Estimation of 3D deformation and rotation rate tensor from volumetric particle data via 3D least squares matching

Jens Kitzhofer¹, Patrick Westfeld², Oliver Pust³, Thomas Nonn³, Hans G. Maas², Christoph Brücker¹

1: Institute of Mechanics and Fluid Dynamics, University of Freiberg, Germany,
{jens.kitzhofer,christoph.bruecker}@imfd.tu-freiberg.de
2: Institute of Photogrammetry and Remote Sensing, University of Dresden, Germany,
{patrick.westfeld,hans-gerd.maas}@tu-dresden.de
3: Dantec Dynamics A/S, Skovlunde, Denmark, {oliver.pust,Thomas.nonn}@dantecdynamic.com

Keywords: Least Squares Matching, Volumetric post processing, Tomographic PIV

In the last decades several investigations have been performed to experimentally describe the motion of fluid. The fundamental theorem by Helmholtz says that every infinitesimal motion of a fluid element can be decomposed in translation, rotation and deformation. Initially, the measurement technique Particle Image Velocimetry (PIV) was used for 2D investigations yielding a planar field with translational velocities in the Eulerian frame. Thus, only one part of the fundamental theorem could be described in 2D.

Developments like Scanning PIV, Holographic PIV and Tomographic PIV expanded the description of fluid motion to 3D. Here, correlation based techniques, like 3D cross correlation, are frequently applied in the post processing on gray value voxel spaces to extract the zero order translational velocity components neglecting the higher order terms of rotation and deformation.

The assumption is that the flow field is smooth and not significantly influenced by rotational or shear displacements, thus yielding the zero-order translational displacement field with an additional uncertainty in measurement due to neglecting the higher-order terms. Reduction of the measurement uncertainty can be achieved by window deformation techniques. The higher-order motion terms are then estimated by finite difference schemes of the velocity field information on discrete grids (indirectly by consideration of the translational velocities of neighbouring elements). The assumption is that the higher order fluid motion of an element is only affected by the translational velocity components of the neighbouring elements.

Another approach for estimation of velocity fields is Least Squares Matching (LSM). In contrast to correlation based techniques, Least Squares Matching (LSM) shifts, rotates and stretches a fluid area. For this purpose, the least squares matching algorithm iteratively compares gray value information of an interrogation area in the first time step with the gray value information in the second time step. This is an iterative least squares procedure applying an affine transformation on the interrogation areas.

In 2D this results in six transformation parameters and in 3D this results in twelve transformation parameters for each interrogation area. This article discusses the estimation of the deformation rate and rotation rate tensor from results obtained by 3D Least Squares Matching (LSM). It is shown that LSM already yields the velocity gradient matrix for computation of the deformation and rotation rate tensor. Vorticity, shear and strain are calculated without applying central differences schemes. For this purpose, this article first discusses the necessary principles (fundamental theorem of fluid motion, Least Squares Matching) and the connection of both principals to calculate the deformation rate and rotation rate tensor via the results obtained by LSM.

A parametric study is performed on a single volumetric cube to show the accuracy and the constraints of the estimation of deformation and rotation rate. A simulated Hill type vortex shows the strength of the method for identifying regions in the flow of high fluid mechanical interest (figure 1). At the end we will show results for the benchmark experiment of a vortex ring.

Fig 1 Visualization of some results obtained by LSM applied to the simulated hill type vortex ring; isosurface of vorticity magnitude (blue) identifies the donut like vortex core, isosurface of positive (red) and negative (blue) strain rates identify stagnation points; 3D streamlines show a spherical shape and the velocity vectors in the symmetry plane y-z characterize the velocity field of the hill type vortex