An investigation of vortex ring formation in strongly forced jet flows by high speed particle image velocimetry

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Abstract The periodic formation of vortex rings in the development region of a round jet subjected to high levels of acoustic forcing is investigated with High Speed Particle Image Velocimetry (HSPIV). Harmonic velocity oscillations of 20 to 100% of the mean flow at the jet exit were achieved for several forcing frequencies determined by the acoustic response of the system. The HSPIV measurements provided a time-resolved history of vortex ring evolution so that the roll-up time, pinch-off location and circulation Γ of the vortex rings could be evaluated. As the forcing amplitude A was increased two main effects were observed. Firstly, the Γ the vortex rings increased linearly with A with many cases exhibiting a trailing jet indicative of an optimal vortex ring as defined by Gharib et al. (1998) and Linden and Turner (2001). Secondly, the location where the vortex ring started to roll-up moved progressively upstream towards the nozzle exit. The critical value of forcing amplitude where this occurred varied approximately between A ≈ 0.2-0.5 for the cases studied and is both Re and frequency (Strouhal no.) dependent. It is shown that a universal formation time-scale exists based on the forcing period, T, determines the amount of vorticity periodically discharged from the nozzle exit that is rolled-up into the vortex ring occurs at t/T ≈ 0.3. This non-dimensional time corresponds to when the acceleration changes sign, i.e. just after the local maxima. Overall, the effect of high amplitude forcing is marked by a transition from amplification of shear flow instability to the periodic break-up of the jet into large-scale vortex rings sharing many of the features of starting jets.

1. Introduction

Although free round jets are self-similar in the far field, it is well known that the initial development region exhibits large-scale orderly structures. Numerous experiments over the years have shown that small levels of forcing applied to the base flow can control how the flow field develops amplifying instability growth rates and the subsequent transition to turbulence, sound generation, jet growth, and mixing characteristics (Becker and Massaro 1968, Crow and Champagne 1971; Gutmark and Ho 1983; Lee and Reynolds 1985; Raman et al. 1989). This paper investigates the effect high amplitude forcing has on the structure of the near field of a jet, particularly the onset of large-scale unsteady behaviour characterized by the periodic formation of vortex rings. Vortex rings formed by starting and unsteady jet flows play a key role in a variety of emerging technologies. For example, unsteady propulsion devices such as pulse detonation engines and pulse combustors generate periodic vortex rings whose formation timescales correlate with improved thrust augmentation (Mason and Miller 2006). In combustion instability, vortices are a source of flame sheet kinematics responsible for the non-linearity that leads to limit-cycle oscillations and is a significant development issue in lean burn gas turbine combustion (Külsheimer and Büchner 2002; Balachandran et al. 2005).

Fundamental investigations into the vortex rings formed by an impulsive slug of fluid have provided a clear explanation of the formation process together with models to predict a variety of ring parameters of interest such as trajectory, circulation Γ and vorticity ω distribution (Pullin 1979; Didden 1979). Gharib et al. (1998) showed that as the length to diameter ratio of a fluid slug L/D was increased the size of the vortex increased rolling-up all the fluid into a single ring conserving circulation, impulse, volume and energy. The authors found that above a critical value of L/D ~ 4 it
was not possible to roll-up the entire fluid slug into a single ring and that beyond \( L/D > 4 \) the excess fluid and circulation formed a trailing jet in the wake of the vortex. This transitional state between a single ring and a ring with a trailing jet occurring at an \( L/D \sim 4 \) is commonly referred to as the \textit{Formation Number}. Linden and Turner (2001) showed that the limiting process described by the formation number corresponds to an ‘optimal’ vortex ring as it possesses the maximum impulse, circulation and volume for a given energy input. The presence of counter and co-flow on the formation number were also investigated by Dabiri and Gharib (2004) and Krueger et al. (2006) respectively. However, no studies have investigated whether or not the existence of such a limiting process exists in forced jets.

The focus of this paper is to study the response of strongly forced jet flows and to characterize the resulting properties and timescales of the vortex rings. There are several issues of interest, for example, when does a forced steady jet become an unsteady jet? What is the amplitude and frequency dependence on vortex ring properties? Does a limiting process exist for forced jets? For this purpose, an experimental investigation has been performed using 2D High Speed Particle Image Velocimetry (HSPIV) for the quantitative examination of the flow field. HSPIV enabled velocity vectors and the derived quantities to be obtained with a high temporal resolution. Time-resolved iso-vorticity contours were used to identify the boundary of the vortex rings and track their evolution from the initial roll-up stage through to pinch-off.

2. Experimental Method

Fig.1 (a) Sketch of the experimental setup (b) Acoustic response of the experimental rig to forcing frequency

2.1 The forced jet apparatus

A sketch of the experimental setup of the acoustically forced round jet is shown Fig.1(a). The air flow rate was set using an Alicat MC series mass flow controller and passed through an oil seeder consisting of double Laskin nozzles which produced an oil mist with diameters of 0.3 to 0.5\( \mu \)m. The seeded air flow entered a 200 mm long plenum with an inner diameter of \( ID = 100 \) mm. A 300 mm long tube with an ID = 35 mm which was fitted with matched cubic nozzle with a jet exit diameters of \( D = 10 \) and 23 mm (although the results presented in this paper focus on the 10 mm nozzle due to
size restrictions). In order to generate harmonic velocity oscillations at the nozzle exit by acoustic forcing, two diametrically opposed loudspeakers were connected to the plenum chamber. A TTi 40 MHz waveform generator provided monochromatic sinusoidal input signals which were amplified to drive the loudspeakers.

The first step was to measure the frequency response of the system by placing a hot wire anemometer (HWA) at the centre of the jet exit to measure the frequency response at constant forcing amplitude. Fig.1(b) shows that the largest amplitude response occurs at forcing frequencies of 40, 150, and 260 Hz. With mean velocities between 5 and 20 m/s velocity amplitudes of $A = 0.2$ to just over 1 were achievable. The mean exit velocity profiles under forced and unforced conditions were obtained from HWA and the HSPIV measurements and are shown in Fig.2(a). The velocity profiles are almost top hat and show good agreement apart from the shear layer where the HSPIV results are not as spatially resolved at the HWA measurements (see next section). Fig.2(b) shows the phase averaged exit velocity is sinusoidal and linear over a wide range of $A$’s.

For a sinusoidally forced jet the exit velocity $U_e(t)$ is defined by the background mean velocity $U_b$, $f$, $A$, and phase angle $\phi$ as shown in equation (1). An analogy to vortex ring experiments using a piston-cylinder arrangement can be made by considering the period of acoustic forcing as a piston stroke and retraction. The mean piston velocity $U_p$ within the time range of interest can then be calculated by integrating $U_e(t)$ according to equation (2).

$$U_e(t) = U_b + AU_b \sin(2\pi ft + \phi)$$  \hspace{1cm} (1)  

$$U_p = \frac{1}{\tau} \int_0^\tau U_e dt = U_b - \frac{AU_b}{2\pi f} \left[ \cos(2\pi ft + \phi) \right]_0^\tau$$  \hspace{1cm} (2)

2.2 HSPIV configuration and post-processing

The 2D HSPIV system was comprised of a Pegasus-PIV dual pulse 527 nm Nd:YLF laser with a maximum repetition rate of 10kHz, a set sheet forming optics (without collimation) and a Photron Fastcam SA-1.1 monochrome camera with a maximum frame rate of 5.1kHz at full 1MP resolution. The camera timing and the synchronization of the laser pulses were achieved using a LaVision high speed controller. A 1 mm thick laser sheet was passed through the jet cross-section as shown schematically in Fig.1(a) with the camera located normal to the imaging plane. The fields of view (FOV) were adjusted to cover the developing region of the jet up to $x/D \sim 6$, giving the images...
spatial resolutions between 0.06-0.09 mm/pixel. The time delays between two image pairs were varied between 10-60µs to ensure particle translations 4-5 pixels depending on inlet conditions. It was aimed to sample 33 image pairs per T which necessitated 1320 and 4950 Hz sampling frequencies at f = 40 Hz and 150 Hz respectively. In order to maintain maximum camera resolution 19 image pairs were obtained at 4940 Hz for jet forcing at f = 260 Hz.

![Fig.3](image)

Fig.3 (a) Raw PIV image (b) Velocity vectors (¼ of the total). Re = 4,250; St = 0.24; A = 0.78

The cross-correlations were performed using Davis software. Firstly, image pre-processing which includes sliding background subtraction and particle intensity normalization was applied to the raw images. Second, cross-correlations with 4 passes of iterations with round form of interrogation windows, 32×32 pixels of window sizes for the first two and 16×16 pixels of window sizes for the final two passes, and 50% overlap of adjacent windows were applied to each image pair providing a flow field resolution of 0.5-0.7 mm. Velocity vectors for each 8 number of pixels in x and r directions were obtained. A vector post-processing step including; median filter, smoothing, and filling up of empty spaces was applied. In Fig.3(a) and (b), a sample raw PIV image, and the corresponding velocity vectors are shown. These two images show the vortex ring forming at the shear layer between the ambient stagnant fluid and the injected fluid.

As we are investigating how the vortex ring properties are affected by the forcing conditions, the instantaneous nozzle exit boundary conditions are calculated at each time step and used to calculate the velocity history and amplitudes as in Fig.2(b). The circulation of the vortex rings and the total circulation are calculated by integrating the vorticity field over the area of interest using Stokes’ theorem in equation (3).

\[ \Gamma = \int_{\Gamma} u \cdot dl = \int_{\Sigma} \omega \cdot dS \] (3)

3. Results and Discussion

3.1 Evolution of the vortex rings

The formation of a vortex ring over a forcing cycle, T for high A at f = 260, 150 and 40 Hz is shown by the non-dimensional vorticity, \( \omega^* \), contours in Fig.4(a), (b), and (c) respectively where \( \omega^* = \omega D / U_b \). The initial roll-up, growth, pinch-off and advection of the vortex rings are clearly seen. For similar Re and A’s the effect of increasing forcing frequency (wavelength) is to generate smaller rings which roll-up and pinch-off closer to the jet exit, at around 1D for f = 260 Hz for example. This is because the effective L/D decreases for increasing frequency. For f = 40 and 150 Hz the
existence of a trailing jet is evident, with the former resembling the type of flow field encountered in the experiments of Gharib et al. (1998) for starting jets with long $L/D$'s. The vortex ring formed by forcing at $f = 150$ Hz shows some interesting phenomena. Once the lead ring has pinches off from the trailing jet, the remaining vorticity starts to roll-up and is entrained into the proceeding vortex ring. For the 40Hz case the ring has pinched-off and advected out of view approaching mid cycle. Although the ring travels outside the FOV of the measurements, the results up to $t = 0.55T$ shows the vortex ring is rapidly deformed preventing the observation of a clear pinch-off. It is interesting to note that, although outside the FOV, the vortex rings did not clearly persist once they advected beyond the developing region of the jet.

The growth of the vortex ring can be quantified by measuring the circulation around the largest iso-vorticity contour which defines its boundary. However, defining the boundary of a vortex ring is difficult given that the process of formation is a transient event. This has an impact on defining other aspects of vortex ring formation such as pinch-off time, and the core size. To this extent we

![Diagrams](image_url)

Fig.4. Velocity vectors and the iso-vorticity contours during one forcing period at $Re = 4250$ (a) $f = 260$ Hz, $A = 0.90$ (b) $f = 150$ Hz, $A = 0.99$ (c) $f = 40$ Hz, $A = 0.96$. Iso-vorticity contours are in the range $\omega^* = -21$ to 21 with increments of 2.

The growth of the vortex ring can be quantified by measuring the circulation around the largest iso-vorticity contour which defines its boundary. However, defining the boundary of a vortex ring is difficult given that the process of formation is a transient event. This has an impact on defining other aspects of vortex ring formation such as pinch-off time, and the core size. To this extent we
adopt the approach used by Gharib et al. (1998) and Krueger et al. (2006) by defining a threshold around an appropriate iso-vorticity contour level. A sensitivity analysis showed that this threshold value can significantly influence the value of vortex ring circulation since it affects the area and the sum of bounded vorticity. Krueger et al. (2006) determined this threshold at a normalized vorticity of $\omega^* = 0.91$ which corresponded to approximately 20% of the peak vorticity in the pinched off ring. In the current study, we found that the smallest value which gave sensible results was $\omega^* = 1$, with the normalization based on the mean exit velocity, making the vortex ring boundaries and the pinch-off events clear and identifiable. This value corresponds to approximately 6% of the peak vorticity which is deemed to be well resolved when compared to Krueger et al. (2006).

### 3.2 A Universal Time Scale for Vortex Ring Formation in Forced Jets

Fig. 5(a) and (b) plots the change of non-dimensional total and vortex ring circulations for increasing amplitude, $A = 0.78$ to 1.17. The circulation is normalized with by the mean shear layer strength of the jet $I^* = I/U_pD$. In these figures the vortex ring circulation, increases with $A$ and remains constant for the remainder of the forcing period yet in all three cases the lead vortex ring is followed by a trailing jet of vorticity. In starting jets, the trailing jet is evidence of excess $\Gamma$ due to vortex rings formed with $L/D$’s greater than the formation number. This implies that the formation of the rings in all of these cases is optimal and that the normalization of the vortex ring circulation should include the contribution of the unsteady velocity terms, $I^* = \Gamma/ U_pD$. When normalized in this way, the vortex ring circulations collapse.

In impulsive starting jets the formation number shows that the vortex ring growth is limited to a universal timescale corresponding $L/D \approx 4$. A closer look at Fig.5(a) reveals a relationship between the total circulation and the resulting vortex ring circulation. Independent of $A$ the intersection between the vortex ring circulation the total circulation curves occur at $t/T \approx 0.28$. Physically this means that the vorticity flux up to $t/T \approx 0.28$, just after the sign change in acceleration, determines the circulation budget of the vortex ring. This explanation is very similar to the universal formation number definition for vortex rings in starting jets although the time scales are different due to the differences in the singular and periodic natures of the two flows. It is worth seeking universality among the data in terms of this definition. For this purpose, total and vortex ring circulation curves for a wide range of data are plotted in Fig.6. As clearly seen on these graphs the formation times consistently range between $t/T = 0.27$ to 0.31 and were consistent with all the data sets. This suggests a formation number exists for forced axisymmetric jet flows.

### 3.3 Dependence of Vortex Ring Characteristics on the Flow Variables

Fig. 7 shows the $\omega^*$ contours for $Re = 4,250$ for (a) $f = 150$ Hz and (b) $f = 260$ Hz from $A = 0$ to maximum at a fixed phase in the forcing cycle. The unforced jet captures the Kelvin-Helmholtz instability which is broken down by the periodic formation of vortices once a critical threshold of $A$ is reached. When forcing at $f = 150$ Hz this occurs $A = 0.28$ whereas for the 40Hz case the transition occurs somewhere between $A = 0.4 - 0.5$. This amplitude dependence with frequency is consistent with the results reported by Külsheimer & Büchner (2002). For increasing $A$ the location where the vortex ring starts to roll-up moves progressively upstream towards the nozzle exit. The transition is easily seen in Fig.7(b) since the forcing frequency is lower than in (a). Calculation of vortex ring circulations for each data set reveals the effect of $Re$, $f$, and $A$. In Fig.8(a) and (b) this dependency is shown for the range of data $Re = 4,250-6,750$; $f = 40-260$ Hz ; $A = 0.4-1.3$. The curves show a linear relationship between $I^*$ and $A$ for all conditions and this dependence does not change when the contribution of the unsteady velocity is taken into account (see previous section). It can be clearly seen that vortex ring circulation values are larger for smaller forcing frequencies for a given $Re$ and $A$. This result can be explained by the larger wave length at smaller frequencies which results in larger amount fluid entrained in the rings.
Fig. 5 (a) Change of total and vortex ring circulations with time. $Re = 4,250$; $f = 150$ Hz; $A = 0.78, 0.99, 1.17$. (b) Iso-vorticity contours are in the range $\omega^* = -21$ to $21$ with increments of 2.

Fig. 6 Comparison of vortex ring circulations with total circulations: (a) $Re = 4,250$; $f = 260$ Hz (b) $Re = 6,750$; $f = 260$ Hz (c) $Re = 6,750$; $f = 150$ Hz (d) $Re = 7,500$, $f = 40$ Hz
3.4 Length and Time Scales of the Formation Process

In the scope of this study, another feature of the vortex ring formation in a forced jet flow investigated is the length- and time-scales associated with the formation process. The evolution of the rings over each forcing period were examined for all the data sets and their pinch-off times, $t_{po}$, were noted. Secondly, the pinch-off distance, $x_{po}$, which is defined as the distance between the vortex ring center and the nozzle exit at the pinch-off event were determined. This was to investigate the effect of the $Re$, $f$ and $A$ on the pinch-off time and distance. Examining Fig.9(a) and (b) these effects in terms of $T$ can be observed. In Fig.9(a), it is seen that an increase in the $A$ and $Re$ results in decrease in the pinch-off time, whereas increasing $f$ causes pinch-off to occur later in the period. Fig.9(b) shows that the pinch-off distance $x_{po}$ is almost constant with $A$, when the jet is fully unsteady. As expected pinch-off occurs further downstream as $Re$ is increased while larger frequencies result in smaller $x_{po}$.
Non-dimensionalisation of pinch-off time, \( t^* = t_{po} U_p / D \), enables all the data sets to be gathered into a single plot where change of pinch-off distance with non-dimensional pinch-off time can be examined. This is shown in Fig. 10. This graph reveals a significant result as it shows that the \( x_{po} \) and \( t^* \) has a linear relationship in the whole range of data presented. Moreover, the slope of the line fitted to the data, which can also be considered as the non-dimensional average vortex ring velocity until the pinch off incidence, is approximately 0.50. This shows that the pinch-off velocity of the vortex rings is \( \sim 0.5 U \) and is consistent with both the most unstable growth rates and disturbance velocity of a jet from linear theory (Drazin 2002).

![Fig.9 Velocity, frequency and amplitude dependence of (a) pinch off time (b) pinch off distance](image)

![Fig.10 Change of non-dimensional pinch off distance with pinch off time](image)

4. Conclusion

In this study, investigation of periodic formation of vortex rings in the development region of a round jet subjected to high levels of acoustic forcing has been performed for a wide range of experimental conditions. Harmonic velocity oscillations of 20 to 130% of the mean flow at the jet exit were achieved for several forcing frequencies determined by the acoustic response of the system. High Speed Particle Image Velocimetry (HSPIV) was shown to be capable of providing required data which was used to characterize the flow field and the vortex ring structures. Time-resolved history of vortex ring evolution enabled the evaluation of circulation, roll-up time and pinch-off location of the vortex rings.
During a period of forcing it was observed that a single vortex ring forms and a trailing jet region followed indicating the presence of an optimal ring. This behaviour is similar to what was reported by Gharib et al. (1998) for the formation of a single ring in starting jets with large $L/D$'s. Although vortex ring size linearly increases with $A$, a limitation was observed with respect to the total circulation. This limitation is specific to the periodic nature of the exit velocity and regardless of $A$. The intersections of the vortex ring circulation lines with their corresponding total circulation curves occur at $t/T \approx 0.30$. The universal characteristics of this time step in the range of experimental conditions investigated leads to a universal formation number for the vortex ring formation in forced jets.

Comparison of the flow fields at different forcing amplitudes with unforced jet flow shows how the jet shear layer is periodically broken down into controlled large-scale coherent structures. As $A$ was increased, the location where the vortex ring started to roll-up moved progressively upstream towards the nozzle exit.

Finally, the length- and time-scales of the formation process based on the completion and pinch off of the vortex ring were shown. Pinch-off distances and times were determined for each experimental condition. Increasing $A$ results in decrease in the pinch off time while pinch off occurs later at larger frequencies. Pinch-off distance is almost constant with $A$, except the $A$ values smaller than 0.50. Non-dimensional pinch-off time collapses all the data showing that the $x_{po}$ and $t^*$ have a linear relationship where the slope, which can also be considered as the non-dimensional average vortex ring velocity until the pinch off incidence, is approximately 0.50.

5. References


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