On the development of a novel low cost high accuracy experimental setup for Tomographic Particle Image Velocimetry

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Abstract This work deals with the critical aspects related to reducing the cost of a Tomo-PIV setup and with the bias errors introduced in velocity measurements by the coherent motion of ghost particles. The basic idea for reducing the cost of the Tomo-PIV system is the use of two imaging systems composed by three (or more) low speed single frame cameras. The two systems are used to reconstruct respectively the first and the second light intensity distributions in the investigated volume (relative to the first and the second exposure, respectively), to be interrogated by cross-correlation in order to obtain the measured velocity field. The cost reduction is possible thanks to the availability of cheap and high accuracy low speed single frame cameras. The increase in accuracy is due to the reduction of the degrading effect of ghost particles in the measured flow fields caused by their mismatch in the correlations, since they are effectively randomly distributed (the distribution of ghost particles is imaging system – dependent). Guidelines for the development and the application of the present method are proposed. Synthetic experiments are presented in order to describe and assess the accuracy of the proposed setup. It is shown that the low cost system proposed herein produces a much lower signal modulation with respect to the three cameras system and that the potential accuracy improvement with Motion Tracking Enhanced MART is much higher than in the case of the correspondent standard configuration.

1. Introduction

Recent efforts in experimental fluid mechanics are dealing with the implementation, the assessment and diffusion of Tomo-PIV, an experimental technique for the measurement of the three-dimensional three-components (3D-3C) instantaneous velocity distribution in a fluid flow (Elsinga et al. 2006). Tracer particles immersed in the flow are illuminated by a pulsed light source within a 3D region of space. The scattered light pattern is recorded simultaneously from several viewing directions using double shutter cameras (generally three or four). The 3D particle pattern is reconstructed as a 3D light intensity distribution from its projections on the camera sensors. The reconstruction is an inverse problem and its solution is not straightforward since in Tomo-PIV it is in general underdetermined: a single set of projections can result from many different 3D objects and the determination of the most likely 3D distribution is the topic of tomography (see Herman and Lent 1976). The particle displacement (hence velocity) within a chosen interrogation volume is then obtained by the 3D cross-correlation of the reconstructed particle distribution at two subsequent exposures (Adrian 1991). The interrogation algorithm is generally based on the cross correlation analysis with image deformation methods (Scarano 2002) extended to 3D. Tomo-PIV has been widely applied as a useful tool in fluid dynamics investigations of predominantly turbulent flows (e.g. Schröder et al. 2008; Hain et al. 2008; Humble et al. 2009; Scarano and Poelma 2009, Elsinga et al. 2010a; Violato and Scarano 2011, Violato et al. 2011). Nevertheless the aspects related to the cost reduction and the measurement reliability (in particular, the tomographic reconstruction) are still concerning the scientific community. Great attention has been devoted to the improvement of the accuracy of tomographic reconstruction (Novara et al. 2010, Novara and Scarano 2011). In fact while the errors introduced in the imaging
and cross-correlation steps of the experimental procedure are in common with planar PIV and have already been documented (for an overview see e.g. Raffel et al. 1998), Tomo-PIV measurements have to deal with the errors due to inaccurate tomographic reconstruction and are being object of recent investigations. The tomographic reconstructions contain spurious light intensity peaks, which are commonly referred as ghost particles (Maas et al 1993). Elsinga et al (2010b) discuss the bias errors introduced in Tomo-PIV velocity measurements due to the coherent motion of the ghost particles, occurring when a ghost particle is formed from the same set of actual particles in both the reconstructed volumes analyzed by cross-correlation. The displacement of the resulting ghost particles pair is approximately the average displacement of the set of the associated actual particles. The bias error does not significantly affect the measured flow topology as deduced in an evaluation of the local velocity gradients; instead, it leads to a systematic underestimation of the measured particle displacement gradient magnitude in the depth direction.

At the same time both the high computational cost and the expensive experimental setup make it difficult the spreading of the technique as a common tool for velocity measurements. Several works are aimed to the reduction of the computational costs of tomographic reconstruction and volumetric PIV while retaining the same accuracy (Worth and Nickels 2008, Atkinson and Soria 2009, Discetti and Astarita 2012a, b).

In this work an idea is proposed for the development of an experimental setup that allows the reduction of the hardware cost of a Tomo-PIV experiment without any significant loss of accuracy. In the authors’ knowledge, in all the Tomo-PIV experiments performed until now, double-shutter or hi-speed cameras are employed as in planar PIV; however, the requirement of double shutter cameras in planar PIV, due to the need to acquire two separate images from the same point of view for the image correlation, ceases to exist in a Tomo-PIV setup. In fact, as previously said, the particle displacement (hence velocity) within a chosen interrogation volume is obtained by the 3D cross-correlation of the reconstructed particle distributions corresponding to two subsequent exposures. The particle distributions can be reconstructed from images acquired from several viewing directions by a different set of cameras at each exposure. Thus, while three double-shutter cameras are required (at least) for the reconstruction of a certain velocity field, the approach proposed in this work requires the use of two (or more) imaging systems of three normal (single shutter) low-cost cameras. This approach is expected not to compromise the accuracy of the measurements because the effect of the ghost particles is easily suppressed since their distribution is strongly camera-orientation dependent, i.e. the ghost particles are not expected to coherently contribute (and hence, produce bias errors) to the cross-correlation map; conversely, their contribution is random, and one can assume to eliminate it easily by using the Motion Tracking Enhancement (MTE) proposed by Novara et al. (2010).

Guidelines for the development and the application of the present method are given and a numerical analysis of the accuracy of the proposed experimental setup is presented referring to 2D numerical simulations and to a 3D simulation of a synthetic experiment with a jet-like flow field.

2. Proposed experimental setup and procedure

As said in the introduction, a reduction of the hardware cost of a Tomo-PIV experiment can be provided using two different imaging systems for the reconstruction of the first and the second exposure used for 3D correlation in order to evaluate the 3D flow field; the number of cameras is doubled, but since the requirement of double shutter cameras does not hold anymore, one can purchase much cheaper single-shutter cameras for the same purpose.

In particular the simplest idea is the use of 2 systems of three single shutter cameras for the reconstruction of the first and the second exposure. It has to be remarked that another easy implementation could be the use of two double shutter cameras (available in all the labs that perform Stereo-PIV experiments) together with two single shutter cameras obtaining two imaging
systems composed by the two double shutter cameras and by the first and the second single shutter camera for the first and the second exposure, respectively. For the sake of brevity this case will not be discussed herein but most of the results presented in the following can be applied also to this condition.

The accuracy of the proposed experimental setup is not expected to be compromised. In fact, considering the experimental setup with 3+3 single shutter cameras (even if each of the volumes is reconstructed by only three cameras, while using six cameras in total), this approach is expected to result in much more accurate estimate of the velocity fields with respect to the simple adoption of three double shutter cameras because it is less affected by the effects of ghost particles since their distribution is strongly camera-orientation dependent. In the proposed approach the ghost particles are not formed in the two subsequent exposures by the same set of actual particles, so they are expected to randomly contribute in the cross-correlation maps; accordingly, the bias effect is completely removed, at the expense of a reduction of the signal-to-noise ratio. For the same reason, the MTE is effective independently of the flow features (i.e. the velocity gradients in the depth direction), and the achievable reconstruction accuracy can be comparable to the case of the configuration with six double shutter cameras, as it will be shown later. Of course, the execution of the MTE algorithm implies an increase of the computational cost; however, by using the solutions proposed by Discetti and Astarita (2012b), one can easily reduce the processing time of the 3D PIV processing algorithm of one or two orders of magnitude, depending on the complexity of the flow field.

3. Numerical simulations

3.1 Spatial resolution

Numerical simulations on 2D synthetic image fields reconstructed by 1D cameras (as in Elsinga et al 2006) are presented in order to analyze the effect of the proposed imaging setup on the spatial resolution, expressed in terms of the Modulation Transfer Function (MTF). The volume considered for the present analysis is a slice of 100x12 mm$^2$ and a spatial resolution of 20 voxels/mm. The synthetic imaging system for the reference Tomo-PIV system is composed by three cameras oriented at -45°, 0° and 45° while the proposed one is composed by 3+3 cameras oriented at -45°, -10° and 30° for the first group and -30°, 10° and 45° for the second group. The results are compared also with those from a setup composed by six double-shutter cameras in the same arrangement as a reference.

Gaussian particles with a diameter of 3 voxels and maximum particle intensity of 200 counts (12 bit images) are randomly distributed in the slice to be reconstructed. A sinusoidal displacement field, with $U=\sin(2\pi z/\lambda)$ and $V=0$ is imposed; the wavelength $\lambda$ is varied between 24 and 248 pixels. The present simulation is performed at three levels of source density (namely 0.15, 0.25 and 0.35) and the modulation transfer function is evaluated on the detected flow field averaged over 500 images. The resolution ratio between pixels and voxels is set equal to 1; the reconstruction is performed by 5 MART iterations with a relaxation coefficient of 1.

Since the flow field is one-dimensional, the PIV process can be performed with interrogation windows elongated along the x-direction (in the present case 500 x 48 voxels with 75% overlap in the gradient direction) in order to guarantee a sufficient number of particles in every interrogation window. The MTF is evaluated with the following equation (Astarita 2006):
where $N$ is the number of computed vectors, $U_i$ is the $i$-th measured vector, and $U$ is the exact displacement value.

For a source density $N_s$ equal to 0.15 (Fig. 1a) no significant differences are found between the proposed methodology (from now on referred as with the acronym LC, i.e. LowCost) and the 3 camera (3cam) system, while the MTF relative to the 6 camera (6cam) system is practically the same of the one obtained by interrogating the original images. One should note that for large wavelengths the 3cam system performs better than the LC one, since it benefits of the ghost particles contribution to the cross-correlation (the bias effect is less important, due to the relatively low gradient along the depth direction), reducing the measurement noise (it is important to point out that Eq. 1 is sensitive to noise, i.e. a MTF lower than 1 is estimated in presence of noise even if the mean profile is not modulated). By increasing the sine wavelength, the bias effect due to the ghost particles motion modulates the gradient in the case of the 3cam system, while the LC system seems not to be affected, as expected.

\[
MTF(\lambda) = 1 - \frac{\sum_{i=1}^{N} (U_i - U)^2}{\left( \sum_{i=1}^{N} \sin \left( \frac{2\pi U_i}{\lambda} \right) \right)^2}
\]  

(1)

\[\sum_{i=1}^{N} (U_i - U)^2\]

\[\sum_{i=1}^{N} \sin \left( \frac{2\pi U_i}{\lambda} \right)\]

Fig 1 MTF at: a) $N_s = 0.15$, b) $N_s = 0.25$, c) $N_s = 0.35$. 
**Fig 2** Evaluated flow field at $\lambda = 248$ pixels and $N_s=0.35$ a) 6cam b) LC c) 3cam

By increasing the source density to $N_s = 0.25$ (Fig. 1b) the MTF of the LC system strongly benefits of the higher number of true particles, and it is comparable with that of the 6cam system, while the MTF of the traditional 3cam system is consistently lower (the higher is the source density, the higher is the number of ghost particles; accordingly the bias effects are more intense); at source density $N_s = 0.35$ (Fig. 1c) this effect is significantly more evident. In fig 2 a-c the averaged flow fields at $\lambda = 248$ voxels are presented: it appears evident as the LC system is able to obtain the same flow field obtained by the 6cam system, even if the slightly lower MTF testifies an increase of the measurement noise on the single realization. On the other hand, the noise level for the LC system can easily be reduced by applying the MTE, as discussed in the next section.

### 3.2 Measurement noise and MTE improvement

In this sub-section numerical simulations on 3D synthetic particle distributions reconstructed by 2D cameras are presented in order to analyze the effect of the proposed imaging setup on the measurement noise and the possibility to improve the quality of the results with MTE-MART. The reconstructed region for the present analysis is a volume of $40 \times 40 \times 15$ mm$^3$ with a spatial discretization of 20 vox/mm ($800 \times 800 \times 300$ voxels).

The synthetic imaging system for the reference Tomo-PIV system is composed by 6 cameras, disposed on two horizontal groups; on each horizontal group the cameras are angularly equally spaced by $30^\circ$ and the two horizontal systems describe an angle of $60^\circ$, so that, in total, an angle of $60^\circ$ is enclosed by the outer cameras in both directions. From now on, the two horizontal groups will be referred as rails on which the cameras are placed (this terminology actually resembles the possible practical implementation of the system).

The comparison is performed between the 6 double shutter cameras system, a system with three double shutter cameras (2 on the edges of one of the two rails and one on the centre of the second rail) and a LC system with 2 imaging systems similar to that one used for the 3 double shutter.
cameras (i.e. the imaging systems for the first and the second exposure describe two triangles, with the vertices on the opposite sides).
Gaussian particles with a size of 3 voxels and a maximum particle intensity of 200 counts (recorded with 12 bit discretization) are randomly distributed in the volume to be reconstructed. A jet like displacement field is imposed:

\[
d(x, y, z) = \frac{d_{\text{max}}}{2} \cdot \left[1 + \tanh \left( \alpha \cdot \sqrt{\frac{1 - \sqrt{r_1^2 + r_2^2}}{r}} \right) \right]
\]

where \(d_{\text{max}}\) is the maximum displacement (equal to 16 voxels), \(\alpha\) is a parameter to modulate how steep is the descent from the peak velocity to zero (in this case it is set equal to 2) and \(r\) is the station where the jet reaches a velocity that is equal to half of the maximum (set to 200 voxels); \(r_1\) and \(r_2\) are the distances from the jet axis along two orthogonal directions in the plane orthogonal to the jet axis itself (i.e., if the jet is directed along the \(z\) direction, \(r_1\) and \(r_2\) are equal to the \(x\) and \(y\) coordinates). Two cases are discussed, i.e. a jet-like profile directed along the \(z\) and the \(y\) direction. In the first case there is no velocity gradient along the depth, so the MTE is expected to be ineffective in the traditional 3cam layout, conversely to the case of the proposed LC system; in the second case the gradient along the depth direction determines the MTE to be highly effective for both the systems.

The present simulation is performed at a source density of 0.24 corresponding to 0.035 particles per pixel. The resolution ratio between pixels and voxels is set equal to 1; the reconstruction is performed by 5 MART iterations with a relaxation coefficient of 1. The first MTE pass is applied after 5 iterations; subsequently, a MTE pass is applied after each 3 MART iterations.

The PIV process is executed over interrogation windows of 40x40x40 voxels with 75% overlap. Since the scope of the present experiments is to estimate the total accuracy of the proposed method no ensemble averaging is performed.

The flow fields relative to a jet aligned with the \(z\) direction are presented in Fig. 3. By comparison of fig 3b and fig 3c, where results relative to the LC and 3cam systems are shown, and by inspection of Fig. 3h, in agreement with previous results, the 3cam system shows a strong modulation with respect to the LC system while the total effect of noise is quite similar for both results.

With the MTE passes, the quality factor of both the LC and 3cam systems increases significantly (Fig. 3g); in this case the MTE is less effective for the 3cam system, as expected. If fig 3d is compared with fig 3a, the isosurface relative to the LC system, with the MTE, is comparable with those of the 6cam system. Despite of a very high quality factor, the LC system still exhibits some noise effects (actually not very relevant) because the particles are reconstructed with different shapes in the two reconstructed objects.

The flow fields relative to a jet aligned with the \(y\) direction are presented in Fig. 4. By comparison of fig 4a, 4b and 4c and by inspection of Fig. 4g, in agreement with previous results, the 3cam system shows a strong modulation with respect to the LC system and of course to the 6cam system. In this case this aspect becomes even more evident, in fact, by observing the \(u\) and \(w\) scatter plot of Fig 4f relative to the LC and 3cam systems it is evident that the 3cam distribution is strongly elongated in the depth direction due to the strong modulation effect.

With the MTE passes the accuracy of LC and 3cam systems is increased (Fig. 4d, e, g) and after only one MTE iteration (i.e. 5 MART iterations + 1 MTE pass + 3 MART iterations) the accuracy of the LC system is practically as good as that of the 6cam system. If fig 4d is compared with fig 4a, after 1 MTE iteration, the results of the LC system are comparable with those of the 6cam system. Also in this case the application of MTE results being more effective for the LC system with respect to the 3cam one.
Fig 3  Numerical simulations on a jet aligned with the z direction: a-e) (isosurface W=15) a) 6cam b) LC c) 3cam d) LC with 3 MTE passes e) 3cam with 3 MTE passes; f) Error scatter plot for the 3cam system and the LC system; g) Quality factor versus MART iterations; h) average W profile
Fig 4  Numerical simulations on a jet aligned with the z direction: a-e) (isosurface W=15)  a) 6cam b) LC c) 3cam d) LC with 3 MTE passes e) 3cam with 3 MTE passes ; f) Error scatter plot for the 3cam system and the LC system; g) average W profile
4. Conclusions

In this work an experimental setup that allows the reduction of the hardware cost of a Tomo-PIV experiment is presented, consisting of two different imaging systems for the reconstruction of the first and the second exposure used for the evaluation of the 3D flow field. Guidelines for the development and the application of the present method are proposed looking at the possibility to use 6 single shutter cameras or 2 double shutter cameras (available in all the lab performing stereo PIV) along with 2 single shutter cameras. The proposed experimental setup does not compromise the accuracy of the measurements as confirmed by the numerical analysis on synthetic images. In fact by using two different imaging systems for the reconstruction of the first and the second object to be correlated for the velocity evaluation, the effect of modulation due to ghost particles is strongly reduced. The measurement noise is expected to be slightly larger, due to the random effect of the ghost particles on the correlation map, and the different reconstructed particles shape. The quality of the method can be further increased with the application of the Motion Tracking Enhanced MART that results to be more effective for the proposed setup than it is for a traditional Tomo-PIV setup composed by double shutter cameras.

References

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