. The pitch is controlled by varying the difference in the angle between the two servo arms. When the two servos move together so that the extension arms connected to them stay parallel, the pitch remains fixed at \( \alpha = 90^\circ \). When there is a differential movement of the two arms, the metal rod in the gill is forced to rotate to compensate for the difference, which causes the pitch of the gill to change. Therefore the angles of the servo arms used to control the pitch are calculated relative to the positions of the stroke servo.

An important aspect of the pumping done by mayfly gills is the effect of combining the gills into an array. For this reason, five gills were used for the experiment to provide three “internal” gills free of end effects. On a live mayfly, there is some slight variation for the distance between the roots of the gills, but for this simplified model the gills are equally spaced with a root separation to gill length ratio of 0.6. The gill plates for the robot were fabricated out of transparent, UV cured acrylic (Loctite 3525).

The flow generated by the live mayfly nymph is bilaterally symmetric, so it was only necessary to construct a single side of the mayfly. The gill plate array (length from gill 1 to gill 5 = 96 mm) was immersed in a small aquarium (260 x 310 x 510 mm) partially filled with mineral oil (depth of fluid = 200 mm), and oriented such that the bilateral symmetry plane of the mayfly body corresponded with the free surface of the liquid. A simplified body shell for the mayfly was constructed, which only models the abdomen of the mayfly nymph to which the gills are attached. This geometry allows for a gimbled motion about the gill root, which provides for variation of the stroke plane inclination angle (\( \beta \)) relative to the fixed body. In the current work, the stroke plane inclination angle was fixed at 60°.

The model was scaled using the oscillating Reynolds number \( Re_l = L_g^2 f / \nu \), where \( L_g \) is the length of the gill from the root to the tip in mm, \( f \) is the frequency that the gills oscillate at in Hz and \( \nu \) is the viscosity in cSt. Considering the velocity ranges of the servomotors and the spacing necessary to present them from colliding, the robotic model dimensions are approximately 54 times those of the original mayfly (\( L_g = 40 \) mm). Mineral oil with a viscosity of 175 cSt was used as the working fluid used for this experiment to achieve a Reynolds of 17.4.

Three dimensional flow data was obtained using stereo PIV, where two cameras (LaVision Imager Pro 4M cameras, 2048 x 2048 pixels, 50 mm lens) equipped with Scheimpflug mounts were placed on the same side of the laser sheet (approximately 2 mm thick) with a 66° angle between them. Because of the shape of the tank and the index of refraction between mineral oil and air, it was necessary to construct prisms so that the camera lenses were normal to the windows (Prasad and Jenson, 1995), giving a resulting field of view of approximately 154 x 90 mm. Data was taken at thirty-two planes (nineteen planes across a single gill) with a 2 mm spacing between planes, in order to obtain a three dimensional volume of flow data around the array of gill plates. Each image was interrogated using a multipass algorithm with a final interrogation window size of 32x32 pixels with 0% overlap, which gave an effective in-plane spatial resolution of approximately 2 mm. The largest uncertainty in the measurements was due to the variability in the motion of individual gills due to tolerances in the individual linkages and the effects of friction (typically less than 1 mm). To minimize the impact of this, 40 images were acquired at the same phase angle (17 phase angles
were acquired over a single cycle), processed individually, and then median sorted to remove outliers from the set. The \( N = 30 \) vector fields which exhibit a minimum deviation from the median vector field were averaged to produce the instantaneous ensemble-averaged velocity field. The uncertainty of this velocity field is taken by the uncertainty of the mean of the ensemble, which found to be less than or equal to \( \sigma_u/N^{0.5} = \pm 0.05 \) pixels or \( \pm 0.18 \) mm/s for all the data, where \( \sigma_u \) is the square root of the variance of the ensemble. See Larson (2011) for details of the analysis.

Figure 3: Stroke kinematics for the robotic mayfly, showing the stroke and pitch program achieved for each of the 5 gills in the array for each of the four intergill phase delays (\( \Delta \Phi \)) tested. Solid lines indicate the measured motion extracted from the stereo images during testing, while the dotted line indicates the intended program.

3. Results

The results of this experiment contain flow field data for four different test conditions. In these four tests, the amplitude of the stroke and pitch are kept consistent for each case, but different phase lags are used in between each gill. The phase lags tested are approximately 0°, 90°, 180° and 270°. The actual stroke and pitch of each gill plate achieved in the experiment will differ from gill to gill from the specified motion due to variations in the tolerance of the actuator construction.

The kinematics realized during the experiment were documented by tracking 38 points on each gill using the images from the PIV measurements. Even though the index of refraction of the gill material (\( n_{\text{gill}} = 1.49 \)) was similar to that of the oil used in the experiments (\( n_{\text{oil}} = 1.47 \)), the edge of the gill still scattered sufficient light to be visible in the PIV image. These points were recorded for the five gills at every phase angle and every plane where the laser sheet intersected with the gill plates. The results of this kinematic tracking for each test case (distinguished by their phase delay, \( \Delta \Phi \)) can be seen in Figure 3. The black circles indicate the position commands that were sent to the servomotors. The same pattern was sent for each gill, just at different times, depending on the phase delay between the gills.

3.1 Instantaneous and Time-Averaged Velocity

The instantaneous velocity fields within the X-Y (coronal) planes for the four different phase delays, as well as the corresponding mean flow, is shown in Figure 4. For all cases, the flow
near an individual gill is fairly similar, with negative vorticity generated at the lateral tip (-y) furthest from the root during retraction (motion towards +x), and a positive vortex during protraction (motion toward -x). A weaker vortex of opposite sign is generated near the root of the gill closest to the medial plane (+y). The primary difference of the flows then stems from the timing of the interactions of these vortices with their surrounding neighbors. For the case of synchronous gill motion ($\Delta\Phi = 0^\circ$), all of the gills peak their circulation at the same point in time, leading to the formation of an array of vortices of the same sign, which may be viewed as a discretized vortex sheet. As it peaks near mid-retraction ($t/T = 0.24$) the two oppositely signed sheets lead to a region
of uniform streaming within the gill region. As the gills decelerate and reverse direction ($t/T = 0.47$), the vorticity diffuses into the outer flow and the external flow is redirected toward the anterior direction. As protraction starts, as similar, but somewhat weaker flow is repeated in the opposite direction. The protraction is weaker due to the asymmetry in the stroke rate (see Figure 3).

The $\Delta \Phi = 90^\circ$ is most similar to the actual animal conditions, and the kinematics is observed to propagate as an antiplectic metachronal wave with a wavelength of $\lambda/L = 2.4$. As noted by Sensenig et al (2010), phasing places the retracting plate in relative isolation of its neighbors during the retracting power stroke, creating a dominant dipole about the root and lateral tip on the same gill, creating an induced flow in the posterior (+x) direction. The posteriorly directed jet follows the metachronal wave anteriorly during the cycle. During the weaker protraction, the plates are in their closest proximity and interfere with one another to prevent significant anteriorly directed backflow.

For $\Delta \Phi = 180^\circ$, operate in a nearly antisymmetric fashion (the pitch program is not symmetric with respect to each half-cycle, and the protraction rate is less than the retraction rate), with plates making their closest approach to neighboring gills the beginning and mid-point of the stroke cycle. This leads to dipole pairs on neighboring gills that produce alternating lateral and medial directed jets, with a dominant lateral (-y) outward direction. Finally, $\Delta \Phi = 270^\circ$ is similar to the $\Delta \Phi = 90^\circ$ condition except that the wave exhibits a symplectic metachronism. This now places the protracting recovery stroke in isolation, producing a weaker anterior (-x) directed jet that travels in the posterior direction. The stronger retraction power stroke occurs in close proximity to its neighboring gills, preventing effectively directed pumping.

The time average of the 17 different equally spaced samples in the cycle gives a representation of the net flow through the gill array. In general the flow is seen to enter the array from the medial plane as well as the anterior ($x < 0$) and posterior ($x > 0$) regions to the left and right of the array. The outflow is generally directed laterally away from the array (transverse to the attachment line of the gills along $y = 0$), but is biased towards anterior or posterior direction depending on the antiplectic (posterior flow bias) or symplectic (anterior flow bias) metachronal kinematics. While the symmetric and anti-symmetric cases should, in principle, produce a symmetric flow, there is a slight posterior-directed bias to the outflow due to the fact that the specified kinematics of the plates are not themselves symmetric.

### 3.2 Flux

In order to properly measure the effective efficiency of the system, one needs to first consider what is being optimized. For living system, such as a mayfly, the oscillating appendages are used to pump fresh oxygenated water close to the skin of the animal to enhance the uptake of oxygen into the animal. In this sense, the animal would like to maximize the oxygen uptake with a minimal expenditure of energy. To calculate this, one would need to measure 1) the scalar absorption rate of oxygen onto the surface of the gill plates and the body of the animal, and 2) the amount work done by the plates on the working fluid.

The first of these points requires tracking a scalar with suitable boundary conditions on the plates and the far field. Such an experiment is beyond the scope of the current work. Instead, we assume the absorption rate would be tied to the local volume flux of fluid through a control volume surrounding the extents of the plate array. Figure 5 shows three of the six surfaces of the bounding control volume used for this calculation, which shows similar trends to the horizontal slices depicted in Figure 4. Specifically for all cases inflow is largely from the dorsal direction and close to the body from the anterior and posterior direction. The outflow is directed laterally and dorsally, but is biased toward the posterior for the antiplectic wave and biased anteriorly for the symplectic wave. The symmetric and antisymmetric cases are slightly biased toward the anterior direction.
Figure 5: Time-averaged volume flux across a control volume bounding the gill array (red = outflow, blue = inflow). Select three-component velocity vectors are shown on the surface, along with their projection onto the surface to convey the net three-dimensional flow. The velocity magnitude is made non-dimensional by \( v/f_L \).

The net flux through the array was then calculated by multiplying the time-averaged out-of-plane velocity components on each surface by the area surrounding each vector, then summing separately all of the flow entering into the Control Volume (CV, defined in this case by the extents of the PIV interrogation domain) and all of the flow moving out of the CV. In the equation below, \( Q_{in,k} \) represents the volume of flow going into one side of the control volume with a unit normal vector in the \( k \) direction:

\[
Q_{in,k} = \int \sigma_{k_in} \, dx_i \, dx_j \cong \sum_{l_{in} = 1}^{N_{in}} \sigma_{k_{in}} \delta x_i \delta x_j
\]

(1)

where \( i, j \) and \( k \) represent the directions relevant to the surface on the volume being analyzed, and \( l \) represents the index over all inward pointing vectors on the surface. \( i \) and \( j \) represent the components in this plane, while \( k \) represent the out-of-plane component. The total volume of fluid moving into the control volume \( (Q_{in}) \) is then given by:

\[
Q_{in} = \sum_{l=1}^{N_{sides=6}} Q_{in,l}
\]

(2)

A similar expression is used for the fluid moving out of the control volume. The results were made non-dimensional by dividing the volumetric flowrate by \( L_g^3 f \), where \( L_g \) is the gill length, 0.04 m, and \( f \) is the frequency, 1.85 Hz. Due to conservation of mass, the sum of the flow in and out of the CV should equal zero, but is not due to the accumulated error in the measurement. The total uncertainty of the measurement was propagated through the flux calculations using the standard error from the variation in the flow field calculated from the ensemble of 30 images acquired at a single phase in the cycle, and results in an uncertainty of about ±1% of the average of the total flow in and out of the control volume, consistent with the observed discrepancy.

The amount of fluid that the array in each test case pumps is given by the average of the magnitudes of the total flow in and the total flow out. Comparing this value for the different test
cases, the second test case, which uses a phase delay of 90°, has the highest value, followed closely by \( \Delta \Phi = 180° (Q_{\text{net,180}} = 0.96Q_{\text{net,90}}) \) and then \( \Delta \Phi = 270° (Q_{\text{net,270}} = 0.94Q_{\text{net,90}}) \). The synchronous case (\( \Delta \Phi = 0° \)) had a distinctly lower volume flux with \( Q_{\text{net,0}} = 0.60Q_{\text{net,90}} \). The marked decrease for the synchronous case is indicative of the pumping benefit provided by the relative isolation and close approach that occurs for the non-synchronous cases. In that the symplectic case is only 5% less than the antiplectic case is somewhat surprising, as the pitch program is biased to favor the symplectic conditions, resulting from basing the kinematics on the live animal. To answer questions of the benefit provided by using a closely-spaced array, it would be of interest to perform future studies with a single gill.

### Table 1: The non-dimensionalized volumetric flow rate going in and out for six sides of a control volume for the five different test cases. These were nondimensionalized by dividing by \( f^3 L g \), where \( f \) is the frequency of the cycle, 1.85 Hz, and \( L g \) is the length of the gill 0.04 m.

<table>
<thead>
<tr>
<th>Phase Lag (( \Delta \Phi )):</th>
<th>0°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In</td>
<td>Out</td>
<td>In</td>
<td>Out</td>
</tr>
<tr>
<td>Surface 1</td>
<td>-0.0926</td>
<td>0.0331</td>
<td>-0.0993</td>
<td>0.0426</td>
</tr>
<tr>
<td>Surface 2</td>
<td>-0.1330</td>
<td>0.1640</td>
<td>-0.2550</td>
<td>0.2770</td>
</tr>
<tr>
<td>Surface 3</td>
<td>-0.0781</td>
<td>0.0503</td>
<td>-0.1580</td>
<td>0.0000</td>
</tr>
<tr>
<td>Surface 4</td>
<td>-0.0364</td>
<td>0.0410</td>
<td>-0.0757</td>
<td>0.1090</td>
</tr>
<tr>
<td>Surface 5</td>
<td>-0.0094</td>
<td>0.0713</td>
<td>-0.0006</td>
<td>0.1780</td>
</tr>
<tr>
<td>Surface 6</td>
<td>-0.0130</td>
<td>0.0086</td>
<td>-0.0146</td>
<td>0.0026</td>
</tr>
<tr>
<td>Total</td>
<td>-0.362</td>
<td>0.368</td>
<td>-0.604</td>
<td>0.609</td>
</tr>
<tr>
<td>Avg. magnitude of flow In and Out, ( Q_{\text{net}} )</td>
<td>0.365</td>
<td>0.606</td>
<td>0.585</td>
<td>0.570</td>
</tr>
</tbody>
</table>

3.3 Dissipation

The second parameter calculated is the amount of dissipation in a fluid system, which is the amount of energy lost due to irreversible viscous shear work done on the fluid. This is calculated by:

\[
\varphi = \mu \left[ 2 \left( \frac{\partial U}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial y} \right)^2 + 2 \left( \frac{\partial W}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right)^2 + \left( \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial W}{\partial y} + \frac{\partial V}{\partial z} \right)^2 \right]
\]

Due to the relatively small inertia of the fluid in these flows, the energy imparted to the fluid by the gill plate motion is dissipated within a short distance of the gill surface. In a companion study to the work of Sensenig et al (2010), direct numerical simulations of a mayfly gill array confirm that the

![Figure 6: Non-dimensional average dissipation and net pumped flux through the control volume.](image-url)
average work done by the mayfly gills over a cycle is within 2% of the total dissipation of energy over a complete cycle in a volume within 1 gill length of the array boundary (K. Abdelaziz et al, personal communication, March, 2011). Using this information, it is possible to compare the average amount of work done by the gill arrays for each test case using the time-average dissipation within the control volume as an equivalent surrogate for the work performed by the gills. The resulting average dissipations were non-dimensionalized by $\rho f^4 L_g^{-5}$, where $\rho$ is the density, $f$ is the frequency of the cycle, and $L_g$ is the gill length. These results are listed in Figure 6. Comparing the time averages of the dissipation shows that the gills in the $\Delta \Phi = 0^\circ$ case did the least amount of work. The $\Delta \Phi = 180^\circ$ degree test case required the most work followed by the $\Delta \Phi = 270^\circ$ and $\Delta \Phi = 90^\circ$ test case, respectively.

Figure 7: Instantaneous dissipation rates during power stroke for each 4 test conditions.

To understand why the average dissipation exhibited the above trend, it is instructive to observe the peak instantaneous dissipation for each case, as shown in Figure 7. The peak dissipation is formed when each gill is performing its power stroke, and the maximum value is largest when the gill performing its power stroke is relatively close to adjacent gills. Observing the results of the $180^\circ$ degree test case, which generated the most dissipation, the gills beginning their power strokes are relatively close to an adjacent gill beginning to move in the opposite direction. When they end their power strokes, they are moving towards a gill once again moving in the opposite direction. These high velocities in opposite directions at close proximity to each other cause high gradients, resulting in the largest amounts of dissipation.

A similar observation can be made when comparing the dissipation generated by the $\Delta \Phi = 90^\circ$ and $270^\circ$ phase delay cases. For the $90^\circ$ case, the antiplectic wave allows the retracting gill to be relatively far away from adjacent gills when performing its power stroke for the $90^\circ$ case, while the symplectic wave for $\Delta \Phi = 270^\circ$ places the plates comparatively closer during the power stroke. This causes smaller velocity gradients for the $90^\circ$ phase lag case, which results in lower peak and total dissipation. The test case with a phase lag of $0^\circ$ resulted in the lowest dissipation for the same reason. The gills are always equidistance from each other and they are always moving in the same direction. This synchronized motion results in the lowest gradients, and therefore produces the least amount of dissipation.

### 3.3 Efficiency

To determine which case exhibits the best overall performance, a metric to describe the “pumping efficiency” as a Mass-Specific Volume Flux (MSVF) is introduced. The mass-specific volume flux was calculated by dividing the net pumped volume flux by the average dissipation (representing the work done by the gills). Uncertainty propagation resulted in uncertainties in the MSVF of ±0.001. The calculated MSVF values are listed in Figure 6. This calculation reveals that the phase lags that yield the highest efficiency are $0^\circ$ and $90^\circ$. With the given uncertainty, there is no statistical difference between the efficiencies for these two phase delays. The lowest efficiency was produce by the $\Delta \Phi = 180^\circ$, which is about 20% lower than the $90^\circ$ phase delay efficiency.

Based on observation of natural pumping systems such as the mayfly, one may hypothesize that the case with a phase delay of $90^\circ$ would function with the highest pumping efficiency. While
this is true, a phase delay of $0^\circ$ resulted in the relative pumping cost per unit of fluid moved over the array. A mayfly nymph uses its gills to circulate water around its body in order to maintain the necessary amount of oxygen in it that allows it to breathe, and hence the absolute magnitude of the amount of pumped fluid is also relevant. Since the amount that the array pumps with a phase delay of $0^\circ$ is 40% lower than the amount pumped with the array that uses the $\Delta \Phi = 90^\circ$, the $90^\circ$ phase delay would be more effective for this purpose. In order to determine if this is a substantial explanation, a study that specifically focuses on the amount of fluid that comes in contact with the body of the mayfly (and therefore delivers oxygen to the body) would have to be performed.

4. Conclusions

In order to further explore the pumping mechanisms of a set of mayfly gills, a two-degree-of-freedom robotic oscillating plate array was constructed, which allowed for variations in the kinematics beyond what is exhibited by the animal, allowing for an examination on the effect of gill phasing on the pumping performance of the array. Stereo PIV was used for four different test cases to measure all three components of the unsteady velocity field over a three-dimensional volume surrounding the array. Data was taken using four different phase delays: the $\Delta \Phi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$, all with the same programmed stroke and pitch amplitude. The quantitative measurements acquired for each case allowed for the net pumping rate, flow induced dissipation and a ratio of these two, representing a specific flux efficiency, to be directly computed.

A number of trends emerged when examining which phase delay resulted in the highest efficiency, highest flux and highest dissipation. Phase delays that cause the gills to move asynchronously (and therefore to generate counter-rotating vortex pairs on adjacent gill tips), were observed to generate significantly higher amounts of flux. The phase delay with the gill performing the power stroke the furthest from adjacent gills, the $\Delta \Phi = 90^\circ$, produced the highest amount of flux. This makes smaller phase delays, which allow higher stroke and pitch amplitudes to be reached without collision, to be advantageous for applications where high flow rates are required. Synchronized motion ($\Delta \Phi = 0^\circ$) produces the least amount of dissipation, which for this low Reynolds number experiment can be equated to the amount of work done within a region near the gill array. The total amount of dissipation increases as the phase delay causes the space between adjacent gills with large velocity gradients between them to decrease.

These trends show that adjacent gills must have velocity gradients between them to achieve high flow rates, but that if the high gradients are accompanied by close proximity to adjacent gills, the amount of work required will increase, and the amount of pumping will decrease. This combination causes a decrease in efficiency. For an optimized pumping device, it would be necessary to determine the best balance for spacing and velocity pattern to achieve beneficial vortex pumping, but minimize detrimentally high gradients.

References


