On the application of a two-frame particle tracking velocimetry with multi-frame approach

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Abstract Novel algorithms for three-dimensional particle-tracking velocimetry are proposed and tested with three-dimensional data. The tracking scheme implements the nearest neighbour cluster matching which demonstrates better performances in terms of spatial adaptivity than fixed volume approaches. The similarity criterion is based on a correlation function built between two successive time steps, which depends on the distance and angles of particles surrounding the particle to be track. The algorithm is enriched with several temporal multi-frame improvements, i.e. extrapolation of expected particle positions in subsequent frames and frame-gap technique. The particle identification procedure is modified respect to the traditional background subtraction, local thresholding and grey level weighted averaging by using the optical flow equation. The local maximum of grey levels around a “feature”, which is associated to the particle, is identified and the barycentre is calculated by using Gaussian fitting. The improvement of the particle centroid identification represents a fundamental step for stereo matching through the epipolar geometry criterion. The spatial matching procedure, based on the epipolar geometry, is improved by using two subsequent frames, in order to reduce the number of ambiguities. Tests on the tracking scheme and the spatial matching reconstruction have been conducted via synthetic data that simulate the Burgers’ vortex. Finally, the entire measurement chain, from centroids identification to stereo matching and temporal tracking, is applied to analyse the non-steady phenomenon of penetrative convection in a stably stratified fluid.

1. Introduction

Particle Tracking Velocimetry (PTV) is a well established technique in fluid mechanics developed to measure velocity fields by tracking tracer particles seeding the flow. In many investigations the description of the flow field in terms of properties of individual particles of the fluid (Lagrangian representation) (La Porta et al. 2001) is more useful than a Eulerian description (velocity is given as a function of time and space). Dispersion is a good example of a phenomenon conveniently described from a Lagrangian point of view (Moroni and Cushman 2001). The basic requirements for a PTV experiment are illumination of the seeded fluid and recording of images of the region of interest. This technique is often defined as low-image-density Particle Image Velocimetry (PIV) (Adrian 1991), because when the tracer density is high, it may be difficult to match particles frame by frame. On the other hand, PTV algorithms have, in general, two principal advantages with respect to classical PIV. First, they may be relatively simple to extend to three –dimensional (3D) flows via stereoscopic analysis (Maas et al. 1993; Malik et al. 1993). Second, Eulerian and Lagrangian statistics may be calculated from Lagrangian type PTV output; in fact, the calculation of Lagrangian statistics from a regular Eulerian data set, as obtained with PIV, is less accurate since it involves the integration in time of the instantaneous Eulerian velocity fields (Cenedese and Querzoli 1997). In the 2D case, PIV and PTV have their advantages and disadvantages, whereas in three-dimensional analysis PTV is the natural choice, because the particle
density in the physical space is below the range of PIV: the particle distances need to be stretch along the whole depth of view (Pereira et al, 2006).

We will focus on 3D-PTV using the photogrammetric technique (Dore et al. 2009). The two most important processing steps in 3D-PTV (i.e. the establishment of spatial and temporal correspondences between particle images in simultaneous acquisitions from multiple cameras) can be handled in different ways:

- 3D particle positions determined for each time step and tracking performed in 3D space (Dore et al. 2009);
- particles tracked in image space and establishment of spatial correspondences between 2D trajectories (Moroni and Cushman 2001);
- combination of image- and object space-based information to establish spatial-temporal correspondences between particle positions at consecutive time steps (Willneff and Gruen 2002).

In this paper new algorithms, developed to strengthen the spatial and temporal results, are presented. First, a new approach for calculating particle barycentres, based on the computation of the Harris matrix, is introduced. This new method permits to replace the threshold binarization and pixel labelling steps shared by the most diffused algorithms by using a feature identification procedure, hereinafter Feature Extraction (FE) guaranteeing a higher number of barycenters identified respect to other techniques (Shindler et al. 2010). FE employs the procedure used in the Feature Tracking (FT) technique (Moroni and Cenedese 2005), which permits to identify the so-called “good features to track” and subsequently to compute the particle’s barycentre with the 1D Gaussian estimator (Ouellette et al. 2006). To reconstruct the three-dimensional particle positions in a world reference system, a physically based photogrammetric calibration of the stereoscopic arrangement is needed. The combination of image- and object space-based information is employed to establish the correspondences between particle positions, by using the epipolar geometry and a newly developed spatio-temporal procedure useful for reducing the unavoidable ambiguities. Once the three-dimensional particle positions are reconstructed, a new algorithm for tracking particles in the volumetric space has been used. It is based on the cluster matching Polar Coordinate System Similarity (Ruan and Zhao 2005) but differs from this for using the concept of topological distance instead of metric distance. This simple modification allows to overcome problems like uneven particle density distribution and particles distributed near the borders of the analysed domain guaranteeing a strong spatial adaptivity. A frame-gap technique is also introduced to overcome loss-of-pair situations and to guarantee long trajectories. The improved 3D-PTV algorithms have been employed to study penetrative convection in a stratified fluid heated from below. Prior to the application to real data, the novel algorithms have been tested on synthetically generated data.

2. Three-dimensional Particle Tracking Velocimetry

2.1 The particle identification procedure: “good feature to track”

A novel technique for identifying tracer particles by using a feature extraction procedure based on the optical flow equation is proposed here to overcome problems like low intensity particles, overlapping particles and presence of random noise. The optical flow equation (Horn and Schunk 1981), sometimes called the “image brightness constancy constrain” (BCC), states that local variations in the image intensity are balanced by convective changes due to the velocity field. If it is computed at a single point, hereinafter defined feature, it only provides one equation for two unknowns, the velocity components \((u, v)\). It is only when the equation is evaluated at each pixel in a region \(W\) surrounding the point, that it provides sufficient information for computing \(U (u,v)\). The problem has to be reformulated as a minimization in a least square sense and the solution will be the velocity vector that better approximates the motion of the interrogation window associated to the
single feature. This implies a purely translational motion model is assumed. The minimization brings to the equation:

$$\begin{bmatrix} \int I_x^2 \text{d}S & \int I_x I_y \text{d}S & \int I_x \text{d}S \\ \int I_y I_x \text{d}S & \int I_y^2 \text{d}S & \int I_y \text{d}S \\ \int I_z^2 \text{d}S & \int I_z \text{d}S 
\end{bmatrix} \mathbf{U} + \begin{bmatrix} \int I_x I_y \text{d}S \\ \int I_y I_z \text{d}S \\ \int I_z I_x \text{d}S \end{bmatrix} = 0$$

or

$$\mathbf{G} \cdot \mathbf{U} + \mathbf{b} = 0 \quad \mathbf{U} = \mathbf{G}^{-1} \cdot \mathbf{b}$$

where $I_x$, $I_y$, and $I_z$ stand for the image intensity derivatives in the direction x, y, and z respectively. The existence of a solution for the system 1 is closely embedded in the definition of feature, which is the prerequisite for particle identification. The existence of the solution depends on the invertibility of the Harrys matrix $\mathbf{G}$, which is invertible if its eigenvalues, $\lambda_1$ and $\lambda_2$, are not null. System 1 suggests that if both $I_x$ and $I_y$ are different than zero, both eigenvalues are real and positive since they are the solution of the characteristic equation of $\mathbf{G}$. This implies system 1 can be solved. The two eigenvalues define the quality of the feature and if it is well-defined, the particle identification will be optimised. The noise requirement implies that both eigenvalues are large, while according to the conditioning requirement they cannot differ by several orders of magnitude. Then, if the smaller eigenvalue is large enough to overcome the noise criterion, $\mathbf{G}$ is consequently well-conditioned because the greater eigenvalue cannot be arbitrary large (Shi and Tomasi 1994). A feature is included in the “good feature to track” set if its minimum eigenvalue is greater than a percentage of the maximum among the minimum eigenvalues calculated for each pixels of the image. A minimum distance between the features, depending on the seeding density, is also introduced. This should guarantee each particle is associated to one feature only. Once the feature is well identified and it is likely associate to one particle only, it is possible to determine its peculiar characteristic, i.e. its barycentre. To do this, the local maximum of the grey levels, inside a square region around the integer position of the feature, is found first. Secondly the barycentre is determined through the best fitting method developed for small particles, i.e. by two 1D Gaussian functions (Ouellette et al. 2006).

2.2 Stereoscopic reconstruction: spatial matching

The purpose of the structure-from-stero step is the reconstruction of the 3D scene from two or more views assuming pinhole model cameras. Two processes are involved: correspondence or matching and reconstruction. Correspondence implies estimating which points in the images are projections of the same scene point. Two cameras are enough to reconstruct 3D information, but additional views increase confidence in the results. The coordinates of corresponding points may be related by the epipolar geometry. Assuming the orientation parameters of the cameras (both intrinsic and extrinsic parameters) are known from the calibration procedure, given a point in one image, its conjugate must belong to a line in the other image, the epipolar line. A tolerance is considered to take into account an error in the calibration parameters producing a shift of the epipolar lines. On the other hand the tolerance may introduce ambiguities in determining the triplets corresponding to the same point in the 3D scene. A multi-choice strategy chooses all the triplets of centroids whose distance from the corresponding epipolars and epipolar intersections satisfies the tolerance criterion. Although this strategy provides 100% of correctly matched triplets (using three cameras) for different stereo arrangements (Dore et al. 2009), it determines a large number of ambiguities that may create implicit difficulties in the subsequent application of a tracking scheme. The way to reduce ambiguities is to strengthen the epipolar constraint. This can be obtained either by increasing the number of cameras or applying the epipolar reconstruction using two subsequent frames (Figure 1).
Figure 1. Simplified sketch of the procedure used to obtain spatial correspondences using two views. \( P_t \) and \( P'_t \) are the particles detected at time \( t \) while \( P_{t+\Delta t} \) and \( P'_{t+\Delta t} \) are the corresponding at the following time step. \( e_{12} \) refers to the epipolar line of particle \( P \) on view 2 from view 1, considering the two time steps \( t \) and \( t+\Delta t \).

The latter has been adopted: the epipolar constrains are then imposed to barycentre locations at a given time and to the expected arrival locations at the following time. This implies that each barycentre is detected as well as its expected displacement in the sampling time interval. The latter information is achieved solving the optical flow equation in the area surrounding the barycentre.

2.3 The Enhanced Spherical coordinate system Similarity: temporal matching

The main advantage in tracking 3D positions as opposed to tracking the projections in image space, is that positions may appear to coincide or cross, and tracking ambiguities increase in the projected image. In this paper a 2F cluster matching tracking technique, the Enhanced Spherical coordinate system Similarity (ESS), suitable for strongly rotating flows and for shear flows is employed. It is based on a correlation function built between two successive time steps, which depends on the distances and angles of particles surrounding the particle to be tracked. Given a particle position in a generic time, its candidates at the following time belong to a spherical volume of fixed radius \( R_f \), that depends on the maximum expected displacement of the particles in the assigned time interval. For each particle belonging to the couple, a fixed number, \( P \), of neighbouring particles is set. The positions of neighbours in the couples are then identified with polar radius, \( r \), and polar angles \( \theta \) and \( \phi \).

A similarity function \( S \) for the two clusters is defined as follows (Ruan and Zhao 2005):

\[
S = \sum_{p_1} \sum_{p_2} H(e_r - |\Delta r|, e_\theta - |\Delta \theta|, e_\phi - |\Delta \phi|)
\]

H is a step function equal to one only if all arguments are strictly greater than zero. \( \Delta r \), \( \Delta \theta \) and \( \Delta \phi \) are the deviations of the neighbours in the transition from the first to the subsequent time step. \( e_r \), \( e_\theta \) and \( e_\phi \) are the tolerances of the relative deviations, to account for a variation from the quasi-rigidity condition of the two groups (Shindler et al. 2010). The choice of the tolerances depends on the type of flow. The stronger the deformations involved, the larger these tolerances should be. The similarity function is calculated for all the candidates and the particle that gives the maximum value of \( S \) is selected. A minimum value of \( S \) (\( S_{\text{min}} \)) defined as the minimum number of neighbours that satisfies the quasi-rigidity condition, is established to take into account loss-of-pair situations and avoid spurious vectors. \( S_{\text{min}} \) is function of the seeding density: the higher the seeding density, the larger the \( S_{\text{min}} \) coefficient.

The ESS algorithm is enforced by following the multi-frame approach of frame-gap in order to strengthen the temporal tracking, to increase the trajectories length and the number of velocity vectors. The frame-gap technique allows enforcing the Lagrangian post-processing statistics and incrementing the number of vectors in the Eulerian flow field description. More specifically, if the particle to be tracked is not identified at a generic time, the same particle is searched at the
subsequent time. When skipping one time, a higher value of $S_{\text{min}}$ has to be assumed to account for larger deformations and higher number of candidates.

3. Data

3.1 Synthetically generated data

To evaluate the performances of the spatial matching reconstruction and of the tracking scheme, synthetically generated data simulating the Burgers’ vortex have been used. The Burgers’ vortex is a steady solution of the Navier–Stokes equation where the action of strain and viscosity balance each other to give a vortex core of steady finite size. The Burgers’ vortex is a laminar but complex structure and displays a range of scales defined by the distance between successive windings of its streamlines.

\[\text{Figure 2. Burgers’ vortex flow trajectories}\]

The equation of Burgers’ vortex is given in Cartesian coordinates as:

\[V = \frac{\Gamma}{2\pi r} \left[1 - \exp\left(-\sigma \frac{r^2}{4v}\right)\right],\]

\[r = \sqrt{x^2 + y^2}\]

where:
- $\sigma$ is the rate-of-strain
- $\Gamma$ is the circulation
- $v$ is the kinematic viscosity.

To simulate three-dimensionality, a linear velocity gradient $\delta = 0.001 \text{ s}^{-1}$ normal to the $X$–$Y$ plane has been considered. The measurement domain is a cubic volume of $100 \times 100 \times 100 \text{ mm}^3$ with $X \in [-50, 50] \text{ mm}$, $Y \in [-50, 50] \text{ mm}$ and $Z \in [-50,50] \text{ mm}$. The parameters of the Burgers’ vortex are: $\Gamma = 200 \text{ mm}^2 \text{ s}^{-1}$, $\sigma = 0.01 \text{ s}^{-1}$ and $v = 1 \text{ mm}^2 \text{ s}^{-1}$. Figure 2 shows a sample of the synthetic trajectories inside the domain.
3.2 Experimental data

The laboratory model consists of a convection chamber containing an initially stable, density stratified fluid, which is then heated from below to cause destabilization and penetrative convection. The test section is a tank with a square base (0.41 x 0.41 m$^2$) and 0.40 m high (Fig. 3). Its lateral sides are insulated by 0.03 m thick removable polystyrene sheets. When images are acquired, the insulation on the sides facing the cameras is removed. Distilled water is used for the fluid phase and pollen particles of about 80 µm mean diameter and 1.06 g/cm$^3$ density (Moroni and Cenedese 2005) are used for the passive tracer to reconstruct particle trajectories. A diffuser, which also acts to insulate the upper surface since it is attached to a polystyrene sheet, floats on the surface of the water as it fills the tank. A tank filled with hot water drains by gravity into a continually stirred colder tank and that tank in turn drains to the diffuser. While the diffuser floats upwards, it fills the test section creating a linear stratification of the fluid, cold to hot from bottom to top (two-tank filling method). Twenty-six thermocouples are vertically spaced in the tank to record temperature. After the fluid stratification within the tank is completed, a cryo statically controlled hot water bath is attached to the metal base plate and the experiment begins. An approximately 0.15 x 0.15 x 0.40 m$^3$ light volume produced by a high power lamp (1000 W) is employed to illuminate the central region of the test section (shaded region inside the test section in Fig. 3). Images of the pollen particles are recorded using three synchronized 764 x 576 pixels CCD cameras with a time resolution of 25 frames/s. Synchronization of the cameras is obtained using the sync signal from one of the cameras as the triggering signal for the others. Each camera is connected to a different colour input of a RGB frame grabber and the full resolution images are stored as single channel bmp files.

![Figure 3. Experimental set-up](image-url)
4. Results

4.1 Tests on synthetically generated data

The novel spatial matching technique has been tested on the synthetically generated data presented before. The test aims are to compare the single time matching procedure employing the epipolar geometry (Dore et al 2009) to the novel spatio-temporal matching algorithm performances in terms of total number and correctly matched number of triplets output by the codes depending on number of particles filling the test section, camera arrangement, wrong evaluation of the calibration parameters and tolerance accounting errors in the calibration parameters producing a shift of the epipolar lines.

The algorithm that performs better is expected to output the highest percentage of correctly matched triplets and the lowest number of total triplets to avoid ambiguities in the following particle reconstruction and tracking steps.

The effect of an increasing number of particles seeding the measurement volume (from 100 particles to 1000 particles) on the matching procedure performances was tested. The maximum number of particles filling the test section (1000) is characteristic of a medium seeding density while the algorithm is supposed to be employed for analysing medium-high seeding density images. The choice to keep it relatively small is motivated by the negligible further information provided by those tests compared to the large amount of computation time and memory storage required. Dore et al. (2009) present tests on synthetic data where up to 2000 particles have been employed. Results show that the algorithm performances for high seeding densities are substantially equivalent to low seeding densities considering the proportional increase of matched triplets.

3D-PTV requires viewing the sample volume from two or more directions. The performances of the matching algorithms have been then tested by using two different three cameras arrangements (labeled A and B; Fig. 4). Arrangement A corresponds to a three camera set-up with optical axes lying on the same plane and forming an angle of 90° with the closer camera; arrangement B presents three cameras with optical axes of couples of cameras forming an angle of 90°. The ideal setup for obtaining highly accurate trajectories (Becker et al., 1999) requires the cameras to be mounted with the distance between them equal to the distance to the center of the measurement volume. But the camera arrangement is usually a compromise between ideal geometrical conditions in the measuring volume and practical restrictions associated with the experiment.

For each arrangement, the three-dimensional particle positions have been projected on the image plane of the three cameras (#1, #2 and #3) replicating the exterior and interior calibration
parameters of the acquisition system employed. The effect of a wrong evaluation of the calibration parameters has been investigated as well. In this case, the image sequences have been created by employing a fixed set of calibration parameters (the correct ones), while the reconstruction has been carried out by using a modified set of calibration parameters by introducing a given error (labeled “err1”, “err2”, “err3”, “err4” or “err5”). The deviation from the original data set has been determined by evaluating the largest errors in terms of positions and angles committed during the system calibration step. The calibration parameters labeled with “err5” present the largest deviation from the original dataset, “err1” the smallest one.

![Figure 5](image)

**Figure 5.** Effects of the tolerance on the total # of matched triplets vs. total # of particles for both arrangements determined via (a) the classical epipolar reconstruction procedure and (b) the novel spatio-temporal matching algorithm

The effect of the tolerance is presented in Figure 5 for both arrangements. The classical epipolar reconstruction procedure and the novel spatio-temporal matching algorithm are both employed. For both arrangements, as the tolerance increases, the number of matched particles grows as well as the corresponding number of ambiguities. In all cases, the set of triplets output by the algorithms contains the correct matches. On the other hand, the novel approach permits to drastically reduce matched triplets and ambiguities for all the tolerances considered and for both arrangements.
A second test have been conducted to evaluate the performances of the Enhanced Spherical coordinate system Similarity (ESS) comparing it with other tracking schemes. Data sets of 5000 vectors, with application points normally distributed in space and with modulus-direction determined by means of the Burgers equation, are synthetically generated. Different time intervals, $\Delta t$, have been imposed in order to test the potential of the algorithm with large particle displacements. Table 1 reports the maximum displacements for each case and the relative time step, together with the tolerances used for the ESS. The initial number of particles, $N_0$, in the measurement volume remains unchanged in each case. To simulate a real case, the number of particles is randomly reduced of 5% between two consecutive frames, creating unpaired particles.

<table>
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<th>$\Delta t$ (sec)</th>
<th>$x_{max}$ (mm)</th>
<th>$\varepsilon_r$ (pixel)</th>
<th>$\varepsilon_\theta$(rad)</th>
<th>$\varepsilon_\phi$(rad)</th>
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</table>

Table 1. Time intervals with the related maximum displacements and ESS tolerances

The performances of the algorithm are measured using the correct matching ratio, $\eta$, defined as the ratio between the number of correctly identified velocity vectors and the actual number of correct velocity vectors. The number of neighbours used for computing the cluster matching is 24 while the number of minimum neighbours satisfying the quasi-rigidity condition is 12.

Figure 6 shows the results as a function of the time step, comparing our algorithm with the well consolidated New Relaxation algorithm (NRX) and the Nearest Neighbour (NN) approach (Pereira et al., 2006).

The Nearest Neighbour scheme suffers the increasing time step, due to the fact that the maximum displacement becomes larger than the mean distance among particles. The relaxation method and the Enhanced Spherical coordinate system Similarity perform equally, showing both a high correct matching ratio also for great displacements. A sample of the velocity flow field obtained with the ESS is shown in Figure 7.

In these tests we have considered only a two-frame scheme, but the original ESS has been implemented including also the frame-gap technique which increases the performances in the analysis of experimental data due to the noise in the acquired images and the unavoidable errors associated to the reconstruction procedure.
4.2 Experimental results

The whole measurement chain has been applied to a laboratory model simulating the evolution of a thermally forced convective boundary layer. Figure 8 shows matched centroids projected on the XZ plane (a) and the YZ plane (b) inside both the stable and the unstable layers as they evolve with time after 440 s from the beginning of the experiment. Darker colours have been linked with later times, while lighter colours with earlier times. From both projections it can be clearly seen that the bottom turbulent region, feeding the mixing layer, is moving upward against the almost quiet stable layer above.

The 3D-PTV procedure is suitable for reconstructing the displacement field (i.e. particle trajectories) in both the mixing and the stable layer. The velocity data obtained from the 3D
trajectories have been used to obtain quantitative results for the Convective Boundary Layer (CBL) height evolution. Since dispersion in turbulent convective phenomena is mostly due to transport by large organized structures, the knowledge of the vertical extension of the structures dominating the flow field, associated to the CBL height is a necessary requirement. The domain is divided into layers to compute the vertical velocity standard deviation profiles at given time from the beginning of the experiment. The horizontal homogeneity assumption allows averaging velocity data in each layer. Values of the velocity standard deviation are expected to be larger inside the mixing region than in the stable layer where it should vanish. According to this expectation, profiles present a typical behaviour with a maximum becoming larger for longer times and an inflection point moving upward. In Fig. 9, the standard deviation profiles are normalized according to the Deardorff mixed-layer similarity (Willis and Deardorff, 1974), where $w^*$ is the convective velocity and $z_i$ the mixing layer height. After normalization, all profiles collapse on the same curve demonstrating the phenomenon is self-similar, hence results given by different experiments and different time intervals can be compared. Standard deviation profiles are also compared to atmospheric models output (Lenschow et al., 1980), tank experiments (Cenedese and Querzoli, 1994; Deardorff and Willis, 1985) and field campaign measurements (Young, 1988). Comparison shows a good agreement with measurements both at bench and real scale, demonstrating the validity of the experimental apparatus and its applicability for the study of the real atmospheric boundary layer for environmental purposes.

![Figure 9. Vertical velocity variance profile normalised according to the mixed layer similarity](image)

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