Computed tomographic X-ray velocimetry for simultaneous measurement of 3D velocity and object geometry in opaque vessels

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Abstract  Computed tomographic X-ray velocimetry (CTXV) has been developed for simultaneous measurement of three-dimensional flow and vessel geometry. The technique uses cross-correlation functions calculated from X-ray projection image pairs acquired from multiple viewing angles to tomographically reconstruct the flow through opaque objects with high resolution. The reconstruction is performed using an iterative, least squares approach. The simultaneous measurement of the object’s structure is performed with a limited projection tomography method. An extensive parametric study using Monte Carlo simulation reveals accurate measurements with as few as 3 projection angles, and a minimum required scan angle of only 30°. Synchrotron experiments are conducted to demonstrate the simultaneous measurement of structure and flow in a complex geometry with strong three-dimensionality. The technique will find applications in biological flow measurement, and also in engineering applications where optical access is limited, such as in mineral processing.

1. Introduction

Many biological flows exhibit strongly three-dimensional flow characteristics. This is especially true of the cardiovascular system. Hemodynamic properties, particularly shear, are the primary driving factor in atherosclerosis (Nesbitt et al. 2009), and have been shown to affect the development of the heart and arterial endothelium (Hove et al. 2003). The common sites for atherosclerosis, including the carotid bifurcation and the aortic arch, are those which exhibit complex geometries and flow structures. It follows that a flow measurement technique capable of measuring blood flow fields in 3D at high resolution would be a valuable tool for research in biological fluid mechanics. Furthermore, the precise mechanical, chemical and physiological conditions in the body are difficult to replicate in vitro, therefore the ability to measure blood flow in vivo is necessary to study the effects of hemodynamic properties on atherosclerosis and cardiac development. This capability also provides a pathway for new techniques of diagnosis and treatment for cardiovascular disease.

Particle image velocimetry (PIV) is an established technique for non-invasive flow field measurement, in which a particle laden fluid is illuminated using a visible wavelength laser, and image pairs are analysed for particle displacement using cross-correlation techniques (Adrian 2005). PIV is favored over other techniques as it provides very high resolution and accuracy. Variants exist for 3D measurement, such as μPIV (Hove et al. 2003, Poelma et al. 2010), Tomographic PIV (Elsinga et al. 2006) and Holographic PIV (Barnhart et al. 1994). These techniques have been applied to in vitro models of the cardiovascular system to characterize and study biological flows (Nesbitt et al. 2009, Vetel et al. 2009), however the requirement of optical access has limited in vivo experiments to either thin walled or transparent animal models (Hove et al. 2003, Poelma et al. 2010). Several groups have attempted to address this limitation by applying PIV analysis techniques to imaging modes capable of seeing through living tissue, most notably Ultrasound and X-ray imaging.
Ultrasound imaging has been combined with PIV, termed echocardiographic PIV (Echo PIV) for non-invasive measurement of cardiac flows (Kheradvar et al. 2010, Kim et al. 2004, Niu et al. 2010). This technology has advanced to application in a clinical setting, however those studies show severe limitations in both spatial and temporal resolution. In fact, when comparing measurements using Echo PIV with those from laser based digital PIV in vitro, it was found that Echo PIV was unable to resolve small-scale features of ventricular flow, and also underestimated high velocity values, resulting in only a qualitative measurement of the flow-field (Kheradvar et al. 2010). Echo PIV has also been used to measure the flow in a rat carotid artery (Niu et al. 2010). Unfortunately, ultrasonic imaging is inherently limited by a trade-off between penetration and spatial resolution. The depth of penetration of an ultrasonic wave is inversely proportional to its wavelength, however the spatial resolution achieved is directly proportional to its wavelength. This trade-off limits high-resolution ultrasound imaging to tissue at a depth on the order of millimeters (Niu et al. 2010) and has resulted in a spatial resolution for velocity measurements of no better than 20mm in clinical Echo PIV applications (Kheradvar et al. 2010).

PIV has also been combined with synchrotron X-ray imaging to produce high-resolution measurement of flows within opaque vessels (Dubsky et al. 2010, Fouras et al. 2007, Im et al. 2007, Irvine et al. 2008, Irvine et al. 2010, Kim and Lee 2006, Lee and Kim 2003). The coherence and brightness of Synchrotron radiation sources have allowed phase contrast X-ray imaging (PCXI) to achieve ultra-fast, high-resolution imaging at higher contrast than is possible with typical absorption based imaging (Fouras et al. 2009). Similar in nature to in-line holographic imaging, propagation based PCXI exploits the slight refraction of X-rays that occurs at the interfaces between materials. By allowing the X-rays to propagate some distance past the sample prior to detection, the transmitted and refracted rays generate interference patterns, resulting in a phase contrast image in which weakly absorbing materials can be distinguished (Fouras et al. 2009, Wilkins et al. 1996). Recent advances have been made imaging and tracer particle technology specific to X-ray PIV applications.

X-ray PIV was first applied in vitro using traditional two-dimensional PIV analysis techniques (Kim and Lee 2006, Lee and Kim 2003). It was then shown that due to the volumetric nature of the illumination, 3D velocimetric information is encoded into the image pair cross-correlation functions (Fouras et al. 2007). This information has been exploited in single projection techniques, along with
certain assumptions, to yield three-dimensional measurement of flows within certain symmetry constraints (Fouras et al. 2007, Irvine et al. 2008, Irvine et al. 2010). The authors have recently extended these methods for general three-dimensional velocity measurement, using tomographic reconstruction based on multiple projections. This technique, Computed Tomographic X-ray Velocimetry (Dubsky et al. 2010), was applied to in vitro blood flow, yielding high-resolution 3D velocity measurement in opaque vessels. As multiple projection-images are taken, tomographic reconstruction of the vessel structure is also possible.

In this paper we describe techniques to increase computational efficiency and robustness of CTXV, as well as the addition of a limited projection tomography method for reconstruction of vessel geometry. We have also carried out an evaluation of the effect of imaging parameters on the accuracy of velocity reconstructions, using an extensive Monte Carlo simulation. Results from in vitro experiments are also presented, demonstrating the application of CTXV on flows with strong three-dimensionality and complex geometry.

2. Description of Computed Tomographic X-ray Velocimetry technique

The imaging setup for CTXV is shown in Figure 1. The monochromatic beam passes through the particle-seeded fluid. X-rays are slightly refracted at the interfaces between materials. The transmitted and refracted rays are allowed to propagate and interfere before being converted into visible light by the scintillator. This is then imaged using a high-speed detector and visible light optics, resulting in a phase contrast projection image. The image results from the superposition of interference fringes generated by the particle liquid interfaces, creating a dynamic speckle pattern that faithfully follows the particles (Fouras et al. 2007, Irvine et al. 2008, Irvine et al. 2010).

![Figure 2. Example of image preprocessing for CTXV. Raw phase contrast image of hollow glass spheres in glycerine (left). Average image subtraction removes stationary structures from the images (middle). Phase retrieval removes effects of interference fringes (right) and prepares images for cross-correlation analysis.](image)

Figure 2 shows a raw phase contrast image of hollow glass spheres in glycerine. The detector used to acquire this image utilised an X-ray scintillator that is fibre-optically coupled to a CCD detector, resulting in the honeycomb structures seen in the image. Horizontally oriented streaks are also apparent, which result from the crystal mono-chromator. Average image subtraction removes these stationary structures from the image, along with any effects due to inhomogenous illumination, or dust on the optics, leaving only the particle interference fringes. The images are then analysed using a phase retrieval algorithm, which is used to recover the projected thickness of the sample from the interference fringes, preparing the images for cross-correlation analysis (Irvine et al. 2008).
Unlike visible light based imaging systems, in which images contain focus or holographic depth information from which velocity can be inferred (Barnhart et al. 1994, Fouras et al. 2009, Willert and Gharib 1992), the nature of PCXI results in a two dimensional volumetric projection image in which the entire volume is in focus, and therefore contains no information of the distribution of velocity in the plane parallel to the X-ray beam. Furthermore, from any single viewing angle only two components of displacement can be determined. This information deficit is overcome by rotating the sample and imaging from multiple projection angles, allowing tomographic reconstruction of the velocity field within the volume. As multiple projections are gathered, simultaneous tomographic reconstruction of the object structure is also possible.

2.1 Forward Projection

As in traditional PIV, particle image pairs are discretised into interrogation regions and cross-correlation is performed on these regions. However, due to the large velocity distribution within the projected interrogation region, the cross-correlation functions will be highly distorted. The resulting projected cross-correlation statistics can be modeled as the velocity probability density function (PDF) of the flow projected onto that sub-region of the image, convolved with the particle image auto-correlation function (Fouras et al. 2007, Westerweel 2008). Therefore, if the flow field and particle image autocorrelation function is known, the cross-correlation functions that would theoretically result from the flow field can be estimated. This represents the forward projection model (see Figure 3). CTXV provides a solution for the inverse problem of de-convolving the measured cross-correlation function and the particle image autocorrelation function to yield the velocity PDF, and back-projecting this to yield the flow-field.

Fouras et al. 2007 demonstrated the effect of a finite exposure time on the cross-correlation function of projection-image pairs. Due to motion of the particle during the exposure, the contribution of each velocity to the cross-correlation function will be stretched along the direction of that velocity, with a magnitude that is linearly proportional to that velocity. As this effect has been well characterized, it can be taken into account in the forward projection model to eliminate any errors due to this phenomenon.

**Figure 3.** Schematic of the forward projection model. Cross-correlation functions are estimated by convolution of the velocity PDF, projected from the flow model, with the auto-correlation function calculated from the projection images.
2.2 Solution to the Inverse Problem

Figure 4 demonstrates the implementation of CTXV. The velocity field is reconstructed in slices orthogonal to the axis of rotation, concurrent with the rows of interrogation regions within the projection images. A rectangular grid model represents the flow-field in the reconstruction domain. The three velocity components are defined at each node point in the model and bi-linear interpolation is used between grid points to define the flow in between node points. Higher degree interpolation schemes may be used, such as spline interpolation, at the expense of computation time and robustness.

Cross-correlation functions are estimated using the method shown in Figure 3. The convolution is effected through an FFT implementation. A Levenberg-Marquardt algorithm is utilised to minimize the error between the cross-correlation functions estimated from the flow model and those calculated from the projection image pairs, resulting in a flow model which accurately represents the flow-field. As the problem is heavily over-specified, a Tikhonov-type regularization scheme is used to ensure convergence of the reconstruction, where the regularization function is equal to the sum of the difference between each node velocity value and the mean of its neighbors.

2.3 One-dimensionalisation of the Cross-Correlation

In order to reduce the number of optimization parameters and memory required for the reconstructions, a one-dimensionalisation of the cross-correlation functions is performed, allowing separate reconstruction of the data for \( v_r \) and \( v_q \). Projection of the cross-correlation data results in

\[ \text{Figure 5. One-dimensionalisation of the cross-correlation functions. Integrating across the rows and columns in the 2D cross-correlation function yields a 1D representation of the velocity PDF in the r and q directions, respectively.} \]
two, one-dimensional representations of the function each of the velocity components, $v_q$ and $v_r$, as illustrated in Figure 5. By separating the two components they can be reconstructed individually, greatly reducing the number of optimization parameters required per reconstruction. Furthermore, the process significantly reduces the amount of data that needs to be stored and analysed.

3. Simultaneous Structure Reconstruction

To correctly model the forward projection of the velocity PDF, the relative particle seeding density within the reconstruction domain must be known. Assuming a homogenous seeding density within the working fluid, this corresponds to knowledge of the sample geometry. There are a number of limited projection CT reconstruction techniques that may be used to reconstruct the object geometry from projection images (Myers et al. 2008, Soussen and Mohammad-Djafari 2004). We propose here a limited projection CT technique that allows the vessel geometry to be reconstructed using the data obtained during the CTXV scan.

In typical CT reconstruction techniques, integrated object density in the projection direction is calculated from the X-ray attenuation, which will be proportional to pixel intensity values on a digital projection image. In the case of a material of constant density, this integrated object density will be proportional to the object thickness.

The contrast of the particle speckle (defined as the ratio of the intensity standard deviation to mean) will increase with the square root of object thickness (Berry and Gibbs 1970), and so this data may be also used for tomographic reconstruction of the object’s structure. This is advantageous, as in many cases, including in vivo imaging of blood vessels, the absorption contrast is insufficient for tomographic reconstruction of the object structure.

The object is reconstructed using a polygon reconstruction method based on the CT technique described in Soussen et al. (Soussen and Mohammad-Djafari 2004), here implemented with a Levenberg-Marquardt optimization. The technique performs a non-linear optimization of a polygon representing the object boundary, minimizing the residual of the projected thickness of the object with the speckle contrast data obtained from the projection images, normalized to the maximum thickness. Regularisation is included to ensure convergence to a smooth reconstruction, as described by Soussen et al. (Soussen and Mohammad-Djafari 2004).

![Flow-fields used for synthetic image generation. Surface and colours indicate $v_z$ component, vectors show the $v_x$ and $v_y$ components. a) Axisymmetric flow-field, with parabolic $v_z$ profile and diverging $v_x$ and $v_y$ velocities. b) Asymmetric flow-field, reversing $v_z$ profile with mono-directional cross-flow.](image-url)
4. Parametric Study

An extensive Monte Carlo simulation was performed to elucidate the effects of the imaging parameters on CTXV reconstruction accuracy. The relevant imaging parameters for CTXV are the number of projections acquired ($n_{proj}$), the number of correlation averages performed at each projection angle ($n_{ave}$), the angles of projection and the particle seeding density. The minimization of $n_{proj}$ and $n_{ave}$ is important to reduce scan time, X-ray dose, computational expense, and in the case of a multiple source/detector configuration, equipment cost. The particle seeding density and angles of projection may be limited by the specific experiment and so knowledge of the effects of these parameters on reconstruction accuracy is also useful.

Synthetic projection image pairs were generated using randomly positioned particles. Reconstructions were performed with varying $n_{ave}$ and $n_{proj}$, and the total scan angle. Each data point in the results represents the statistics of 128 individual reconstructions. The root mean square (RMS) error is calculated as,

$$RMS = \sqrt{\frac{\sum_{i=1}^{N} (E_x^2 + E_y^2 + E_z^2)}{3N}},$$

where $E_x$, $E_y$, and $E_z$ are the absolute error in $v_x$, $v_y$, and $v_z$ respectively, and $N$ is the total number of vectors used in the ensemble. Reconstructions that exhibit an RMS error of less than 1px are deemed acceptable. Two flow cases were studied, an axi-symmetric flow and an asymmetric flow (see Figure 6). The axisymmetric flow is made up of a parabolic $v_z$ profile with a diverging $v_x$ and $v_y$ flow. The asymmetric case exhibits a reversing $v_z$ profile with mono-directional cross-flow.

Figure 7 shows the combined RMS error, plotted against both $n_{ave}$ and effective seeding density. Here, effective seeding density is defined as the seeding density for a single image multiplied by $n_{ave}$. It has been shown that correlation averaging will have an equivalent effect on the quality of the cross-correlation function as an increase in seeding density (Westerweel et al. 2004). This metric can be seen as a measure of image quality. Projections were equally spaced over 180°. As expected, an increase in $n_{ave}$ gives a reduction in error. Importantly, acceptable results are obtained with as few as 3 projections. The RMS error continues to reduce as effective seeding density increases past 1 particles per pixel (ppp). Due to the volumetric nature of PCXI, these high seeding densities can be easily obtained in X-ray PIV experiments.

![Graph showing RMS Error vs. $n_{ave}$ for axi-symmetric case and asymmetric case](image)

**Figure 7.** RMS Error vs. $n_{ave}$ for a) axi-symmetric case and b) asymmetric case, for (■) $n_{proj} = 3$, (▲) $n_{proj} = 6$, (●) $n_{proj} = 9$, and (●) $n_{proj} = 18$. Results show acceptable accuracy using as few as 3 projections, with high effective seeding density.
Figure 8 shows the same data, plotted against effective dose, which is defined as the number of image pairs acquired over the total scan (that is, $n_{\text{proj}} \times n_{\text{ave}}$). The data converge onto a single curve, indicating that the reconstruction accuracy is proportional to the effective dose, and that this relationship is largely independent of the number of projections. This is an important result as it

![Graph](image1)

(a) Axisymmetric case

![Graph](image2)

(b) Asymmetric case

**Figure 8.** RMS Error vs. Effective dose ($n_{\text{ave}} \times n_{\text{proj}}$) for a) axisymmetric and b) non-axisymmetric flowfields, for (■) $n_{\text{proj}} = 3$, (▲) $n_{\text{proj}} = 6$, (●) $n_{\text{proj}} = 9$, and (◇) $n_{\text{proj}} = 18$. Data collapse onto a single curve for all numbers of projections, indicating that the error is proportional to effective dose, and independent of the number of projections.

![Graph](image3)

(a) 3 Projections

![Graph](image4)

(b) 6 Projections

**Figure 9.** Total scan angle vs. RMS error for the axisymmetric test case for a) $n_{\text{proj}} = 3$, and b) $n_{\text{proj}} = 6$, $n_{\text{ave}} = 128$. Results indicate that an equal spacing over 180° is optimal (120° for 3 projections, and 150° for 6 projections).
guides the experimentalist in the choice of imaging parameters. For example, if fewer projections are available, one must increase image quality, number of correlation averages, or seeding density, or conversely, the addition of more projections may be used to overcome poor image quality. The minimum RMS error achieved for the axisymmetric case is lower than for the asymmetric case, indicating that the complexity of the flow-field has an effect on the accuracy achieved.

Figure 9 shows the effect of the total scan angle on the RMS error for 3 and 6 projections. To eliminate any effects of directionality in the flow-field, this study was performed only on the axisymmetric flow case, however the trends will be qualitatively valid for general flow fields. An optimal scan angle of 120° was found for 3 projections and 150° for 6 projections, representing to equal spacing over 180° (exclusive). This is not surprising, as projections spaced 180° apart are equivalent, and therefore the inclusion of a 180° projection is redundant. It should also be noted that the minimum scan angle for acceptable results to be obtained is ~30°, which may be important where access to the sample is limited.

5. Experimental Application

Experiments were performed to demonstrate the application of CTXV to the simultaneous measurement of structure and velocity. Experiments were conducted at the Spring-8 Synchrotron, Hyogo Japan, on the medical imaging beam-line BL20B2.

The sample used was an opaque plastic vascular model, with a complex three-dimensional geometry, manufactured using Objet™ 3D-printing technology. The geometry was constructed as the union of a cone and a helically swept, vertically oriented circle, resulting in corkscrew geometry, with a decreasing cross-sectional area. The geometry was chosen to exhibit a strongly three-dimensional flow. The working fluid: glycerine seeded with 35µm (nominal) solid glass spheres (Spheriglass A, Potters Inc.); was pumped through the model at 0.1ml/mn using a syringe pump (Harvard Apparatus, PHD 22/2000). The propagation distance, defined as the distance from the front face of the object to the scintillator, was optimised for signal to noise ratio of the glycerin/glass mixture at 6m.

The BL20B2 beamline uses a bending magnet insertion device. An X-ray energy of 25KeV was selected using an Si-111 monochromator. A fast X-ray shutter was used to minimise sample dose, and also to protect the P43 scintillator from the high flux X-ray beam. An EMCCD detector (Hamamatsu) was used for its sensitivity and low-noise characteristics. The optics used resulted in an effective pixel size of 9.5 x 9.5 µm², allowing a field of view of 9.5mm x 9.5mm. Images were acquired at 19 angles, evenly spaced over 180° (inclusive). The 180° projection was included to allow the calculation of the centre of rotation of the sample, however this may be excluded in place of a simple calibration/alignment process. The detector acquired images at 28.5fps with an exposure time of 30ms. Ensemble averaging was used with 99 image pairs averaged at each angle to produce the correlation data. The object structure was reconstructed using the method described in Section 3. For the velocity reconstruction, cross-correlation functions were calculated using 64×64 pixel interrogation windows with 75% overlap. The 69 axial slices were individually reconstructed on a rectangular grid of approximately 300 node points, depending on the size of the object within each slice, interpolated onto a 128×128 voxel sub-grid. The resulting structure and velocity fields are shown in Figure 10. As expected the flow follows the helical geometry, increasing in speed as the vessel constricts through the cone section. The results illustrate the ability of the technique to accurately measure complex 3D flows, even with few projections.

To demonstrate the statistical convergence in the correlation ensemble, the number of image pairs used to generate the correlation data at each projection (nave) was varied from 1 to 75. These reconstructions were then compared to those using the full set of 99 image pairs (Figure 11). The trend shown in the Monte-Carlo simulation is confirmed by the results (Figures 7 & 8). The results
Figure 10. CTXV reconstruction of flow through helical geometry. Colours represent velocity magnitude. Upper: Structure reconstruction shown in translucent surface. One quarter of the velocity reconstruction is shown. Vector resolution reduced by 4× in x, y, and z. Lower: Velocity profiles in isolated x-z and y-z planes, plotted with reduced vector resolution to aid visualization. The results illustrate the ability of CTXV to simultaneously measure the 3D structure and velocity of flow through complex geometries.
show a small decrease in the RMS error of the reconstructions for an increase in $n_{\text{ave}}$ of 50 to 75, when compared with the reconstructions using 99 averages, indicating an adequate number of image pairs in the analysis. The result also gives an indication of the uncertainty of the reconstructions, which is shown to be approximately 0.25px RMS.

![Figure 11. RMS Error vs $n_{\text{ave}}$ for experimental data. RMS Error is calculated by comparison with reconstructions using $n_{\text{ave}} = 99$.](image)

### 6. Summary and Conclusions

We have developed and applied the CTXV technique for simultaneous velocity and structure measurement for flow in opaque vessels. Through Monte Carlo simulation, it was demonstrated that accurate reconstructions can be obtained with as few as 3 projections and as little as 30° scan angle, provided good image quality is achieved. It was shown that accuracy is determined by the effective dose ($n_{\text{ave}} \times n_{\text{proj}}$), and that this relationship is largely independent of the number of projections acquired. Accurate results were demonstrated in a complex flow with strong three-dimensionality in both structure and velocity.

The few projections required for accurate velocity reconstruction opens the possibility of instantaneous measurement using a multiple source/detector experimental setup. Phase-locked reconstructions of periodic flows are also possible using a single source/detector configuration. With recent developments in ultrafast synchrotron imaging (Lee et al. 2010, Wang et al. 2008) and X-ray PIV tracer particles, the ability to image flows at physiological rates will allow application of this technique to in vivo measurement of blood flow. Engineering applications may also be found where optical access is limited, for example in the mineral processing or automotive industries. With little adaptation the technique may also be used in conjunction with imaging modes other than PCXI such as visible laser light or infrared imaging.

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8. References


