Higher order 4-frame Particle Tracking Velocimetry

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Abstract In fluid mechanics particle tracking methods have a great potential for enhancing the spatial resolution and measurement precision compared to correlation-based methods (PIV) [10]. They are not biased do to inhomogeneous seeding concentration close to walls [11] or volume illumination and out-of-plane gradients [4]. However, at high seeding concentrations the reliable particle pairing is challenging and the measurement precision decreases due to noise caused by overlapping particle images. In this paper it is shown, that the particle image information acquired at four time steps can be used for a reliable particle pairing at high seeding concentrations. Furthermore, it is shown that the accuracy can be increased by using vector reallocation and displacement estimation via a fit of the trajectory in the case of strong spatial gradients.

1. Introduction

Particle Image Velocimetry is a well-established technique frequently applied for fluid dynamics research. The velocity is estimated by a cross correlation of the images of small tracer particles recorded at time t and t+Δt. For a robust and precise cross correlation, interrogation windows covering 6-10 particle images are usually required to obtain accurate results. Since 6-10 particle images have to be present, the size of the interrogation windows depends also on the particle image diameter and the seeding concentration. The spatial resolution of the vector field is therefore limited by the size of these interrogation windows and often not appropriate to resolve the flow gradients properly [10]. To avoid these errors and increase the spatial resolution, single pixel ensemble correlation can be used to estimate velocities and Reynolds stresses accurately and robustly [20]. Furthermore, the velocity vector can be determined with up to third-order accuracy [21]. However, the real resolution, i.e. the distance between two independent vectors, is limited by the particle image size [10] and a large amount of images with a high particle image density is necessary. To resolve small flow features, the optical magnification is often increased and the seeding density becomes too sparse for cross-correlation methods. Furthermore, correlation based methods often show bias errors due to changing particle image density close to walls. As particle tracking techniques are not affected by these bias errors at all [11], the technique is often better suited to resolve flow gradients. Modern particle tracking algorithms as the one published by Ohmi and Li [15] work very robustly even at high seeding concentrations. The accuracy and the robustness are however limited by the fact that only two representations at t and t +Δt are present. Thus, only a first order approximation of the velocity can be estimated. To further enhance the precision in estimating the flow velocity multi-pulse or multi-frame techniques were already developed in the early days of digital PIV as summarized by Adrian [1] and references herein. Most of these techniques use equidistant time intervals between the images. However, a different time-stepping scheme can have some advantages as presented in the following. The paper will be organized as follows: first the limitations of PTV algorithms will be presented using synthetic data, second the concept of a multi-pulse system will be discussed. The main benefits of such a system are the enhanced accuracy due to the estimation of the velocity by integrating the particle image path line, the lower relative error due to larger absolute displacements and a possible vector relocalization in the case of strong gradients. These benefits are discussed in detail using synthetic particle distributions. Next, the algorithm is used to evaluate experimental 2D data of a water jet experiment. Finally, 3D data obtained in a microfluidic experiment are analyzed to show the performance of the approach
for volumetric data sets.

2. Limitations for particle tracking algorithms

For particle tracking algorithms at high seeding concentrations the two biggest sources of errors are the error related to the determination of the particle position in the plane or volume and the wrong particle pairing. The first one is related to the imaging itself and the detection algorithms a discussion can be found in Cierpka and Kähler [3]. However, if the seeding concentration is low enough to avoid a large number of overlapping particle images, the position of particle images can be determined with an uncertainty lower than 1/100 px [11].

If one assumes that the locations of the particles have been determined reliably, the related particles pairs have to be identified by the tracking algorithm. The simplest case to match corresponding particle images is a nearest neighbor PTV algorithm; others are artificial neural networks or relaxation methods to minimize a local or global cost function [18]. Okamoto et al. [16] presented their so called spring force model in 1995. Particle pairs were identified by searching for the smallest spring force calculated over particles in a certain neighborhood. Another method to improve the detection of corresponding particle pairs is the use of a predictor. This predictor can significantly decrease the search area in the second frame and thus improve the match probability of particles. Such predictors can be theoretically known velocity distributions or be experimentally obtained by PTV of previous images (for steady flows) or a coarse PIV evaluation [2],[7],[12],[22]. For higher seeding concentrations, probabilistic approaches that take the motion of neighboring particles into account show a very high vector yield. Such an algorithm was presented by Ohmi et al. [15]. The user has to define a case sensitive search radius in the second frame to find possibly matching particles. This will be done for all particles found in the first frame. For each possible match the algorithm adds the probabilities of similar neighbor vectors using an iterative approach, where the threshold for the common motion of the neighboring particles is another parameter specified by the user. This two-frame method showed quite superior results for high seeding concentrations. Another method for higher seeding concentrations is the tracking of particles over more than two successive frames. One of the first approaches was presented by Nishino et al. [14] who used four consecutive frames. The smoothness of a particle trajectory was evaluated to check if a particle path was valid or not. Hassan and Canaan [9], for example, used a nearest neighbor approach with four different frames with equidistant time intervals to enhance the results for bubbly flow. Ouellette et al. [17] used criteria as the minimal acceleration for the third frame, or minimized change in acceleration for the fourth frame where a modified version of the latter criteria showed best results. Guezenne et al. [8] applied a penalty function to prove the path coherence of particle trajectories the Euclidian distance of probable particle images to the predicted position from previous frames was weighted and added to the penalty function. Thus the particle track with the lowest penalty is the most likely one. As a compromise between computational costs and accuracy, the authors proposed to use 5 frames. Malik et al. [13] also developed a four-frame method to detect 3D particle trajectories in a volume. The velocity estimates were used to decrease the search radius for the nearest neighbor search in the next frame. For increasing seeding concentrations more than one possible link was determined and the velocity or acceleration was used to select the most likely one. In Figure 1 the schematic of a particle tracking algorithm with predictor is shown. The algorithm first determines the displacement between the particle images in frame one and two. The time interval between frame one and two should be small enough to allow for nearly 100% of valid particle links (cmp. Figure 2). The predictor is now used to decrease the search area in frame three. This allows a larger time interval and thus a larger displacement. For valid particle pairs the error associated with PTV is only given by the uncertainty in the centroid estimation and thus the relative error for the displacement will decrease. The predictor is also used later in the fourth frame.
The benefits and differences for different temporal schemes will be described later. First, the algorithm was tested for different particle image densities that can be best specified by the mean inter particle image distance $\Delta x_o$. Please note that the mean inter particle image spacing was used and not the mean minimal particle image spacing. A suitable criterion to compare the performance of a PTV algorithm is the ratio of this mean particle image distance to the maximum displacement of the particle images $p = \Delta(x_o)/\Delta(x_{\text{max}})$. For a displacement of 2.31 times the inter particle spacing, Malik et al. [13] could detect about 90% of simulated particle pairs, $R_1 = N_{\text{detected}}/N_{\text{possible}} = 0.9$, whereas the yield, the number of valid vectors divided by the number of detected vectors was $R_2 = 0.97$.

Figure 1) Schematic of the working principle of the four-frame method. The circles indicate the search area for the corresponding frames.

In this study we used the probability approach [15]. This method has the benefit, that it can be used for two, four or even multiple frame particle tracking. To validate the performance of the different cases, a Monte-Carlo simulation of a uniform flow without a gradient was used. Random particle image positions were simulated on a 256x256 pixel space with the number of particle images increasing from $N_{\text{particles}} = 10$ to 20000. The mean inter particle image spacing ranges from $1.8$ to $57$ pixel, where $N_xN_y$ corresponds to the image size. The particle image density for this case would be $N_{\text{particles}}/A = 0.00015$ to 0.3 pp. At these high particle image densities the final limitation of the particle tracking is not anymore the tracking algorithm but the ability to determine the particles’ centers reliably. On the left side of Figure 2 $R_1 =$ the number of detected vectors divided by the number of particles is shown for a simple nearest neighbor algorithm (NN), a nearest neighbor algorithm with weighting functions (NNW), the probability approach for two frames (P2F) and the probability approach for four frames (P4F). For P4F, the time interval between frames 2 and 3 was five times the time interval of the others. It can be clearly seen that both nearest neighbor approaches give a vector for nearly each particle image as $R_1$ is almost unaffected by the inter particle distance. However, with increasing seeding concentration, i.e. smaller distance between particle images, wrong particle links result in an underestimation of the velocity. The ratio $R_2$ drops significantly; for $p = 2.3$ only 70% of good vectors are determined. If physical knowledge of the flow is available, one could increase the detectability by using weighting factors. That means the distance in the predominant direction is multiplied by a factor lower than one, to artificially increase the particle spacing in the other direction. Using that approach, about 90% of good vectors can be found at $p = 2.3$. Since the flow is uniform for this synthetic test case, the probabilistic algorithm’s performance is superior. For $p > 1$, over 90% of possible vectors were found. In addition the ratio of valid vectors is nearly 100%. The four-frame
algorithm determines a predictor from the displacement between frame 1 and 2 and, thus, shows a better performance in finding the right links in the next frames. It also becomes clear from Figure 2 that in the case of a uniform flow, nearly every detected vector represents the right velocity ($R_2 \approx 1$).

For the next investigations, the ratio between the mean inter particle spacing to the maximum velocity was chosen to be unity, which would result in 2170 particle images per frame and thus an image density of $N_{\text{particles}}/A = 0.033$ ppp. For particle images with a diameter of 3, this would already lead to 20% of the image area covered. This shows that the four-frame algorithm works very well, even for particle image densities that are optimal for correlation methods. Since a velocity vector is found for each individual particle, the resolution is 6-10 times larger for PTV.

3. Vector reallocation and velocity estimation by the trajectory

An inherent limitation of PIV and PTV algorithms using two frames is the fact that the velocity can only be estimated up to the second order. Using multiple frames, a higher order approximation of the velocity is possible. So the question arises of how the additional information from representations must be used to further increase the accuracy of the methods.

For a two-frame (double laser pulse) representation in PIV or PTV the basic assumption is that the
particle path between the two positions is linear. Often, strong spatial gradients are present and this assumption is not valid. To overcome this problem, the time interval between the two frames must be reduced. Unfortunately this results in a smaller displacement and thus the relative error of the displacement estimation increases as the absolute uncertainty $\sigma_x$ for the displacement estimation stays constant. The same particle path as shown in Figure 2 is shown in Figure 3 to illustrate the bias error. The time interval between $t_1$ and $t_2$ is rather short so that non-linear effects can be neglected. However, the relative error is large for the small displacement. Using the representations of the particle position for $t_1$ and $t_4$ gives a larger displacement, and helps to reduce the relative error. However, a bias error $\varepsilon_{\Delta x}$ for $\Delta x_{14}$ can be clearly seen due to the linear assumption compared to the integral particle path. Another error arises for the vector positioning. As can be seen from the figure, the vector is usually positioned in the middle between the two positions at $t_1$ and $t_4$ for a two-frame representation. In this case a substantial error $\varepsilon_x$ is made compared to the positioning at the trajectory. In order to quantify the benefits from the above mentioned vector reallocation and displacement estimation by the particle trajectory, the flow field of a Lamb-Oseen vortex was used. The maximum tangential displacement was chosen to be $\Delta_{\tan} = 5.5$ pixel at a radial position of $r = 31.7$ pixel between $t_2$ and $t_3$.

In Figure 4, the tangential and radial displacement distributions are shown for the case without any noise or uncertainty related to the particle positions. Therefore, the errors are purely systematic. The
red circles represent the simple central differences between $t_2$ and $t_3$. As expected, the tangential displacement is underestimated as can be seen in the lower part of the figure, where the difference from the analytical solution is shown. This underestimation due to the curvature of the particle path reaches 0.034 pixel at the maximum. However, the error related to the radial displacement is much stronger and reaches 0.45 pixel. For the four-frame method, the particle positions in $x$ and $y$ were fitted by a second and third order polynomial function $x(t)$ and $y(t)$. Two different methods were used for the displacement estimation. First, the displacement was estimated to be the gradient of this polynomials at the $t = t_2 + (t_3-t_2)/2$. This method is indicated by 'grad' in the figure. The second method uses the integration of the trajectory to get the path length and is indicated by 'int'.

As can be seen, the best match is achieved for the third order polynomial which represents the circular trajectories of the particles best. Almost no errors can be seen for the gradient based displacement estimation in the tangential and radial directions. However, using the integration method, the tangential displacement is underestimated by about 0.013 pixel whereas the radial component does not show significant errors. Using a second order polynomial fit results in an error of 0.017 pixel for the tangential displacement and a negligible error for the radial displacement using both, the integration and the gradient method.

Figure 5) Tangential radial displacement distribution for a Lamb-Oseen vortex estimated with central differences for 2 frames and a third and second order polynomial using four frames (top) and difference to the analytical velocity (bottom). The particle positions are subject to Gaussian distributed noise with a standard deviation of 0.02 pixels.

However, in reality, an uncertainty is related to the particle position. Since a third order polynomial that fits all positions exactly will be found, the displacement estimate using such a fit would show a
larger random error. This can be seen in Figure 5, where the displacement estimates are shown for particle positions associated with a Gaussian distributed error with a standard deviation of 0.02 pixels.

These error levels can be easily reached for elaborate particle detection algorithms and high quality imaging [11]. A large random scatter of the two-frame method data can be seen on top of the systematic error. For the four-frame algorithms, the error of the tangential displacement is not clearly visible from the profile but can clearly be seen in the lower part of the figure. The two-frame method still shows the systematic errors but also shows a random scatter in the same order of magnitude. For the second order polynomial, integration and gradient estimation perform equally. For the fit with the third order polynomial, the scatter is much larger due the above mentioned issue of the capability to exactly fit the four successive positions. Here, the smoothing that is inherent with the second order fit is beneficial for the displacement estimation. In comparison to the gradient method the integration of the path length tends to decrease random errors. On the left hand side, the error for the radial velocity also shows much less scatter for the integration method. Since experimental data always shows uncertainties, it can be concluded that the second order polynomial provides the best trade-off in avoiding bias errors due to the curvature of the trajectory and random errors due to the uncertainty in the particle image position detection.

Since the exact positions are known, the error made in the position estimation can be evaluated and is listed in Table 1. The first value always corresponds to the case without noise for the particle positions. As can be seen from the table the maximum error made with the central differences is already 0.5 pixel although the maximum displacement, and thus the curvature, was low for the synthetic vortex. For the second and third order polynomial, the maximum error is in the order of 0.15 pixels. In general, the radial position is underestimated by the central difference scheme whereas the use of the fit functions does not show such a trend.

Table 1) Maximum, mean and standard deviation of the differences between the exact radial position and the estimated position for the central differences, the third order polynomial fit and the second order polynomial fit. First value is without noise, the second with noise for the position.

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<thead>
<tr>
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<th>central differences</th>
<th>3rd order polynomial</th>
<th>2nd order polynomial</th>
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<tbody>
<tr>
<td>(\text{max}(r_{\text{sim}} - r)) in px</td>
<td>0.426 / 0.495</td>
<td>3.61 E-04 / 0.147</td>
<td>3.61 E-04 / 0.137</td>
</tr>
<tr>
<td>(\text{mean}(r_{\text{sim}} - r)) in px</td>
<td>-0.045 / -0.045</td>
<td>1.04 E-05 / 7.97 E-04</td>
<td>1.57 E-05 / -0.001</td>
</tr>
<tr>
<td>(\text{std}(r_{\text{sim}} - r)) in px</td>
<td>0.079 / 0.081</td>
<td>4.46 E-05 / 0.035</td>
<td>6.03 E-05 / 0.035</td>
</tr>
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4. Experimental validation

4.1 2D measurements for a macroscopic water jet

The algorithm was first tested on a 2D macroscopic water jet experiment. 76,000 vectors (between \(t_2\) and \(t_3\)) were found in 500 images using the four-frame method, whereas 100,000 vectors were found by the two-frame method. This effect is related to the loss of pairs, which is three times larger for the four-frame method hence becoming a limitation of the method for low out-of-plane motion in the case of 2D imaging. In Figure 6, the stream wise flow is represented by color on the left side using the two-frame method (top) and the four-frame method (bottom). Note that only the first 10,000 data points are shown. Vertical profiles are presented in the middle of the figure. The high spatial resolution clearly shows the edge of the jet as expected for PTV. However, the larger scatter for the two-frame method can already be seen. For the four-frame method, the loss of pairs,
especially in the region close to the exit, can also be clearly seen. The histogram for dx is shown on the right side. Peak locking can be clearly seen for the two-frame method. Since the experiment was inherently set-up for PIV, the particle images are small and of rather low SNR of about 2-3. This causes peak locking since the particle detection algorithms are known to work less efficiently for small particles [3]. Using the four-frame method, the peak locking decreases strongly without altering the overall appearance of the histogram. This effect is due to the fact that information from four different particle images is taken into consideration. Although each position is attributed to errors that cause peak locking, this error is spread out for the final displacement estimation.

Figure 6) Color representation of the dx displacement for a water jet (left), scatter plot for regions close to the exit (blue y=102 +/- 15 px) and further away (red y=205 +/- 15 px) and histogram of the dx displacement for the two-frame method (top) and four-frame method (bottom).

4.2. 3D measurements in a microfluidic channel

Another test was the analysis of a 3D measurement of an acoustically driven flow in a microchannel [19]. The flow was observed from the top using astigmatism PTV (APTV) which enables the measurement of the three-dimensional velocity field in a volume with a single camera [5]. The data was projected on the xz-plane for a better visualization, where z is the viewing direction of the microscope. In Figure 7, the particle positions are indicated in blue. The four-frame PTV algorithm was used to find matching particle quadruples. The displacement was then evaluated using the second and third frame for the standard central differences approach and is indicated in red (2F). As discussed above, the best compromise for the four-frame method was achieved by using a second order polynomial fit and the integral displacement estimation (indicated in green, 4F). In the middle part of the figure, the differences for the position and the displacement using both methods becomes quite obvious. The data using the four-frame method also shows less outliers since the velocity estimation is based on four data points instead of two. So even when a wrong particle is chosen by the matching algorithm, the influence on the final displacement estimation is less severe. On the right side of the figure, the difference between the two-frame and the four-frame method is highlighted. The red vectors indicate the difference in the displacement and the blue vectors the
difference for the position. For the position, the standard deviation between both methods is 0.3 µm with a maximum of 2.28 µm. For the displacement, the mean absolute difference is 0.5 µm which corresponds to 2% of the maximum displacement. The standard deviation is 0.9 µm with a maximum difference of 12.3 µm. From these figures, it becomes quite obvious that the use of a higher order displacement estimator could strongly enhance the accuracy of the data.

Figure 7. Measured displacement for a microfluidic experiment (left and middle) evaluated with the four-frame method. Central difference estimation (red vectors) and the use of a second order polynomial fit with the integration method (green vectors). Difference between the two evaluation methods for the position and the displacement (right).

5. Conclusion and Outlook

A PTV algorithm based on the work of Ohmi and Li [15] was adapted for time-resolved imaging or multi-pulse imaging in general. The main benefits of taking information from various time-steps into considerations are:

• the reliable determination of a predictor which allows for very high seeding densities,
• the more accurate velocity determination by fitting the particle path,
• the vector reallocation in case of large spatial gradients,
• the determination of Lagrangian velocities.

The analysis of synthetic particle distributions reveals that the algorithm works well even with particle concentrations that are higher than what can be resolved reliably on an image sensor. Hence, the limitation is the particle detection. The improvement in the position and displacement estimation was tested on a synthetic particle distribution representing a Lamb-Oseen vortex flow. The systematic and random errors can be decreased using higher order curve fitting. Finally, the algorithm was tested on experimental data of a micro flow and a water jet experiment. In conclusion, time-resolved image acquisition should be used to enhance particle tracking data. If the time resolved imaging is not possible, e.g. for high speed flows, a four-frame setup can be built using two lasers with polarization filters and two PIV cameras. The benefit of such setup would overcome the effort in many cases. However, the technique is limited in the case of strong out-of-plane motion that causes a loss of pairs for 2D measurements. Nevertheless, PTV in general is very well suited for the velocity estimation in the case of tomographic or other 3D approaches, since the data handling is much easier once the particle positions are known, compared to the data handling for the volume representation in the voxel space. Furthermore, techniques can be developed to estimate turbulence spectra and gradients from velocity data on unstructured grids [3],[6],[7].
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7. References


