Multi-Scale Stereoscopic PIV measurements of a Jet in a Cross-Flow

Nicolas Lanitis¹*, James R. Dawson¹

1: Department of Engineering, University of Cambridge, Cambridge, UK
* correspondent author: nl258@cam.ac.uk

Abstract  Multi-scale Stereoscopic PIV measurements were performed in the far field of a cross-flow jet. Measurements were taken in the spanwise-wall normal (y-z) plane containing the Counter-rotating Vortex Pair (CVP). Two stereoscopic setups were used: one having a large Field Of View (FOV) which contained all of the CVP and thus providing information on the large-scales and another small (FOV) which zoomed in on the right core of the CVP to capture smaller-scale features. Measurements were taken with a jet Reynolds number of \( \text{Re}_{\text{jet}}=2\times10^5 \) with jet to cross-flow velocity ratio of \( V_s=10 \) in the far field at three downstream positions. The mean vorticity field of the large FOV shows that the cores of the CVP are elongated in the z direction. All three components of normal stress (rms) exhibit a characteristic ‘kidney’ shape with peak values located on each core of the CVP, i.e. the locations maximum vorticity magnitude. The high rms values in the cores together with the instantaneous data from the small FOV suggest that the structure of the CVP is composed of a number of smaller vortex tubes. Profiles of the mean velocity field, rms velocity, Reynolds stresses and spatial correlations in both in-plane directions are presented and show good agreement between the large and small FOV. Overall, the results demonstrate that multi-scale measurements offer to improve the dynamic range and is an effective method to study multi-scale interactions.

1. Introduction

The jet in cross-flow has been an integral part of turbulence research form many years due to its wide range of applications and superior mixing properties compared to a normal jet. From a more fundamental perspective the complex vortical structures involved, specifically the formation of a Counter-rotating Vortex Pair (CVP) in the far field, allows for a mechanistic approach to understanding turbulence. A schematic of the different vortical structures in the flow formed from the interaction between the jet and the cross-flow are shown in Figure 1. Early research concentrated on qualitative measurements of the cross flow jet using flow visualization to look at the jet trajectory and structures that existed within the the different regions of the flow. Using pressure probes measurements and hot-wire anemometry showed the existence of the counter rotating vortex pair (CVP) in the cross-plane far field (Fearn and Weston, 1974). Due to the complexity of the flow a number of studies tried to model the flow using various techniques (Broadwell and Breidenthal, 1984; Fearn and Weston 1974; Hasselbrink and Mungal, 2001). The simplest one is the one by Broadwell and Breidenthal (1984) who modelled the average far-field as a pair of counter rotating vortex lines which form due to the momentum impulse of the jet on the cross-flow, similar to those generated by the wing tips of an airfoil. With the introduction of laser techniques, Particle Image Velocimetry (PIV) and Planar Laser Induced Fluorescence (PLIF) researchers began to look at the evolution of the near field and the mechanisms involved in the formation of the CVP (Kelso et al, 1996; Lim et al, 2000; New et al, 2006).

More recently work has focused on the mixing field by studying the scalar concentration field and the turbulent velocity field in various cross-sectional planes measuring Reynolds fluxes and scalar probability distribution functions for various boundary conditions (Smith and Mungal, 1998; Su and Mungal, 2004; Shan and Dimotakis, 2006). Interestingly PIV measurements in the plane containing the CVP have not yet been reported in the literature. The CVP entrains and mixes fluid between the
jet and the surrounding fluid. The aim of this paper is to study the turbulent field of the large-scale CVP and the smaller-scales within the CVP simultaneously, by employing a multi-scale measurement technique. The CVP is a large scale structure and therefore PIV measurements of the whole CVP will restrict the analysis to large-scale turbulent structures only. In order to improve the dynamic range of the scales involved multi-scale measurements was performed which involves having two PIV systems taking simultaneous measurements at different FOV and spatial resolutions. Turbulence is a well known to be a multi-scale problem with the difference between the largest scales and the smallest scales (i.e the Kolmogorov scale) separated by two orders of magnitude. Therefore this technique is particularly suited to the jet in cross-flow so that the effect of large-scale dynamics of the CVP on the underlying small-scale turbulent structures can be studied, potentially answering the question why the cross-flow jet is a better mixer.

In section 2 the experimental set up is described, including the facility used and the optical set up. Section 3 then presents the results from the measurements. Specifically section 3.1 characterises the flow presented here by looking at the trajectory and the CVP. The following sections compare the measurements between the two field of views in order to validate the technique, with section 3.2 comparing the mean velocity field, section 3.3 the rms velocities and Reynolds stress and 3.4 the spatial correlations. Section 4 presents a discussion on the conclusions drawn from the experiment presented here.

2. Experimental Set Up

2.1 Experimental Facility

Measurements were performed in a recirculating open water channel with height and width of 400×900mm. The inlet of the channel has a series of sieves and flow straighteners of reducing size followed by a third order polynomial contraction. The working section of the channel is 7m long and made out of Perspex thus providing full optical access. The jet was created using a nozzle having a 5th order polynomial contraction profile with an area contraction ratio of 25, to ensure a top hat jet-exit velocity profile, and an exit diameter of \( d_j = 4\text{mm} \). The jet was supplied with water from the channel from a position far downstream of the jet exit using a pump. The flow rate through the nozzle, and hence the jet exit velocity, was controlled using two gate valves and measured using an orifice plate. The orifice plate was designed and constructed to the British Standard specifications of EN ISO 5167-2:2003 for an orifice to pipe diameter ratio \( \beta = 0.5 \). To ensure that the jet velocity remained constant throughout the experiments the output from the pressure transducer was monitored and recorded using a data-logger. The jet was injected into the cross-flow through a 900×800×15mm Perspex plate which was submerged just under the free surface. The leading edge of the plate had an elliptic profile and was tapered to ensure the oncoming cross-flow did not separate. The jet exit was located halfway along its length 450mm downstream of the leading edge.

Measurements were taken in the y-z plane, as shown in Figure 1, at different 3 different downstream x locations in the far-field of the cross-flow jet as defined by Smith and Mungal (1998) as being after the 'branch point' located at \( x_{bp} = 0.2V_r^2d_j \). Throughout this paper the instantaneous velocities in the streamwise \( x \), spanwise \( y \) and wall-normal \( z \) direction are denoted by \( U, V \) and \( W \) and a Reynolds decomposition is applied to obtain the mean and fluctuating components. The different inlet parameters studied at these locations are summarized in Table 1. \( V_r \) is defined as the momentum ratio between the jet and the cross-flow, which reduces to the jet to cross-flow velocity ratio as \( \rho = \text{constant} \). Broadwell and Breidenthal (1984) defined the circulation Reynolds number \( \text{Re}_r = \Gamma/\nu \) as
\[ \text{Re}_i = \left( \frac{x}{V_r d_j} \right)^{-\frac{1}{3}} \text{Re}_\text{jet} \]  

which, given that the far field is dominated by a CVP, can be used to characterise the far field flow and is also shown in the table.

<table>
<thead>
<tr>
<th>( V_r )</th>
<th>( x/d_j )</th>
<th>( X_{bp}/d_j )</th>
<th>( U_{\text{jet}} )</th>
<th>( U_\infty )</th>
<th>( \text{Re}_i )</th>
<th>( \text{Re}_f )</th>
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<td>10</td>
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<td>5.7</td>
<td>0.285</td>
<td>2×10^4</td>
<td>1.23×10^4</td>
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</table>

Table 1: Flow Conditions

### 2.2 Multi-scale measurements

Velocity measurements were performed using Stereoscopic PIV in order to obtain all three velocity components. A schematic of the set up is shown in Figure 1(b). Two stereoscopic systems were used, one in which the field of view captured the whole of the CVP, the Large-scale Field of View (LFOV), and the second which zoomed in on one of the vortex cores, the Small-scale Field of View (SFOV). Simultaneous overlapping measurements were obtained, each covering a different range of scales. Two Photron Fastcam SA1.1 high-speed cameras with 1MP resolution were positioned on either side of the channel with an angle of 90° between them. Prisms filled with distilled water were used to minimize optical distortion due to the change in refractive index. The flow field was illuminated using a 120mJ, 512nm, Nd:YAG New Wave Gemini low speed laser. The laser beam was formed into a sheet using a spherical and cylindrical lens with a thickness of approximately 3mm in the measurements plane. The relative orientation of the laser sheet with respect to the cross-flow made the large thickness necessary in order to get a sufficient particle displacement in the recorded images whilst the particles remained inside the laser sheet. This is bigger than the in plane resolution of the Small-Scale field of view and would normally compromise the resolution of the results but, since in the far field the CVP is aligned with the cross flow, the turbulent structures present are also expected to be oriented with the channel flow (x-direction).

The time between images resulting in an image pair was \( dt = 1ms \), with a 1s gap between image pairs to ensure statistical independence. One of the main issues with multi-scale PIV measurements is the particle images themselves. Since the same particles are imaged in the two FOVs their size and density will be different. Specifically the particles in the small-scale FOV will have larger diameter and the seeding density will be lower for a given large-sale FOV. As a result the relative sizes of the two FOVs were chosen by finding a balance between the seeding density and the particle size so that image correlations can be performed with a window size of 32×32 pixels in the SFOV measurements and avoid peak-locking in the LFOV measurements. Increasing the ratio between the two FOVs would then introduce problems such as over seeding the flow or having particles which are less than a pixel in size.
The flow was seeded with spherical silver-coated hollow glass spheres with a mean particle diameter $d_p = 10\mu m$ and a specific gravity of 1.4. Using $Re_T$ as the turbulent Reynolds number and the size of the jet $\delta_o$ (defined later) as a turbulent length-scale, the estimated Kolmogorov timescale is $\tau_\eta = 2.79 ms$. The particle response time is calculated using the expression derived from Stokes drag law $\tau_p = d_p^2 \rho_p \mu \over 18 \mu$ (Raffel et al, 2007), which yields $\tau_p = 6.82 \mu s$. The Stokes number is therefore $St=2.44 \times 10^{-3}$ verifying that the particles accurately follow the flow.

<table>
<thead>
<tr>
<th>$V_r$</th>
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<th>15</th>
<th>20</th>
</tr>
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<td>$x/d_j$</td>
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<td>85</td>
<td>85</td>
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<td>Jet exit diameter</td>
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<td></td>
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<tr>
<td>$\delta_o/d_j$</td>
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<td>34.3</td>
<td>39.7</td>
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<tr>
<td>Kolmogorov length scale ($\eta$)</td>
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<td>0.136mm</td>
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<td>1755\eta×1377\eta</td>
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<tr>
<td>Small-Scale Field of View</td>
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<td>13.75d_i×12.5d_i</td>
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<td>Resolution</td>
<td>1.8mm</td>
<td>437\eta×397\eta</td>
<td>404\eta×368\eta</td>
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</table>

Table 2: Experimental conditions for a jet Reynolds number of $Re_{jet}=2 \times 10^5$. The FOV details given for $V_r=10$ are for the downstream measuring position at $x/d_j=85$.

All of the data was processed using *Davis 7.2* and a multi-level calibration plate was used. For the LFOV a 3rd order polynomial calibration was used with a resulting average rms fit error between the different runs of 0.1 pixels. For the SFOV the pinhole model was used. A self calibration procedure was applied to all data sets with a resulting average disparity error of 2 pixels. For the cross-correlations an interrogation window size of $32 \times 32$ pixels and 75% overlap was used (Raffel et al, 2007; Elsinga and Westerweel, 2011), to improve the accuracy of the resulting vectors from the correlations.
Table 2 summarizes the experimental conditions for both fields of views. The Kolmogorov length scale, \( \eta \), was estimated using the circulation Reynolds number, \( Re_{\Gamma} \), as the turbulent Reynolds number in Kolmogorov’s equation \( \eta = l_{turb} Re_{turb}^{-3/4} \). The width of the CVP, \( \delta_w \), was defined as the distance between the points where the vorticity drops to 5% of its maximum value along the y-axis through the cores (maximum vorticity magnitude) and was used as the turbulent length scale \( l_{turb} \).

3. Results

3.1 Jet Trajectory and the CVP

The jet trajectory was one of the first properties of the cross-flow jet to be measured, and is the most frequently quoted property in the literature. Using the scaling law in equation 2 that Pratte & Baines used to collapse the trajectories from visualizations of oil aerosol jets, the trajectory was estimated by tracking the position of maximum/minimum vorticity and fitting the data with a least squares method using the following equation

\[
\frac{z}{V_r d_j} = A \left( \frac{x}{V_r d_j} \right)^m.
\]

(2)

The values of A and m reported in the literature range from \( 1.2 < A < 2.6 \) and \( 0.28 < m < 0.38 \) (Hasselbrink and Mungal, 2001). The values of A and m from the curve fit, shown by the red line in Figure 2, were found to be \( A = 1.62 \) and \( m = 0.28 \).

![Figure 2](image)

Figure 2. (a) Plot of the normalized jet trajectory. The solid red line represents equation 2 with \( A = 1.62 \) and \( m = 0.28 \). The different symbols represent the different inlet parameters. \( \square \) : \( V_r = 10 \) at \( x/d_j = 30 \), \( \bullet \) : \( V_r = 10 \) at \( x/d_j = 55 \), \( \triangle \) : \( V_r = 10 \) at \( x/d_j = 85 \), \( \times \) : \( V_r = 15 \) at \( x/d_j = 85 \), \( \star \) : \( V_r = 20 \) at \( x/d_j = 85 \). (b) Plot of the downstream evolution of the vorticity field using the measurements performed for \( V_r = 10 \). The solid blue line represents the trajectory of the jet found from the curve fit performed above.

As mentioned before the far field of the cross-flow jet is dominated by a counter-rotating vortex pair. Figure 2(b) shows the vorticity field at the 3 downstream measurement locations for the case where \( V_r = 10 \). The figure clearly shows the presence of the CVP, and that the mean shape of the cores is elliptical, rather than circular, elongated along the wall normal z direction.
Figure 3 shows the vorticity field for the case of $V_r = 10$ at $x/d_j = 55$. Figure 3(a) shows the mean vorticity field showing in detail together with the SFOV measurements, whose position within the LFOV is indicated by the dashed square. The agreement between the field of views is better observed by taking profiles along the $y$ and $z$ direction, as shown in Figure 3(c), through a given point within the field (e.g. peak or lowest value). The solid blue lines represent data from the LFOV measurements. Since the grids of the two field of views are different the SFOV measurements are represented by two colors. The solid red line is the profile through the point chosen, found from the SFOV data, and the solid green line is the line closest to the $y$ and $z$ values of the blue/LFOV profiles. The same color scheme will be used for all profiles plotted. The data shows a very good agreement between the two FOV measurements, with a small shift between the two FOVs is noted but this lies within the resolution of the large-scale field of view measurements, shown by the blue error-bars, indicating that the misalignment is due to the limited dynamic range of the LFOV measurements.

Figure 3. The vorticity field for the case where $V_r = 10$ at $x/d_j = 55$. (a) The mean vorticity field showing both the large-scale (left) and small-scale (right) measurements. (b) Profiles along $y$ (top) and $z$ (bottom) direction through point of minimum vorticity. (c) Example of an instantaneous vorticity plot. The top left contour plot is an extract of the large-scale FOV data from the area where the small-scale FOV measurements where performed, indicated by the dashed square. The bottom right contour plot is from the small-scale FOV measurements.
Figure 3(c) shows an example of the instantaneous vorticity field. The plot on the left shows contours of the instantaneous vorticity. The top right contour plot simply shows a close up of the LFOV measurements from this area and the bottom right contour plot is the vorticity field from the SFOV measurements. The SFOV measurements enhance the right core of the CVP and this is the case for all inlet conditions studied. Looking at the position of the strong vorticity events in the LFOV measurements, the two field of views show a good agreement. However, the SFOV locates the position of the peaks much better and importantly manages to capture smaller regions of high vorticity present within the CVP which are smoothed out in the LFOV. This clearly illustrates a major advantage of performing multi-scale measurements in that smaller-scale events can also be measured and characterized in context with the large-scale flow.

3.2 Mean velocity field

Figure 4 shows the mean velocity fields of $U$, $V$ and $W$ for the case where $V_r=10$ at $x/d_j=55$ from both the LFOV (top row) and SFOV (bottom row). At first glance good agreement is observed. Both the $V$ and $W$ velocity fields are consistent with what is expected from the CVP. The mean contours of $V$ alternate sign and a large positive upwash between the cores is seen for $W$. Contours of $U$ exhibit a wake like structure across the CVP with a momentum deficit between the two high momentum cores.

![Figure 4](image)

Figure 4. The mean velocity field of all three components (a) $U$ (b) $V$ and (c) $W$. Figures on the top row represent the data from the large-scale FOV measurements. The black dashed box represents the position where the small-scale measurements were performed, shown in the bottom row.

The mean velocity profiles across the cores of the CVP are plotted in Figure 5. The error bars correspond to the spatial resolution of the LFOV. In all cases the SFOV exhibit good agreement as shown by their degree of overlap and magnitudes fall within the spatial resolution of the LFOV. As expected the out of plane velocity, $W$, shows a slight increase in discrepancy.
Figure 5. Profiles along the y and z directions through the point of maximum velocity for all three components (a) $U$, (b) $V$ and (c) $W$. The solid blue lines represents data from the large-scale FOV measurements and the solid green and red lines represent data from the small-scale FOV measurements.

3.3 The underlying turbulent field

Due to the highly three dimensional nature of the cross flow jet due to the formation of the CVP the underlying turbulent field is very complex. Figure 6(a), (b) and (c) shows the three components of the rms velocity field, $\sqrt{u'^2}$, $\sqrt{v'^2}$ and $\sqrt{w'^2}$, of the LFOV (top row) and SFOV (bottom row). Both $\sqrt{u'^2}$ and $\sqrt{w'^2}$ clearly show the characteristic ‘kidney’ shape frequently noted in the literature. The $\sqrt{v'^2}$ velocity field is similar to the ‘kidney’ shape of the other two components but is upside-down.

Figure 6. The rms velocity field of the cross flow jet for $V_r=10$ at $x/d_j=55$. Figures (a), (b) and (c) on the top row show the $\sqrt{u'^2}/U_\infty$, $\sqrt{v'^2}/U_\infty$ and $\sqrt{w'^2}/U_\infty$ fields from the large-scale FOV data with streamlines superimposed. The bottom row shows the respective data from the small-scale FOV measurements.
Figure 7 (a), (b) and (c) shows profiles of the rms velocities dissecting both cores of the CVP along the y-direction (top) and through a single core in the z-direction (bottom). Again good agreement between the different FOV’s is found and within the spatial resolution of the large-scale FOV measurements.

The profiles from the SFOV however result in a slightly larger peak values. As mentioned in section 2, the orientation of the laser sheet plane where the measurements where made was aligned perpendicularly to the flow direction. The value of $dt$ between image pairs was set to minimize particle pairs loss as they are advected through the laser sheet. Since the value of $dt$ is the same in both large and small-scale FOVs, this means that in the small-scale measurements the particles displacements are greater. The resulting vectors from the cross-correlation will therefore be more accurate, and smaller displacements within the image will be picked up thus increasing the dynamic range of imaged particle displacements. For this reason it is expected that the results from the small-scale FOV measurements will be slightly different in magnitude from the large-scale FOV measurements.

The maximum rms velocity in all three components occurs at the position of maximum vorticity, i.e. they lie on cores of the CVP as shown in Figure 8. Furthermore, the position of minimum rms velocity for all components occurs at the center of the CVP where the vorticity is minimum. This, combined with the instantaneous vorticity plot in Figure 3(c) suggests that the structure of the CVP is made up of a number of vortex tubes which wrap around each other which on average appears as a single vortex core.
Figure 9. (a) Plot of the Reynolds stress field of $\overline{v'w'}$. Top figure is from the large-scale FOV measurements and the bottom figure is from the small-scale FOV measurements. The $\cdots$ line in the figures represent the lines along which profiles are taken. (b) Profiles along the $y$ and $z$ direction through the position of maximum $\overline{v'w'} U_\infty^2$.

3.4 Mean Strain field and spatial correlations

Figure 10 shows the two point auto-correlations, with the zero position taken as the point maximum $\sqrt{\overline{v'^2}}$ (i.e the core of the CVP), for all velocity components $u$, $v$ and $w$. A good agreement between the two FOV measurements is observed. The plot shows that the LFOV captures the whole correlation function, whereas the SFOV captures approximately 1 to 2 integral length-scales. Interestingly the auto-correlation function $R_{ww}$ in the $z$-direction (bottom right plot) drops off at a slower rate compared to the other correlations functions. This, combined with the elongation of the cores in the $z$ direction, indicates that the large-scale strain field created by the CVP has an effect on the turbulent field.
4. Conclusions

The cross-flow jet is a very dynamic and complicated three-dimensional turbulent flow. Therefore in order for a better understanding of the flow to be achieved advanced measurement techniques have to be used. The technique of multi-scale PIV was employed in order to perform the study. Multi-scale measurements are ideal for the flow and experimental set up in question as it allows for an increase in the dynamic range of scales measured. Furthermore due to the restricted \( dt \) between image pairs that can be set, the technique allows for an increase in measurement accuracy for the smaller scales by increasing the mean particle displacement and particle size. Measurements between the two FOVs agree very well with each other, after comparison of the instantaneous vorticity fields and profiles of the mean velocity, rms velocity, Reynolds stress and spatial correlations showing the validity of the technique. The trajectory of the jet using all the measurements with various inlet conditions studied collapse on the curve \( z = A \left( \frac{x}{V_{c,d}} \right)^m \) with the constants being equal to \( A = 1.62 \) and \( m = 0.28 \), when tracking the position of maximum vorticity magnitude. The vorticity field of the cross-flow jet is made up of two cores which are elongated in the \( z \) direction (wall normal). The Reynolds normal stresses (rms velocity field) illustrate the characteristic ‘kidney’ shape of the cross-flow jet. All of the components have two peaks which coincide with the position of maximum and minimum vorticity (cores position) and a cress which coincides with the position of zero vorticity within the CVP. This combined with results of the instantaneous vorticity field from the small-scale measurements suggest that the CVP is made up of a number of small vortex tubes which wrap around each other around the position of the core, which on average appears as a single vortex core. The in plane Reynolds stress \( \overline{v'w'} \) shows a very robust and distinct structure indicating a very specific structure to the in-plane turbulent field. Overall, the results demonstrate that multi-scale measurements offer to improve the dynamic range and is an effective method to study multi-scale interactions.
5. References


