Efficient estimation of burst-mode LDA power spectra

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The estimation of power spectra from LDA data provides signal processing challenges for fluid dynamicists for several reasons: acquisition is dictated by randomly arriving particles, the measured velocities are biased towards higher values and the signal is highly intermittent. The signal can be interpreted correctly by applying residence time weighting to all statistics and using the DFT to compute the Fourier transform. The algorithm is applied to two experiments; one with high data density and one with relatively low. These are compared to corresponding sample-and-hold and hot-wire spectra.

1. Residence time weighting of the statistics

Considering the particle motions in Lagrangian space, the velocity as sampled by the LDA can be defined by [1,3,4]

\[ u_0(t) = \iiint_{all space} u(\hat{a}, t)g(\hat{a})w(\hat{x}[\hat{a}, t])d^3\hat{a} \]

where \( u(\hat{a}, t) \) is the velocity of the particle with initial position \( \hat{a} \). \( g(\hat{a}) \) is a sampling function that describes whether a particle is present or not at position \( \hat{a} \) at the arbitrarily chosen instant. The positions of the particles are given by the displacement field \( \hat{x}[^{2}\hat{a}, t] \). \( w(\hat{x}[\hat{a}, t]) \) is a weighting function that accounts for the finite extent of the measuring volume, and effectively ‘turns on’ when the particle enters the volume and ‘turns off’ when it leaves.

\[ S(f) = \frac{T}{(\sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{-2\pi j f(t_m-t_n)} u_m u_n \Delta t_n \Delta t_n})} \]

where \( \Delta t \) are the residence times and \( T \) is the record length. Note that the frequencies can be chosen arbitrarily.

Figures 2 and 3 show these uncorrected and noise corrected burst-mode LDA spectra along with the hot-wire and sample-and-hold spectra for an axisymmetric jet and a cylinder wake, respectively. The LDA and HWA measurements of the cylinder wake were acquired at slightly different Reynolds numbers. The peaks do not line up in the wavenumber spectrum since the disturbance is temporal, not convected [4]. The hot-wire spectrum and burst-mode algorithm compare favorably for both experiments, but the sample-and-hold are very different for the low data density.

2. Practical spectral estimator

Through a series of arguments, the spectral estimator is given by:

\[ F_1(\omega \Delta t) = \frac{1}{T} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} e^{-i \omega (t_m-t_n)} u_m u_n \Delta t_n \Delta t_n \]

where the frequencies can be chosen arbitrarily.

Fig. 2 Comparison of burst-mode LDA, HWA and S/H spectra for an axisymmetric wake.

Fig. 3 Comparison of burst-mode LDA, HWA and S/H spectra for a cylinder wake.