An advection model to increase the time-resolution of PIV time-series

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Abstract A simple post-processing technique based on the advection equation is investigated for the enhancement of the temporal resolution of PIV data sets obtained at a sufficient or barely sufficient recording rate with respect to the flow time-scales. The method is based on the application of the advection equation, invoking Taylor’s hypothesis of frozen turbulence. When such approach can be applied, the requirement on flow sampling rate is based on the Lagrangian evaluation of the flow velocity at intermediate time instants with respect to the measured samples. The applicability is verified by means of planar and 3D tomographic experiments conducted at temporal resolution higher than that dictated by Nyquist criterion. The flow past the trailing edge of a NACA0012 airfoil closely approximates frozen turbulence and the largest ratio between Lagrangian and Eulerian temporal scales is found. As a result, an order of magnitude reduction of the needed acquisition frequency is possible. In the separated flow around a bluff body a separated shear layer produces unstable waves shed at approximately 270 Hz. The time super-sampling technique applied also in the shear layer region shows a smaller (due to a smaller ratio between Lagrangian and Eulerian flow time-scales), yet substantial increase in effective measurement frequency. In the separated shear layer region a separated shear layer produces unstable waves shed at approximately 270 Hz. The time super-sampling technique applied also in the shear layer region shows a smaller (due to a smaller ratio between Lagrangian and Eulerian flow time-scales), yet substantial increase in effective measurement frequency. Three-dimensional time-resolved tomographic PIV measurements of a transitional jet are considered. In this case, the Lagrangian approach can be implemented following three-dimensional trajectories. The qualitative inspection of the flow field and vortex topology indicates possible reduction of the needed frequency from 1,000 Hz to approximately 100 Hz. All experiments reveal that the current requirements for time-resolved PIV experiments can be revised when spatial information can be poured into temporal, i.e. the instantaneous convective velocity is used to perform temporal super-sampling. A yet important side effect of this approach is that the frequency spectra become significantly less affected by aliasing.

1. Introduction
The increase in performance of high-speed PIV hardware with CMOS imagers (Hain et al., 2007) and diode-pumped solid state lasers has marked an important advancement in fluid mechanics research. Such developments enable nowadays to investigate unsteady and turbulent flows processes with temporal resolution for moderate flow velocity (typically up to 10 m/s). Most documented experiments, are conducted at repetition rate in the range between 3,000 and 5,000 Hz, which is largely sufficient for slow water flows, but becomes insufficient in aerodynamic problems where the air flow velocity may approach 100 m/s. Moreover, the pulse-energy/repetition-rate trade off often imposes lower recording rates in order to obtain a scattering signal sufficiently high to be detected by CMOS imagers. PIV measurements of acceptable spatial resolution often lack sufficient temporal information, especially when compared with traditional point-wise techniques like hot-wire anemometry (HWA). This limitation hampers both qualitative and quantitative evaluation of the flow field i.e., when the dynamical evolution of the flow is to be visualized e.g. as an animated sequence and when temporal correlation functions or spectra are required.

The recent increased interest in simultaneous evaluation of velocity and pressure field from time-resolved PIV (PPI, planar pressure imaging, Liu and Katz, 2002; de Kat et al., 2008) and its applications for aeroacoustics (Lorenzoni et al., 2009) requires measurement conditions at the limit of PIV hardware specifications. As a result, PIV has been commonly intended for the description of the spatial flow organization and when flow time-scales are to be measured at high resolution laser Doppler velocimetry (LDA) or (HWA) are preferred for high-order statistics and frequency analysis.

In this scenario it is not taken into account that a time-resolved PIV measurement contains correlated information both in time and space, along the fluid trajectories and the use of a velocity field from PIV as
an ensemble of separate velocity measurements does not exploit the full potential of the information content provided by the experiment.

This is performed for instance by the use of Taylor’s hypothesis of frozen turbulence, already proposed in various forms by several works. In most cases, the hypothesis is invoked to extend the measurement domain from planar to 3D (Brede et al., 1996; van Doorne and Westerweel, 2007; among others) inferring useful information on the spatial organization of the flow and the development of dominant modes. When the time-separation between successive measurements produces a particle displacement comparable with the light sheet thickness, Ganapathisubramani et al. (2007) showed that it is possible to also obtain the out-of-plane components of the velocity derivatives, for the measurement of the complete velocity gradient tensor in a turbulent flow.

Other approaches that make use of physical models to improve the tracking accuracy of imaged tracers motion have been proposed as the optical flow method (Horn and Schunk, 1981) and other techniques specifically tailored for PIV applications (Okuno et al., 2000). Recently a hybridization of cross-correlation and dense estimators for fluid motion was investigated by Heitz et al., 2008.

In contrast, the use of Taylor’s hypothesis in time-resolved PIV sequences with a configuration such that the measurement plane contains the dominant convec tion does not produce any increase of dimensions of the measurement domain, but it can be used to refine the temporal information, in short pouring Space in Time. There are two major consequences: 1) the temporal resolution of PIV time-sequences can be increased beyond the measurement rate of the system; 2) time-aliasing effects can be largely reduced when using the combined spatial and temporal information.

The present work deals with the first of the above points and the second is left for further investigation in future studies. The work methodology consists in the verification of a time super-sampling technique making use of PIV time-sequences from experiments where all temporal fluctuations from the resolved spatial fluctuations are sampled at a rate fairly beyond the Nyquist criterion. Moreover, the experiments are conducted under well-controlled laboratory conditions with favorable illumination and imaging and the image sequences are analyzed with multi-frame correlation to reduce random errors. This approach provides PIV time-series that can be considered as ground truth to verify the validity of the method. Sub-sampled time series are extracted from the raw-data and then the reconstructed full time-sequence is compared with the reference data.

The paper contains first a short discussion on Taylor’s hypothesis and its relation to the flow time scales and required measurement frequency. Then a section is devoted to the description of the mathematical approach followed for the time super-sampling of the PIV sequence. The investigation of the applicability of the method in real experiments is structured in three parts with planar and three-dimensional cases.

2. Background on Taylor’s hypothesis

Before discussing the mathematical aspects of the super-sampling procedure and its numerical implementation, a more fundamental aspect related to the temporal resolution of PIV measurements and its possible enhancement through the advection model is discussed here.

For a time-resolved PIV measurement, the acquisition frequency $f_{\text{seq}}$ of image pairs (double-frame mode) or single exposures (continuous single-frame) is the parameter governing the temporal resolution. If a frequency analysis of the flow is aimed at, the common criterion to be obeyed is the individual measurements to be separated by a time interval smaller than the duration of temporal fluctuations. Nyquist criterion dictates that for a faithful reconstruction of the signal temporal sequence, the maximum frequency of the measurable flow fluctuations $f_{\text{flow}}$ should not exceed half the acquisition frequency. Considering a local convective speed $V_{\text{conv}}$ a flow fluctuation of length scale $\lambda$ will transit through the measurement point in a (Eulerian time scale) time $\tau_{\text{Eul}} = \lambda / V_{\text{conv}}$. Therefore, the acquisition frequency for the given experiment with PIV (or any other system with a stationary probe), in the hypothesis that the measured fluctuations are dominated by convected turbulence will depend upon the flow speed. The required measurement frequency for the considered length scale in an Eulerian frame of reference must be chosen obeying Nyquist criterion and reads as:
As an example, in aerodynamic problems, the flow speed attains values in the order of 100 m/s, where it is nearly impossible to perform, with currently available systems, any time-resolved measurement complying with the Nyquist principle for velocity fluctuations with length-scale below the centimeter ($f_{\text{Nyquist}} \sim 20 \text{ kHz}$). In contrast, for the same case, PIV measurements can be easily designed in such a way that the spatial resolution enables to resolve spatial fluctuations of the velocity field in the order of a millimeter. Knowledge of the instantaneous spatial information allows, in principle, to predict the flow pattern at future time instants if the convective motion in the fluid is known or measured. This concept is at the basis of Taylor’s hypothesis of frozen turbulence. The correspondence between temporal delay and a spatial shift under such hypothesis, leads to the following property of the space-time correlation function (Hinze, 1975):

\[
R\left(X = U_{\text{conv}} \cdot \tau, t = \tau\right) = 1,
\]

where velocity fluctuations are considered at different points in physical space at distance $X = (\Delta x, \Delta y, \Delta z)$ and delayed by a time interval $\tau$. When the velocity fluctuations can be considered as passively advected ($\sqrt{u'^2 / |U_{\text{conv}}|} < 1$), the above property is rewritten as:

\[
u(X, t + \tau) = (X - U_{\text{conv}} \cdot \tau, t).
\]

Eq. 3 is valid only in an approximated sense and its accuracy depends upon the amplitude of relative velocity fluctuations as well as upon the considered time delay $\tau$. Nevertheless, it poses the basis for the evaluation of the flow velocity from the spatio-temporal analysis of the velocity field from the measurement of the local convection velocity by PIV and of the spatial distribution of the velocity fluctuations. This information can be combined and produce an estimate of the velocity fluctuation at time instants delayed with respect to the actual measurement time. Interestingly, three-dimensional velocity measurements can benefit particularly from this property in that the convective velocity vector is known in a volumetric domain and the fluctuation field can be transported in all three spatial coordinates. The extension of this approach to three-dimensional measurements, can be regarded as an important generalization of previous attempts, limited to single-sensor or planar measurements. The validity of Taylor’s hypothesis has also been extensively studied, for many flow regimes. In a highly turbulent flow Cenedese et al. (1991) verified the validity of the hypothesis setting up a two-points Laser Doppler anemometer, concluding that the hypothesis is essentially correct for small separations and large structures. Instead, eddy deformation is the most responsible for the decrease in space-time correlation. It should be added that a drop in space-time correlation is introduced by the three-dimensional convection field in case of a two-points or planar reference system. The latter limitation can be solved again when the convective velocity vector is determined in all its components by a three-dimensional measurement.

The time-scale of temporal fluctuations in a Lagrangian frame of reference can be significantly longer that the corresponding fluctuation observed from an Eulerian standpoint. Following Koeltzsch (1999) the ratio between the Lagrangian and Eulerian time scales is dependent upon the relative amplitude of turbulent fluctuations with respect to the convective velocity:

\[
\frac{\tau_{\text{Lag}}}{\tau_{\text{Eul}}} = \frac{\alpha}{\sqrt{u'^2 / |U_{\text{conv}}|}},
\]

with $\alpha$ varying between 0.35 and 0.8. The resulting increase of the time-scales for convected turbulence going from the Eulerian frame of reference to the Lagrangian one indicates the potential increase in effective measurement frequency for a PIV time-sequence ($f_{\text{Lag}} < f_{\text{Eul}}$).

### 3. Pouring Space in Time by Time Super-Sampling

The physical and mathematical aspects of Lagrangian flow evaluation will first be covered, followed by
the description of the numerical implementation of the method. Let us consider a three-dimensional unsteady flow field, where a fluid parcel $P$ moves along a trajectory $\Gamma$. A flow property $G$ (e.g., velocity or vorticity) convected by such parcel is considered twice on its trajectory at stations $X_1$, $X_2$ and $X_3$ corresponding to two time instants $t_1$ and $t_3$ separated by the inverse of the measurement rate $\Delta t$. The situation is schematically depicted in Figure 1-top, where the property $G$ travels along the single coordinate $X$. The spatial distribution of $G$ is transformed from the time instant $t_1$ (red line) to $t_3$ (green line). The property of the parcel at location $X_2$ and at an intermediate time instant $t_2$ is considered unknown.

A simple prediction of the location of the parcel may be obtained by the backward projection starting from location $X_2$ by the local advection velocity $V(X_2,t_1)$ ($V_{\text{conv}} = \text{const}$ in the simplest case of frozen turbulence) during the time interval $t_2-t_1 = \Delta t/2$. The latter corresponds physically to the passive transport of the fluid parcel along the local streamline crossing the location $X_2$ and estimated at time $t_1$.

The problem is symmetric and the same information can be obtained by forward projection from location $X_2$ by the local advection velocity $V(X_2,t_3)$ in the time interval $t_3-t_2 = \Delta t/2$. It should be stressed out that the hypothesis of frozen turbulence does not need to be strictly valid in that the convection of velocity fluctuations is not made by a constant and uniform velocity field.

![Figure 1: Schematic principle of temporal super-sampling under advection assumption. Left: spatial distribution of $G$ at two time instants. Right: temporal evolution of $G$ at a location $X_2$.](image)

Therefore, in a two- or three-dimensional problem, where the convection velocity varies in space and time, the prediction of the position at the intermediate time step is not as straightforward as in the case of frozen turbulence transported by a uniform stream. The position of the fluid parcel at time $t_2$ may be estimated from the local convective velocity known in $X_2$ at time $t_1$ and $t_3$. In mathematical terms the origin and arrival positions to $X_2$ can be expressed by a Taylor expansion truncated at the first order that reads as:

$$X_1 = X_2 - V_{\text{conv}}(X_2,t_1) \cdot \frac{\Delta t}{2}$$

$$X_3 = X_2 + V_{\text{conv}}(X_2,t_3) \cdot \frac{\Delta t}{2}$$

When the velocity at position $X_1$ and $X_3$ and time instants $t_1$ and $t_3$ respectively is obtained by spatial interpolation, the above equations can be used to estimate the velocity of the fluid parcel in $X_2$ at $t_2$, which simply reformulates eq. 3:

$$V(X_2,t_2) = V \left( X_1 + V_{\text{conv}} \cdot \frac{\Delta t}{2} , t_1 \right) ,$$

$$V(X_2,t_2) = V \left( X_3 - V_{\text{conv}} \cdot \frac{\Delta t}{2} , t_3 \right) .$$

A closed-form of the estimate is proposed here that provides temporal continuity. A linear combination of eq. 6a and eq. 6b is obtained introducing weighting coefficients based on the temporal distance from the actual sample. The forward and backward predictions of the flow property are put together as follows:

$$V(X_2,t_2) = \frac{1}{2} V(X_1,t_1) + \frac{1}{2} V(X_3,t_3) ,$$

Eq. 7 is easily generalized to the case when the velocity estimate does not need to be exactly at half the sampling time step. The result reads as:
Also the hypothesis of frozen turbulence does not need to be strictly valid to apply eq. 8 in that no assumption is made on the uniformity of the convective velocity field. Although the approach by linear weighting of forward and backward schemes, ensures that the velocity obtained after super-sampling is continuous, its time derivative may exhibit discontinuities at the time instants corresponding to the original samples, when the switch from one sample to the next is made. The procedure outlined above, however cannot yet be directly applied to data measured by PIV in that the velocity of a fluid parcel is only known at a single time instant and the fluid trajectory is not directly available. In fact that the position and the velocity of the fluid parcel at the time $t_2$ obtained from the forward and backward scheme will differ, due to the linearization of the trajectory. The discrepancy is alleviated by an iterative procedure that evaluates first eqs. 5 and then the result ($X_1$ and $X_3$), is used to evaluate eq. 8. Once the velocity at $X_2$ and $t_2$ is estimated one can refine the evaluation of the positions of the fluid parcel and in turn its velocity. The velocity is obtained by spatial interpolation with 2D or 3D cubic splines. When this procedure is repeated typically two to three times, the result does not exhibit any residual variation, which enables to consider the iterative process as converged. As a result, one or more intermediate temporal samples of the flow velocity are obtained between the actual measurements making use of spatial and temporal information. This aspect may be regarded as transferring redundant information from the spatial to the temporal domain or pouring Space in Time. When this procedure is applied with a time step $\delta t = \Delta t / \text{SSF}$ (with the super-sampling factor $\text{SSF}$ being a positive integer), the flow sequence is sampled with a higher temporal resolution. The method will be referred with the acronym TSS (time super-sampling) in the remainder.

4. Experimental verification

Three flow cases are considered for the experimental assessment, which involves planar as well as 3D (tomographic) measurements of free and wall-bounded turbulent shear flows. The first case ($A$) is a planar PIV experiment around a square section prism at $Re = 5,000$. The second case ($B$) is a circular jet ($Re = 5,000$) measured by tomographic PIV. The third experiment (case $C$) is the near wake of a NACA0012 airfoil at cord Reynolds number of 370,000. In the first two cases the temporal resolution could be chosen one to two orders of magnitude in excess of that required. In the third case the temporal resolution is slightly in excess of that needed according to the Eulerian sampling criterion. However, the airfoil wake flow provides the closest conditions to Taylor’s hypothesis of frozen turbulence as it will be shown by the results. The condition of temporally resolved measurements from the raw experiments enables to consider them as accurate reference data for the proposed technique. In all cases the raw data is used to quantify the accuracy of the TSS method. The analysis is first performed qualitatively, by inspection of the flow topology and velocity time-history at a chosen location. The statistical error estimate of the reconstructed data by TSS relative to the raw data is then quantified by the spatial distribution of standard deviation. The frequency domain analysis is performed at a chosen location by means of amplitude spectrum. The main parameter varied throughout the analysis is the super-sampling factor $\text{SSF}$.

4.1 Planar PIV around a square prism (case $A$)

The flow around a square section prism is often considered as a benchmark case for experimental and numerical techniques, in relation to bluff bodies with sharp separation. Several works have documented such flow. In particular Lyn and Rodi (1994) have investigated the behaviour of the flapping shear layer separating from the sharp edge of the prism undergoing Kelvin-Helmholtz instability. The wake flow downstream develops into a Bénard-von Kármán wake with three-dimensional flow structures visible on the back-side of the prism. Experiments are conducted in a low-speed wind tunnel of $40 \times 40 \text{ cm}^2$ cross section. The prism has a side $D = 30 \text{ mm}$ and spans the entire wind tunnel section. The air flow speed is 2.5 m/s resulting in a Reynolds
Two-component PIV measurements are conducted in the mid-span of the prism on a single side of it as shown in Figure 2. Particles of 1 micron diameter produced by a fog machine are used to seed the flow at a concentration of about 10 particles/mm². Illumination is provided by a Quantronix Darwin-Duo laser (Nd:YLF diode pumped, 2×25mJ at 1,000 Hz, λ = 528 nm). PIV images are acquired at 13,500 Hz with a Photron FASTCAM-SA1 (CMOS, 1024×1024 pixels at 5,400 Hz, 10 bits, pixel pitch 20 μm) equipped with a Nikon objective with 60 mm focal length (f# = 4.0). The active sensor size at this framing rate is reduced to 640×512 and the resulting digital resolution of the imaging system is 13.3 pixels/mm. Additional details of the experimental apparatus are described in previous works by de Kat et al., (2008). At the chosen frame rate, the reference particle image displacement is 2.5 pixels in the free stream and an interrogation area is 1.2×1.2 mm² (16×16 pixels) is chosen. The velocity field is obtained by an adaptive multi-framing cross-correlation technique averaging the cross-correlation map from 4 subsequent image pairs, where the time separation within each pair is chosen to average 10 pixels displacement. By this approach, the precision error is greatly reduced and inspection of the temporal sequence is estimated to be below 0.01 pixels. The present experiment clearly over-samples the large-scale dynamical events occurring in the free shear layer as well as downstream in the wake. The shedding of Kelvin-Helmholtz vortices in the shear layer occurs at approximately 270 Hz and is measured by 50 samples in time. The Karman shedding is approximately 11 Hz (>1000 samples/cycle).

The analysis is performed sub-sampling the raw data by a factor 10, 30 and 60. The sub-sampled data is considered as the starting point for the TSS technique which is applied with a super-sampling factor SSF if 10, 30 and 60, such to produce data series at the original rate. The most critical case is considered, where the reconstructed sample at mid point of the measurement interval, (δt = Δt/2) is shown in Figure 2.

At SSF = 10 no clear difference can be observed between the reconstructed field and the raw measurement. Instead at SSF = 30 some artifacts become visible in the core of the vortices formed in the shear layer and in the smaller flow structures downstream of the prism. When SSF = 60 the distortion of the flow field is largely inaccurate also in the steady part of the shear layer, which is ascribed to the effect of streamlines curvature, which is not taken into account by the present numerical procedure. It may be concluded that in the present case a recording rate 10 to 20 times lower (f_acq ~ 1,000 Hz) would be sufficient to accurately reconstruct the flow by TSS at a higher time-resolution. However, with a measurement performed at 1 kHz, the shear layer vortex shedding would be represented by 3 to 5 samples, which may be considered barely sampled, leading to significant modulation of the fluctuations amplitude. In this respect, the application of a moderate super-sampling factor (e.g. SSF = 5) would bring the super-sampled series to an equivalent frequency up to 5,000 Hz, with approximately 20 samples/cycle, which eliminates the above problem, as also shown in the diagrams of Figure 3, where the time-sequence of the vertical velocity component is shown (solid black line) for a point located in the rear of the separated shear layer (X=450,Y=250). For the chosen location, the flow exhibits an intermittent behavior: the point is initially on the high-speed side of the shear layer and due to the shedding from the counter-clockwise vortex, the shear layer progressively moves outside, such that the chosen point falls within the separated flow region where relatively small frequency fluctuations are observed (in the final part of the time sequence). Starting from sub-sampled data series, the signal is reconstructed at the original measurement rate by the TSS method. The result obtained by SSF = 10 follows the raw data with excellent agreement. In this case also the linear time-interpolation does
provide an acceptable level of accuracy. A more visible departure between the linear interpolation and the raw data is observed for $SSF=30$, nevertheless, the TSS reconstruction exhibit a pattern closer to the reference data, especially reducing the clipping of peak values. Increasing the sub-sampling factor to 60 the temporal fluctuations in the initial part are sampled far below Nyquist criterion and the linear interpolation is also introducing strong aliasing effects. In these conditions also the TSS reconstruction also shows large differences from the original data, nevertheless, the pattern of the time fluctuations is followed to a better extent.

![Graph of displacement vs time for different sub-sampling factors](image)

**Figure 3:** Time-series of the vertical velocity component in the rear part of the separated shear layer from Figure 2.

### 4.2 Tomographic PIV of a circular jet (case B)

A laminar water jet is issued at a velocity of 0.45 m/s from a circular contoured nozzle of 10 mm exit diameter ($Re=5,000$). Vortex rings developing in the free shear layer undergo vortex-pairing followed by the transition to three-dimensional regime a few diameters downstream. Further details of the experimental apparatus and flow conditions are given in Violato et al., (2010). The high-speed tomographic system is composed of a Quantronix Darwin-Duo Nd:YLF diode-pumped laser illuminating a cylindrical domain of 30 mm diameter and operates at 1,000 Hz repetition rate. Four LaVision High Speed Star 6 CMOS cameras compose the imaging system. The concentration of Latex particles of 56 µm diameter is approximately 1 particle/mm$^3$. The particles displacement between recordings is 8 pixels (voxels). Tomograms of 600×600×1000 voxels are produced with DaVis 7.4 and analyzed with volume deformation iterative multigrid ($VODIM$, Scarano and Poelma, 2009) making use of ensemble correlation averages over four exposures (3 pairs). The interrogation volume is $2\times2\times2$ mm$^3$ (40×40×40 voxels). The vortex rings are formed at a rate of approximately 29 Hz, therefore each shedding process is described by more than 30 snapshots. The flow pattern is visualized by an axial data slice where velocity vectors and velocity axial component contours are shown. The vortices 3D pattern visualization is given in terms of iso-surfaces of $Q$-criterion (Hunt et al., 1988). Two axi-symmetric vortex rings are present in the region close to the nozzle exit, whereas the ring above ($Y/D = 2.5$) exhibits azimuthal waves and is contracted under the action of a larger structure downstream, which will later involve it in pairing/leapfrogging. The axial flow is accelerated inside vortex rings causing inward radial flow at their trailing region.
The visual inspection of the reconstructed velocity pattern at $SSF = 6$ (Figure 4) shows no visible difference with respect to the raw data. At $SSF = 12$ the coherence of the circular motion in the vortex cores is slightly lowered, however the topology of the vortex pattern based on the $Q$-criterion visualization is essentially retained. Only at $SSF = 24$ important artifacts are introduced, both in terms of velocity vector field as well as $Q$-criterion; more prominently, in the upper part of the flow, where smaller flow scales are formed, which are visibly reduced in strength. Also in the lower part of the domain, the vortex shedding is not well modeled by advection and non-linear effects such as strong streamlines curvature and acceleration cannot be compensated for by the advection model. This is not surprising, when one considers that the sampling time interval becomes $\Delta T = 24$ ms, which largely exceeds half the time taken by a shedding event ($\Delta T_{shed} = 34$ ms).

The time history of the axial and radial velocity (in terms of particle image displacement) allows to directly observe the effect of data sub-sampling and reconstruction by using the advection model and compare it with the point-wise linear interpolation in time (Figure 4). For $SSF = 6$ both the reconstruction based on linear interpolation (dashed blue line) as well as the super-sampled series follows well the reference data. Instead, at a sub-sampling factor of 24, the linear reconstruction becomes largely inaccurate as opposed to the $TSS$ reconstruction that still shows good agreement with the raw data.

Lowering the measurement rate in time-resolved tomographic PIV experiments allows operating the system at higher pulse energy, in turn enabling to measure over a larger domain. This is an important factor, considering that, time-resolved tomographic PIV measurements in air flows are currently limited by the insufficient energy scattered by the particle tracers.

A global error norm is introduced to quantify the discrepancy between the data reconstructed by the $TSS$ method and the reference data. The quadratic difference $\varepsilon_{TSS}$ of the local velocity normalized by the jet exit velocity $V_{jet}$ is displayed in Figure 6 for the chosen values of $SSF$.

\[
\varepsilon_{TSS} = \frac{1}{V_{jet}} \left[ \frac{1}{N} \sum_{n=1}^{N} (u_{TSS,n} - u_n)^2 + (v_{TSS,n} - v_n)^2 + (w_{TSS,n} - w_n)^2 \right]^{1/2}
\]  

(9)

The largest discrepancy between the velocity reconstructed by super-sampling and the reference data is
observed along the jet shear layers (Figure 6).

Figure 5: Measured velocity time history (expressed as voxels displacement) evaluated at \(X/D = 0\) and \(Y/D = 3.5\). Axial displacement (top) and radial displacement (bottom) for \(SSF = 6\) (left), 24 (right).

Vortices shedding and pairing cause large fluctuations and strong streamline curvature, which introduces errors increasing for large values of \(SSF\). The data series at \(SSF=6\) exhibits a relative error within 3%, which may be considered acceptable in view of the fact that it is in the same order of the typical measurement uncertainty for tomographic PIV. At \(SSF=12\) and 18 the discrepancy attains approximately 6% and 10%, which could be justified for flow-visualization purposes.

![Figure 6: Relative error (\(\varepsilon_{TSS}\)) in the axial plane as a function of the super-sampling factor. Left-to-right \(SSF = 6,12,18,24\).](image)

4.3 High-speed 2C PIV of the near wake of a NACA airfoil

The experiments are conducted in a low-speed wind tunnel at free-stream velocity of 14 m/s over a NACA-0012 airfoil of 40 cm chord length placed at zero incidence. The chord based Reynolds number is 370,000 and the boundary layer transition is forced at 30% chord by a tripping device. The measurement region covers the trailing edge to investigate the dynamical phenomena at the origin of acoustic noise emission. The measurements are conducted utilizing the same hardware components as in the previous cases.
Figure 7: Instantaneous velocity field after a NACA-0012 airfoil (case C). Velocity vectors and color coded streamwise velocity component. The black circle indicates the point where the velocity time history is considered. Raw data (top) and reconstructed with $SSF = 12$ (bottom). Data on the edge is not valid since it is reconstructed taking values from outside the measured domain.

The acquisition rate of 20,000 Hz is achieved firing each of the two lasers at 10,000 Hz with a time delay of 50 µs. A recording sequence of 1s duration is considered for this case. The region of interest indicated in Figure 7 is the near wake of the airfoil, where the two boundary layers interact. In these conditions, the hypothesis of frozen turbulence holds reasonably well in comparison with the other two cases where free shear layers with one stagnant flow side was present. The digital resolution of the measurement is 20 pixels/mm and the interrogation is performed with a window size of 20 pixels and an overlap factor of 75%. As a result, velocity fluctuations of wavelength not smaller than approximately two millimeter are spatially well resolved. The needed temporal resolution according to an Eulerian sampling that avoids aliasing is 7,000 Hz, (eq. 1). Thus, frequency spectra from measurements performed at 20,000 Hz are not expected to be affected by aliasing and can be considered as reference. The velocity time history illustrates that from the sub-sampled series most of the velocity fluctuations can retrieved with the TSS approach. A notable exception is the fluctuations at highest frequency and small amplitude, ascribed to measurement noise, which is uncorrelated between subsequent measurement samples. Amplitude modulation becomes clearly noticeable only at $SSF = 24$. The absolute and relative standard deviation of the super-sampled series is summarized in table 1. The difference between the raw data and the super-sampled series is in the order of 0.2 pixels, which is slightly in excess of the expected measurement errors of the raw data. An estimate of the measurement error is obtained comparing the raw data with the data series super-sampled with $SSF = 2$, where both series are sampled at a frequency above Nyquist criterion and their difference can only be ascribed to uncorrelated measurement noise. This result indicates that a significant part of the discrepancy (approximately 70%) is due to the noise-averaging effect of the super-sampling procedure, rather than a spatio-temporal filtering. This is supported by the evidence that the spectral content of the time series remains practically unaltered up to $SSF = 24$. The length of the present dataset allows performing signal analysis in the frequency domain by means of amplitude spectra estimated by the Welch algorithm (Welch, 1967). The time series composed of 20,000 samples is divided into 50 overlapping blocks of 800 samples and each one is weighted by a Hamming window. The resulting frequency resolution is $\Delta f = 50$ Hz.

All the super-sampled data series follow the raw data with good agreement up to a frequency of 4,000 Hz, where the latter, approximately two decades below the maximum amplitude, begins to flatten under the effect of noise. Instead, all super-sampled series continue the decay to a lower level. This behavior is ascribed to the weighted averaging between successive samples (eq. 7).
Interestingly, in this case no visible limit is reached for the maximum value of \( SSF \), meaning that Taylor’s hypothesis holds well (\( \tau_{\text{lag}} > 10 \tau_{\text{Eul}} \)). As a result, the spectra relative to the the data reconstructed with TSS from the sub-sampled series extend far beyond the Nyquist limit dictated for the Eulerian sampling approach. Considering 4,000 Hz as point of departure between raw data and super-sampled ones, an increase of temporal resolution by a factor of 2.5, 5 and 10 has been obtained for the data at \( SSF=6,12 \) and 24 respectively.

5. Conclusions
An advection model is proposed to increase the temporal resolution of time-resolved PIV. The model, based on Taylor’s hypothesis of frozen turbulence, enables to effectively increase the temporal resolution of the measurement system. The maximum increase depends upon the ratio between the time-scales of turbulent fluctuations evaluated from a Lagrangian and an Eulerian reference frame. The effectiveness of the approach is assessed by experiments conducted at a measurement time-rate well
beyond the maximum frequency in the flow, such to provide a solid reference for the temporal evolution of the flow. The analysis of the flow around the square prism shows that the TSS technique is suited for planar data of separated shear flows in presence of strong vortices. However, in this case, the method becomes inaccurate for large time separations in the regions of when strong streamline curvature occur. The TSS method effectiveness is also evaluated on 3D data obtained by tomographic PIV, where the issue of reducing the needed repetition rate is particularly relevant in view of both, current laser power and computational effort needed to produce and evaluate tomograms. The results show that the reference data can be accurately reconstructed starting from a time series taken at a measurement rate approximately one order of magnitude lower than the original.

The analysis of the NACA-0012 airfoil wake flow provides the opportunity to evaluate a case where Taylor’s hypothesis of frozen turbulence holds well. In this case, even the largest sub-sampling factor considered (SSF=24) returns an amplitude spectrum in good agreement with the raw data, clearly indicating the suitability of the method to obtain higher frequency information from the TSS method applied to PIV time series.

Table 1: Standard deviation of super-sampled time series \((X,Y) = (20 \text{ mm},0)\)

<table>
<thead>
<tr>
<th>Raw data</th>
<th>Std((u)) [pixels]</th>
<th>Std((v)) [pixels]</th>
<th>Std((u-v_{\text{SST}})) [pixels]</th>
<th>Std((v-v_{\text{SST}})) [pixels]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSF = 6</td>
<td>0.706</td>
<td>0.630</td>
<td>0.188</td>
<td>0.184</td>
</tr>
<tr>
<td>SSF = 12</td>
<td>0.697</td>
<td>0.608</td>
<td>0.221</td>
<td>0.212</td>
</tr>
<tr>
<td>SSF = 18</td>
<td>0.691</td>
<td>0.604</td>
<td>0.250</td>
<td>0.244</td>
</tr>
<tr>
<td>SSF = 24</td>
<td>0.679</td>
<td>0.583</td>
<td>0.300</td>
<td>0.279</td>
</tr>
</tbody>
</table>

References


flow using Stereoscopic-PIV, *Exp. Fluids* 42, 259-279