Estimation of Reynolds Stresses from PIV Measurements with Single-Pixel-Resolution

Sven Scharnowski, Rainer Hain, Christian J. Kähler

Institute of Fluid Mechanics and Aerodynamics, Bundeswehr University Munich, 
Germany, sven.scharnowski@unibw.de

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A method for estimating Reynolds normal and shear stresses from a PIV image sequence with single–pixel resolution is presented. This work describes how the correlation function can be used to identify the probability–density function of the velocity and how to calculate all Reynolds stress components from it in a 2–D regime.

1. Purpose and method

The two–point ensemble correlation improves the spatial resolution for the estimated velocity significantly, but does not give any temporal information about the flow. Thus, the estimation of Reynolds stresses is not possible in general (Kähler et al 2006). However, in 2006 Kähler and Scholz suggested a method for estimating (symmetrical) normally distributed turbulence from the size of the correlation peak. This idea is refined and improved in this work.

In order to determine the accuracy of the developed method, synthetic PIV images with different probability–density functions were generated and the estimated Reynolds stresses are compared to the simulated values. This allows for a quantitative accuracy assessment.

2. Results

For a Gaussian probability–density function with an elliptical cross section (major axis \( p_x \), minor axis \( p_y \) angle of rotation \( \alpha \)) the Reynolds stresses can be computed from the following Equations:

\[
\bar{u}^2 = \frac{1}{16} \cdot \left( \cos^2 \alpha \cdot p_x^2 + \sin^2 \alpha \cdot p_y^2 \right), \quad (2.1)
\]

\[
\bar{v}^2 = \frac{1}{16} \cdot \left( \sin^2 \alpha \cdot p_x^2 + \cos^2 \alpha \cdot p_y^2 \right), \quad (2.2)
\]

and

\[
\bar{u} \cdot \bar{v} = \frac{1}{16} \cdot \cos \alpha \cdot \sin \alpha \cdot (p_y^2 - p_x^2). \quad (2.3)
\]

The parameters \( p_x, p_y \) and \( \alpha \) can be estimated by applying a Gaussian fit function to the correlation peak and taking the particle image diameter into account. Since the correlation function can be computed for each single pixel of the PIV images using the two–point ensemble correlation, the suggested method allows for the estimation of the Reynolds stresses with single–pixel resolution.

Figure 1 shows exemplarily the comparison between simulated and estimated stresses for small images with homogeneous stresses in each case. The particle image diameter was 3 pixel. The probability–density function had an elliptical cross section and was rotated by the angle \( \alpha \). A cross section through the correlation function is sketched in the upper part.

3. Conclusions

The presented work illustrates how to estimate Reynolds normal and shear stresses with single–pixel resolution. It is shown, that the accuracy of the computed values depends on the particle image diameter, the occurring stresses, velocity gradients and the number of PIV images. Nevertheless, for a data set of several thousand PIV image pairs all Reynolds stresses in a 2–D regime can be computed with single pixel resolution with an error of only a few percent. This is of great importance for the analysis of small–scale flow phenomena appearing at large Reynolds and Mach numbers for instance.

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References
