Estimation of Reynolds Stresses from PIV Measurements with Single-Pixel Resolution

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Abstract  A method for estimating Reynolds normal and shear stresses from PIV images with single–pixel resolution is presented. This work describes how the correlation function can be used to identify the probability–density function and how to calculate the Reynolds stresses from it in a 2–D regime. In principle the procedure can also be applied to sum of correlation peaks computed with conventional PIV. The influences of the particle image diameter and the velocity gradients on the shape of the correlation function are also discussed here. Synthetic PIV images were generated and a two–point ensemble correlation was applied. The resulted correlation maps and the extracted Reynolds stresses were compared to analytical calculations.

1. Introduction

In the last years the analysis of PIV recordings with single–pixel resolution (two–point ensemble correlation) has been developed in order to enhance the spatial resolution and to increase the measurement accuracy of small scale flow phenomena (Westerweel et al 2004, Adrian 1988). In micro–fluids, this approach was first applied because one of the assumptions was the stationary nature of the flow. Later, this approach was extended for the analysis of periodic (Billy et al 2004, Scholz and Kähler 2006) and also for fully turbulent flow (Kähler et al 2006, Bitter et al 2009). The boundary layer measurements in Kähler et al 2006 as well as those performed by Bitter et al 2009 using long–range micro–PIV for the flow along a generic rocket, at supersonic Mach numbers, showed the potential of this approach. As this measurement technique is frequently applied in the turbulent flow regime, the following question arises: Can the Reynolds normal and shear stresses also be computed with single pixel resolution?

With conventional PIV the stresses are directly computed from the velocity fluctuations in a (time) series of vector fields. These vector fields are computed from cross–correlation functions obtained from interrogation the windows of each two (single exposed) PIV images. The maximum–position of a cross–correlation function represents the mean velocity in the interrogation window. Thus, the resolution of the Reynolds stresses is limited to the size of the windows used for the cross–correlation. The computed values only represent an average over the corresponding area and strong spatial changes are smeared out. The window size in conventional PIV cannot be reduced below a certain minimum: the optimum particle image diameter is around 2 pixel and 5 to 10 particle images are needed for computing a valid vector (Raffel et al 2007). The typical outcome of this is a window size of $16^2$ pixel to $32^2$ pixel depending on the particle image density. Reducing this size usually leads to increased uncertainty.

The two–point–ensemble correlation improves the spatial resolution for the estimated velocity, but does not give any temporal information about the flow. Although this information is lost, statistical variables can be extracted from the shape of the correlation peak. Kähler et al. suggested a method for estimating (symmetrical) normally distributed turbulence from the size of the correlation peak (Kähler and Scholz 2006). W. Arnold et al. already showed in 1986 that turbulence studies can be done by visibility analysis in speckle velocimetry (Arnold et al 1986). The probability–density function of the velocity (rather than temporal information) is used in both examples to extract statistical information about the flow. This idea is refined and improved in the following pages.
2. Mathematical description

This work suggests a method for computing all components of the Reynolds–stress tensor from the correlation–function. Since the correlation–function can be computed for each single pixel of the PIV images using the two–point ensemble correlation, the suggested method allows for the estimation of the Reynolds stresses with single–pixel resolution.

For a sufficient amount of PIV image pairs, the shape of a correlation peak is mathematically similar to the convolution of the particle image and the probability density function of the velocity. Without turbulence this peak is narrow and has a distinct maximum. In case of Reynolds–normal stresses the correlation peaks become broader and shear stresses lead to an unsymmetrical shape. The connection between the shape of the correlation peak and the existing Reynolds stresses is discussed in this paragraph.

The two-dimensional velocity field is described as follows:

\[ \mathbf{v}(x,y,t) = \mathbf{v}_0(x,y) + \mathbf{v}'(x,y,t) = \left( u_0(x,y) + u'(x,y,t), v_0(x,y) + v'(x,y,t) \right), \]

(2.1)

where \( \mathbf{v}_0 \) and \( \mathbf{v}' \) are the mean velocity and the corresponding velocity fluctuation respectively.

The probability–density function \( \mathcal{P}(v) \) of the velocity field includes all occurring velocities for a given point \( (x,y) \) over a certain time (or for an ensemble of measurements) and allocates the related probability. Hence, it is possible to compute the Reynolds normal and shear stresses using the probability–density function of the velocity field:

\[-\rho \cdot \overline{u'(x,y)^2} = -\rho \cdot \int PDF(u,v,x,y) \cdot u'^2 \, du \, dv, \]

(2.2)

\[-\rho \cdot \overline{v'(x,y)^2} = -\rho \cdot \int PDF(u,v,x,y) \cdot v'^2 \, du \, dv \]

(2.3)

and

\[-\rho \cdot \overline{u'(x,y) \cdot v'(x,y)} = -\rho \cdot \int PDF(u,v,x,y) \cdot u' \cdot v' \, du \, dv. \]

(2.4)

The only remaining task is to identify the probability–density function from the correlation peak. Therefore, the shape of the correlation peak will be analyzed in the following paragraphs.

2.1 A general analytical inspection of the correlation function

For an ensemble correlation with single–pixel resolution the correlation function \( C(\Delta x, \Delta y, x, y) \) can be computed from the first and the second PIV images \( A(x,y) \) and \( B(x,y) \) as follows (Westerweel et al 2004):

\[ C(\Delta x, \Delta y, x, y) = \frac{\sum_{n} [A_n(x,y) - \bar{A}(x,y)] [B_n(x+\Delta x,y+\Delta y) - \bar{B}(x+\Delta x,y+\Delta y)]}{\sigma A(x,y) \cdot \sigma B(x+\Delta x,y+\Delta y)}, \]

(2.5)

with the standard deviation:

\[ \sigma A(x,y) = \frac{1}{\sqrt{N-1}} \cdot \sum_{n=1}^{N} [A_n(x,y) - \bar{A}(x,y)]^{2}. \]

(2.6)

\( N \) is the total number of PIV image pairs and \( n \) the corresponding control variable, \( (x, y) \) are discrete coordinates of the pixel of interest in the two images. The shift vector \( (\Delta x, \Delta y) \) is directly related to the velocity via the time \( \Delta t \) between the acquisition of each two PIV images and the magnification of the optical setup.
In order to analyze the shape of the correlation peak for a fixed point \((x, y)\) in the image plane, Equation 2.5 is converted into an analytical expression. All images \(A_n\) are replaced by one single Gaussian peak:

\[
A_n(x, y) \rightarrow A(x, y) = \exp \left( -\frac{x^2 + y^2}{b^2} \right) \cdot 8. \tag{2.7}
\]

Instead of having several particle images on random positions (like in real PIV images), only one analytical function describes the intensity distribution. For the purpose of uniform particle image distribution, the Gaussian peak is moved to all possible positions, and the sum of Equation 2.5 becomes an integral. This is equivalent to an infinite number of PIV images of infinite size with infinitesimal small particle image density. The parameter \(b\) in Equation 2.7 is the particle image diameter at \(1/e^2\) of the maximum intensity.

The images \(B_n\) are also replaced by a single Gaussian peak but two things are different compared to \(A_n\): first, they are shifted due to the particle motion and secondly, they are convolved (denoted by \(\otimes\)) with the probability–density function PDF of the velocity:

\[
B_n(x, y) \rightarrow B(x - u_0(x, y), y - v_0(x, y)) \otimes \text{PDF}(x, y), \tag{2.8}
\]

with

\[
B(x - u_0(x, y), y - v_0(x, y)) = \exp \left( -\frac{(x-u_0(x,y))^2+(y-v_0(x,y))^2}{b^2} \right) \cdot 8. \tag{2.9}
\]

For the sake of simplicity, the time \(\Delta t\) between the acquisition of each two PIV images and the magnification of the optical setup is left out of Equation 2.8 (and in the following) and \((u, v)\) identify the motion of the particle image in coordinates of the images. Using Equation 2.7 and 2.8 the correlation function from Equation 2.5 can be transformed into an analytical expression:

\[
C(\Delta x, \Delta y, x, y) \sim \frac{\int A(x - \Delta x, y - \Delta y) \cdot [\int B(\xi - u_0(\xi, \psi), \psi - v_0(\xi, \psi)) \otimes \text{PDF}(\xi - x, \psi - y) d\xi d\psi] d\Delta x d\Delta y}{\text{convolution of } B \text{ and } \text{PDF}}. \tag{2.10}
\]

where the sum of Equation 2.5 was replaced by an integral. This ensures that all possible locations of the particle image with respect to the point of interest were considered. This simulates a perfectly uniform particle image distribution. Thus, no normalization by the standard deviation is required for the integral. The Greek letters \(\xi\) and \(\psi\) are the control variables for the convolution integral. The coordinates \((x, y)\) are in the image plane and \((\Delta x, \Delta y)\) corresponds to the correlation plane.

In general, one must only compute the theoretical Equation for the correlation function and apply this as a fit–function to the experimental data, in order to get the Reynolds stresses.

### 2.2 Reynolds stresses for a specified PDF

Equation 2.10 gives a universal expression for a correlation function. However, in order to solve the integral it is necessary to specify the PDF. For the following procedure the probability–density function \(\text{PDF}(x, y)\) is assumed to be Gauss shaped. The Gaussian distribution has an elliptical cross section (major axis \(p_x\), minor axis \(p_y\)) and is rotated by angle \(\alpha\):

\[
\text{PDF}(x, y) = \frac{8}{\pi p_x p_y} \exp \left( -\left( \frac{\cos \alpha x - \sin \alpha y}{p_x} \right)^2 - \left( \frac{\sin \alpha x + \cos \alpha y}{p_y} \right)^2 \right) \cdot 8. \tag{2.11}
\]
The factor before the exponential term in Equation 3.1 ensures that the integral of the PDF, over all velocities, always equals one. In general, the shape of the probability-density function can change for each pixel of the image plane, hence $p_x$, $p_y$ and $\alpha$ are functions of the position vector $(x,y)$. Equation 2.2, 2.3 and 2.4 are valid for any function $PDF(x,y)$, hence the challenge that remains lies in solving the integrals.

Solving Equation 2.10 for the specified PDF from Equation 2.11 gives an analytical expression for the correlation function $C$. The correlation peaks of the image pairs computed using Equation 2.5 can then be compared to the analytical expression of $C$ in order to find the parameters $p_x$, $p_y$ and $\alpha$ of the probability-density function. Finally the Reynolds-stresses can be calculated from Equation 2.2, 2.3 and 2.4 by using Equation 2.11:

$$
\overline{u'^2} = \frac{1}{16} \cdot \left( \cos^2 \alpha \cdot p_x^2 + \sin^2 \alpha \cdot p_y^2 \right) \quad (2.12)
$$

$$
\overline{v'^2} = \frac{1}{16} \cdot \left( \sin^2 \alpha \cdot p_x^2 + \cos^2 \alpha \cdot p_y^2 \right) \quad (2.13)
$$

and

$$
\overline{u'v'} = \frac{1}{16} \cdot \cos \alpha \cdot \sin \alpha \cdot \left( p_y^2 - p_x^2 \right) \quad (2.14)
$$

### 2.3 Influence of velocity gradients

Keane and Adrian reported in 1990 an analysis of correlation peaks and their dependence on velocity gradients in conventional PIV. For a shear layer they found that the amplitude decreases with increasing velocity gradients and that the diameter is broadened (by the same factor) in the direction of shear. This paragraph investigates the shape of the correlation peak computed with 2-point ensemble correlation in a similar manner.

In order to compute the correlation function analytically, it is essential to find an expression for the velocity in the surrounding area of the point of interest. For almost all PIV experiments, the velocity will not be constant over the whole image area. However, for many cases the mean flow has a preferred orientation and strong gradients occur only in the perpendicular direction. Hence, it seems to be a good approach to develop the velocity in a Taylor series and neglect all higher terms except the first derivative of $u$ with respect to $y$:

$$
\nu_0(x_0, y_0) = \left( \frac{\partial u_0(x_0, y_0)}{\partial y} \right)_{(x_0, y_0)}. \quad (2.15)
$$

$(x_0, y_0)$ is the point of interest in the image plane for which the correlation function has to be computed. Taking this velocity gradient into account leads to a rather complex solution of Equation 2.10. The structure of the solution is as follows:

$$
C(\Delta x, \Delta y) \sim \exp \left( - \frac{f(\Delta x, \Delta y, u_0, v_0, \partial u/\partial y(x_0, y_0), p_x, p_y, \alpha, \beta)}{g(\partial u/\partial y(x_0, y_0), p_x, p_y, \alpha, \beta)} \right), \quad (2.16)
$$

where $f$ and $g$ are functions with linear and quadratic dependency of all arguments except the
Figure 2.1 Contour plot of the analytical correlation–function for different velocity gradients $\partial u/\partial y ([−1, 0, +1]$ pixel/pixel from top to bottom) and different angles $\alpha ([−20°; 0°; +20°]$ from left to right). The parameters $p_x$ and $p_y$ of the probability–density function are 5 pixel and 1 pixel respectively and the particle image diameter was chosen to be 3 pixel.

angle $\alpha$, which appears trigonometric–function–like.

Figure 2.1 shows the correlation function from Equation 2.16 for different conditions. The PDF was non–isotropic ($p_x = 5$ pixel, $p_y = 1$ pixel) in all cases. The angle of orientation, $\alpha$, was $0^\circ$ for the peak in the center and no gradient $\partial u/\partial y$ was applied. For the left and the right of the middle row, $\alpha$ was changed to $−20^\circ$ and $+20^\circ$ respectively. The resulting correlation peaks look similar to the applied $PDF$. The upper and lower row in Figure 2.1 shows the influence of an additional velocity gradient $\partial u/\partial y$. Here the correlation peaks are stretched in the direction of $p_x$ and rotated due to the velocity gradient. Thus, the shape of the PDF is not directly related to the shape of the correlation function in the case of strong velocity gradients. Therefore, it is essential to include the information of the surrounding velocity field in order to compute the Reynolds stresses for a given point. Additional investigations showed that the influence of velocity gradients on the shape of the correlation function is strongly dependent on the size of the particle images. The deformation gets stronger with increasing particle image size. This is demonstrated for synthetic images in paragraph 3.4.

The broadening of the correlation peaks under the influence of strong velocity gradients is in qualitative agreement with the considerations for conventional PIV from Keane and Adrian.
3 Analysis of synthetic PIV images

In order to determine the accuracy of the method presented above, synthetic PIV images with different probability–density functions were generated and the estimated Reynolds stresses are compared to the simulated values. This allows for a quantitative accuracy assessment.

The calculation procedure is described in section 3.1. Section 3.2 and deals with homogeneous and isotropic stresses and investigates the effect of several parameters on the accuracy of the estimated turbulence. The ability of estimating shear stresses is analyzed in section 3.3 and the influences of velocity gradients are studied in section 3.4. At the end of this section the enhanced spatial resolution of the presented method is demonstrated on a set of small synthetic images with different Reynolds stresses in each row.

3.1 Calculation procedure

All synthetic PIV images are generated and analyzed using MatLab functions. A maximum intensity of 1000 counts was applied for the intensity of the particle images. The fraction of illuminated pixel was kept constant at 25% for all generated images, meaning that 25% of all pixels had an intensity of more than \(1000\) counts. Hence the number of particle images changes with the particle image diameter. The average number of particle images in a 10 pixel \(\times\) 10 pixel image is, for example, 3.5 for a particle image diameter of \(b = 3\) pixel. With respect to the finite pixel-size of the camera sensor, each pixel’s gray value is computed from the integral of the intensity over the corresponding area, instead of simply transferring the analytical point-wise values to the pixel. This procedure causes a difference between the applied particle image diameter \(b\) and the resulting diameter computed with a Gaussian fit–function. Figure 3.1 shows the comparison between the input value and the estimated diameter. The factor \(\sqrt{2}\) only holds for particle images bigger than 5 pixel, but for smaller values the particle image size is estimated to be too large.

For each particle in each first PIV image, the position was randomly chosen. The position in the second image changed with respect to the first one due to the velocity and its fluctuations. The velocity fluctuations were random numbers with a Gaussian distribution that fulfill the probability–density function. The theoretical Reynolds stresses are computed from the applied fluctuations. Upon the generation of the synthetic PIV images, the correlation function for each pixel was computed using Equation 2.5 on page 2. In the next step each correlation function was reviewed using a Gaussian fit–function:

\[
C_{\text{fit}}(\Delta x, \Delta y, c_x, c_y, \phi) \sim \exp \left[ - \left( \frac{\cos \phi \Delta x - \sin \phi \Delta y}{c_x} \right)^2 \cdot 8 - \left( \frac{\sin \phi \Delta x + \cos \phi \Delta y}{c_y} \right)^2 \cdot 8 \right]. \tag{3.1}
\]

This fit–function is fairly simple compared with the rather complex one from Equation 2.10 and leads to a stable and fast fitting procedure. However, for gradients which are not too strong, the parameters \(p_x\), \(p_y\) and \(\alpha\) can easily be estimated from \(c_x\), \(c_y\) and \(\phi\):

\[
p_x \approx \sqrt{c_x^2 - 2 \cdot b^2} \tag{3.2}
\]

\[
p_y \approx \sqrt{c_y^2 - 2 \cdot b^2} \tag{3.3}
\]

\[
\alpha \approx \phi \tag{3.4}
\]
Equations 3.2, 3.3 and 3.4 are only exact solutions for a symmetric PDF ($p_x = p_y$) without velocity gradients (see also paragraph 2.3). The particle image diameter can be estimated from the auto-correlation peak. The latter can be computed using Equation 2.5 and simply exchanging image $B$ by image $A$. (For better accuracy one may also estimate $b$ from the auto-correlation of the images $B$.) In theory, the auto–correlation peak of a Gaussian function is also a Gaussian function, but the width is increased by factor $\sqrt{2}$ (Raffel et al 2007). Hence, fitting the auto–correlation peak gives an estimation for the particle image diameter.

With the parameters $p_x$, $p_y$ and $\alpha$, obtained from the PDF, the Reynolds stresses are then computed from Equations 2.12, 2.13 and 2.14.

### 3.2 Homogeneous flow with isotropic stresses

In this paragraph PIV images with homogeneous normal stress distribution are evaluated. The applied probability–density function is constant for all pixels in the image plane. Isotropic stress means that the parameters $p_x$ and $p_y$ are equal and Equation 2.11 becomes:

$$PDF(x, y, p_x = p_y) = \frac{g}{\pi p_x^2} \cdot exp \left( -\frac{x^2 + y^2}{b^2} \cdot 8 \right). \quad (3.5)$$

In this case the parameters $c_x$ and $c_y$ of the fit–function 3.1 should also be equal. Thus, a symmetrical fit–function can be applied. Equation 3.2 gives the solution for $p_x$ and the normal stresses are computed using Equations 2.12 and 2.13. The turbulence $Tu$ is used to evaluate the computed stresses and is defined as follows:

$$Tu = \sqrt{\frac{\nu'^2 + \nu''^2}{2}}. \quad (3.6)$$

Furthermore, with Equation 2.12 and 2.13 it can be reduced to:

$$Tu = \frac{p_x}{\sqrt{b}}. \quad (3.7)$$

Figure 3.2 shows (on the left) the accuracy of the estimated turbulence with respect to the particle...
Figure 3.2 Influence of the particle image diameter on the accuracy of the computed turbulence (left) and velocity (right). Each point represents the average value of $5 \times 5$ correlation peaks computed from 20,000 synthetic PIV image pairs using Equation 2.5. The error-bars correspond to the standard deviation of turbulence or velocity for the 25 correlation-peaks.

Figure 3.3 Influence of the applied turbulence (left) and of the number of synthetic PIV images (right) on the accuracy of the computed turbulence. Again, $5 \times 5$ correlation peaks were analyzed in each case.

image diameter $b$. Three different values of $p_x$ were applied. A systematic error for small particle images ($b \approx 0.5 \ldots 3$ pixel) is observed, which is most likely due to the wrong estimation of the particle image size (see Figure 3.1). For particle image diameters larger than $b \approx 5$ pixel the random error increases consistently. The reason for which is mainly the decreasing number of particle images (the fraction of illuminated pixels was kept constant at 25%). The accuracy of the estimated velocity is shown on the right plot of Figure 3.2. Particle images smaller than $b \approx 1$ pixel are not suitable for accurate velocity measurements. The minimum error is smaller than 0.01 pixel for a particle image diameter of $b \approx 3$.

The accuracy of the estimated turbulence is not only dependent on the particle image diameter, but also on the number of PIV image pairs and the turbulence itself. The influence of the latter is shown on the left hand side of Figure 3.3 for three different particle image diameters. For a turbulence smaller than 0.2 pixel, the turbulence itself tends to be overestimated and, for larger values, the error changes the sign. The estimation of the turbulence seems to be affected by a systematic error. However, the accuracy is (except for very small turbulence) within 5%. Figure 3.3 illustrates (on the right) how the number of PIV images affects the estimated turbulence. The fraction of illuminated pixels was still 25%. As expected, the error decreases with an increasing number of images.

For the tested cases, 10,000 image pairs appear to be a sufficient number. The optimal
3.3 Homogeneous flow with non–isotropic stresses

So far, only synthetic PIV images with symmetric probability–density functions were analyzed. Now, the more general approach from Equation 2.11 is applied. Therefore, the normal stresses $-\rho \cdot \overline{u'^2}$ and $-\rho \cdot \overline{v'^2}$ are not necessarily equal and the shear stress $-\rho \cdot \overline{u' \cdot v'}$ can differ from zero.

However, the simulated stresses are still constant over the whole image plane. Figure 3.4 shows the comparison between applied and estimated Reynolds–stresses for different shapes of the PDF. On the left, the cross section had a constant size but was rotated by angle $\alpha$. The stresses for a constant angle of rotation and increasing ratio of $p_y/p_x$ are shown on the right of Figure 3.4. Each point was computed from 20,000 PIV image pairs with constant stresses. $10 \times 10$ correlation peaks were analyzed. The estimated stresses tend to be somewhat smaller than the applied ones for values higher than 0.2 px$^2$, which agrees with Figure 3.3. However, the difference between estimated and applied stress lies within a few percent in all cases. Further tests with different probability–density functions showed similar results.

According to the previous analysis, the developed method is suitable for estimating Reynolds stresses, in particular shear stresses. Although the stresses are computed for each single pixel, only mean values over 100 pixel are plotted above. Hence, it cannot be called real single–pixel resolution. However, results with true single–pixel resolution are presented in paragraph 3.5.

3.4 Influence of velocity gradients

In paragraph 2.3 (Figure 2.1 on page 4) the deformation of the correlation peak due to a gradient in the velocity was shown. This effect is quantitatively analyzed here. Therefore, PIV images with
Figure 3.5 Influence of the velocity gradient $\frac{\partial u}{\partial y}$ on the accuracy of the computed normal– (left) and shear–stress (right).

Figure 3.6 Computed correlation–peak for each 10,000 synthetic PIV image pairs with different velocity gradients $\frac{\partial u}{\partial y}$. A particle image diameter of $b = 3$ pixel was applied. The parameters $p_x$, $p_y$ and $\alpha$ of the PDF were 2 pixel, 0.5 pixel and 0° respectively. The solid lines indicate size and orientation of the fitted Gaussian peak.

non–isotropic stresses were generated. The parameters $p_x$, $p_y$ and $\alpha$ of the PDF were chosen to be 2 pixel, 0.5 pixel and 0° respectively. The velocity gradient $\frac{\partial u}{\partial y}$ varied from $-1$ to $+1$ pixel/pixel. Figure 3.5 on the following page shows the computed normal stress in the $x$–direction on the left and the shear stresses on the right. Three different particle image diameters were tested. The graphs show a strong biased error that depends on the particle image size and the velocity gradient. The computed stress in the $x$–direction (not shown in the Figure) seems not to be affected by the gradient for the applied parameters. Figure 3.6 shows the correlation peaks for the first, last and middle point of the data set from Figure 3.5 ($b = 3$ pixel). These peaks are computed using Equation 2.5 and are in good agreement with the analytical functions of the middle row in Figure 2.1, on page 4. A positive velocity gradient $\frac{\partial u}{\partial y}$ rotates the correlation peak in the counter clockwise direction and vice versa. The reason for the artificial increase of the stresses is the fact that the correlation peak is stretched and rotated in the presence of velocity gradients.

Figure 3.5 and 3.6 clearly shows that the simple fit–function from Equation 3.1 is not sufficient if strong velocity gradients are present. In order to overcome this problem, it is either necessary to find a correction for Equations 3.2, 3.3 and 3.4 or to use the more complex fit–function from Equation 2.10 to extract the right values for $p_x$, $p_y$ and $\alpha$. However, the errors are systematic and show a smooth behavior. Thus, it should be possible to estimate the right values from the computed stresses. In order to find an efficient method for correcting the extracted data, further research needs to be done.
3.5 Shear flow with non–homogeneous and non–isotropic stresses

This paragraph should demonstrate the enhanced spatial resolution for an example of non–homogeneous Reynolds stresses. Therefore, 10,000 synthetic PIV–image (100 pixel × 100 pixel) with different stresses in each row have been generated. The stresses are controlled by the PDF. The parameters \( p_x, p_y \) and \( \alpha \) of the PDF are \( p_x(y) = y/100 \), \( p_y(y) = 3 \), and \( \alpha(y) = (y – 50 \text{ px})/100 \text{ px} \) respectively. Additionally, a sinusoidal profile for the horizontal velocity was simulated: \( u(y) = \sin (y \cdot 2\pi/100 \text{ px}) \cdot 1 \text{ px} \). These values were kept constant in the \( x \)–direction over the 100 pixel. Figure 3.7 shows the computed results for the horizontal velocity and the three different Reynolds stresses. The velocity distribution is smooth and has a standard deviation \( \sigma < 0.013 \text{ pixel} \), whereas the Reynolds stresses show higher random errors. Here, the standard deviations are 0.024, 0.031, and 0.043 for \( \overline{u'^2} \), \( \overline{v'^2} \), and \( \overline{u'v'} \) respectively. Although the images for the stresses in the upper part of Figure 3.7 are somewhat noisy, it is clearly shown that small changes can be resolved. Even if the exact values for each single pixel cannot be computed, it should be possible to identify small scale flow phenomena that occur only in a region of a few pixels.

4. Conclusion

The presented work illustrates how to estimate Reynolds normal and shear stresses with single–pixel resolution. It is shown that the accuracy of the computed values depends on the particle image diameter, the occurring stresses, velocity gradients and the number of PIV images. Nevertheless, for a data set of several thousand image pairs all Reynolds stresses in a 2–D regime can be computed, with single pixel resolution, with an error of only a few percent. This is of great importance for the analysis of small–scale flow phenomena appearing, for instance, at large Reynolds and Mach numbers. In principle, the developed method can also be used to analyze correlation peaks from sum–of–correlation procedures, computed with conventional PIV. However, the received information in this case is averaged over the interrogation window. Research with further synthetic images and experimental PIV images is still going.
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