Adaptive MLOS-SMART improved accuracy tomographic PIV

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Abstract To improve the accuracy of tomographic particle image Velocimetry (Tomographic PIV) an adaptive reconstruction method has been implemented based on the accelerated MLOS-SMART reconstruction technique. Adaptive MLOS-SMART (AMLOS-SMART) attempts to reduce the presence of ghost particles and reconstruction noise by iteratively adapting the reconstructed intensity fields based on an estimate of the velocity field, retaining only those particles that are present across consecutive exposures. Non-coherent intensity peaks are removed from the solution, which reduces the influence of ghost particles and their associated contamination of the velocity field. This paper describes the working principles of this method with a performance assessment based on the use of simulated images, including the influence of image noise and calibration error.

1. Introduction

Tomographic PIV (Tomographic PIV) (Elsinga et al. 2006) is built around the use of the multiplicative algebraic reconstruction technique (MART) (Herman and Lent, 1976) and has become a popular, albeit computationally intensive, method for measuring three-component three-dimensional (3C-3D) velocity fields. Algebraic techniques such as MART are favored for reconstructing 3D particle intensity distributions from multiple simultaneous 2D images, owing to their ability to handle a small number of arbitrary non-linear distorted views and multiple point-source intensity fields. This process of reconstructing particle locations from limited two-dimensional views requires the solution of a highly ill posed system of equations with multiple possible solutions and particle locations (Michael and Yang, 1991). Consequently a broad range of algebraic reconstructions techniques (Atkinson and Soria, 2007) have all been shown to produce fields consisting of a combination of both true and erroneous ‘ghost’ particles, with a division of recorded intensity across all possible particle locations. Ghosts particles are often more numerous than true particles and as such pose a significant, thought not insurmountable, challenge to particle tracking (Schröder et al. 2008). When cross-correlation is used to determine particle displacements, these ghosts contribute noise to the correlation peak and in some cases can also contaminate neighboring vectors and smooth velocity gradients (Elsinga et al. 2009; Atkinson et al. 2009, Atkinson et al. 2010).

A method to reduce the intensity and influence of these ghost particles was recently presented by Novara et al. (2010) using the information contained in subsequent exposures of a particle intensity field during the reconstruction process. This involves performing the standard 5 MART iterations to reconstruct two intensity fields then cross-correlating them to determine an estimate of the displacement between each field. These intensity fields are then deformed on to each other and averaged in order to reduce the intensity of particles that don’t align with themselves in the previous exposure. Results indicated improvement in both the reconstruction quality and the resulting velocity fields, yet involved the use of up to 50 MART iterations. This requires an order of magnitude increase in computational time, with the reconstruction of a single 1000×1000×200 pixel volume object requiring an estimated 10 hours on a single processor, not including the multiple correlation and volume deformation steps.
Fortunately similar order of magnitude improvements in reconstruction speeds can be achieved when a Multiplied First Guess (MFG) (Worth and Nickels, 2008) or a Multiplied Line Of Sight (MLOS) (Atkinson and Soria, 2009) initial solution is used. In these cases the iterative algebraic reconstruction process is limited to a series of pre-identified potential particle locations, hence reducing the number of calculations required. The memory requirements can also be reduced if the reconstruction is performed using a simultaneous MART implementation as described by Atkinson and Soria (2009), in the form of the MLOS-SMART reconstruction. 10 iterations of MLOS-SMART have been shown to produce similar result to the 5 MART iterations, yet require just over 10 minutes for a 1000x1000x200 pixel volume.

In this paper we present a method to adapt the MLOS technique to improve reconstruction accuracy by using the estimated displacement field to reduce the intensity and the number of ghost particles. The performance of this Adaptive MLOS-SMART (AMLOS-SMART) is evaluated using a series of simulated particle fields and images. The effects of seeding density, image noise and calibration errors are considered.

2. AMLOS-SMART Method

The AMLOS-SMART approach (see Fig. 1) involves dividing the measurement domain into a series of sub-volumes for each exposure or snapshot. In this case sub-volumes will be considered consecutively by a single processor but could easily be distributed between multiple parallel nodes. Multiplied line of sight (MLOS) estimations of all possible particle locations are then independently performed for each sub-volume. Projected images \( \sum_{i} W_{i} I_{n}^{k} \) are calculated for each camera, based on the estimated particle locations and intensities \( I_{n}^{k} \) in all sub-volumes and the weighted contribution of each possible particle location \( (n) \) for every \( k \)th iteration. The ratio of the recorded \( (P_{i}) \) and projected images is then used to establish a correction for the intensity \( I_{j} \) at each particle location \( (j) \) using a simultaneous multiplicative algebraic reconstruction technique (SMART) as outlined in equation 1, Where \( N_{i} \) is the number of camera pixels that observe a given potential particle location and \( \mu \) is a relaxation parameter set to unity. For further details of the MLOS-SMART implementation see Atkinson and Soria (2009).

\[
I_{j}^{k+1} = I_{j}^{k} \prod_{i}^{N_{i}} \left[ \frac{P_{i}}{\sum_{n} W_{i} I_{n}^{k}} \right]^{\mu} \quad (1)
\]

The accuracy of this reconstruction can then be improved in a manner analogous to that described by Novara et al. (2010). The average displacement of each particle in a sub-volume is determined by cross-correlation of sub-volume A with an associated sub-volume B, representing a reconstruction from a subsequent image. Where necessary correlation is performed on a larger interrogation volume, which is assembled from possible particle locations in neighboring sub-volumes. The displacement of this sub-volume is validated using a maximum displacement limit and a normalized mean vector validation (Westerweel and Scarano, 2005). Alternate peaks and linear interpolation are used to replace invalid vectors before smoothing the velocity field. A second correlation pass with a sub-pixel shift is applied in an attempt further improve the estimated displacement of the sub-volume. The intensity of each potential particle is then adapted based on whether or not it has a corresponding shifted particle in the opposite exposure.
Two methods for adapting the particle intensities are considered. In both cases the estimated displacement for each particle in the sub-volume is linearly interpolated based on the estimated displacements of neighboring sub-volumes. The first method is a direct search, where for each particle in sub-volume A its corresponding shifted location in a region around sub-volume B is checked for the presence of potential particle. A search radius is specified and the region around sub-volume B is included so as not to remove true particles that might pass outside the sub-volume between exposures. If a match is found the intensity of this particle is left unchanged. If no matching particle is found then the intensity of the particle in sub-volume A is reduced via multiplication with correction factor $\kappa$, typically $\kappa = 0.1$ to 0.5 such that if the displacement estimate is incorrect the particle will not be completely removed. This correction is also applied to the particles in sub-volume B, where the search is now performed in sub-volume A.

The second method resembles that used by Novara et al. (2010), where the particle intensity is set to the average intensity of the particle in sub-volume A and the intensity at the equivalent location in sub-volume B. Here rather than deforming the entire volume or sub-volume onto a common grid and averaging the intensity, we instead consider just the particle locations in sub-volume A, determine where they map to in a region around sub-volume B, then interpolate for the intensity at these points based on surrounding zero and potential particle intensities. In this paper we will concentrate on the results for linear interpolation as a faster method to determine the intensity, however interpolation based on the more computationally intensive cardinal function has also been used with only minor differences. This average is also performed in the converse direction for sub-volume B. Adapted intensity fields and particle locations and then fed back into the SMART algorithm for further iterative correction based on the recorded images.

**Fig 1.** Schematic of the AMLOS-SMART process for 2 consecutive volumes A and B
3. AMLOS-SMART Simulations

In order to evaluate the performance of the AMLOS-SMART approach and the use of a direct search or average intensity correction, simulations were performed as two-dimensional 1000x200 pixel reconstructions from 1 dimensional images of 1000 pixels (see Fig. 2). Four cameras were used, located in the same plane as the reconstruction volume at angles of -30, -10, +10 and +30 deg respectively. A sinusoidal longitudinal displacement was applied to the particles through the depth direction, \( z \), with a wavelength of 200 pixels and displacement amplitude of 3 pixels as in the simulations of Novara et al. (2010), such that the flow was homogeneous in the longitudinal direction. A particle image diameter of 3 pixels was used with a peak intensity of 4095 counts corresponding to 12-bit images. Seeding density was varied between 0.05 and 0.4 particle per pixel (ppp), with 0.05 ppp being the optimal seeding density for standard MART Tomo-PIV (Elsinga et al. 2006). Calibration errors were considered by applying a constant shift of \( \varepsilon_{\text{calib}} \) pixels to the horizontal image coordinates of each camera relative to the simulated images, with the directions of each shift indicated in Fig. 2. The effect of image noise was considered by adding Gaussian white noise to each image with a peak intensity set as a percentage of the peak particle intensity.

Reconstructions were performed using 5 MART iterations, 10 or 40 MLOS-SMART iterations, and AMLOS-SMART with intensity adapted every 10 or 40 MLOS-SMART iterations. In practice 10 MLOS-SMART iterations have been shown to produce results within the error margin of the standard 5 MART iterations, however previous simulations have suggested that closer to 40 MLOS-SMART iterations are needed with reconstruction times that are still less than those required by MART. In all cases a relaxation parameter of unity was used.

The quality of these reconstructions were assessed based on the correlation or reconstruction coefficient \( (Q) \) between the simulated \( (I_{\text{sim}}) \) and the reconstructed particles volumes \( (I_{\text{rec}}) \):

\[
Q = \frac{\sum_{j} I_{\text{rec},j} I_{\text{sim},j}}{\left( \sum_{j} I_{\text{rec},j}^2 \sum_{j} I_{\text{sim},j}^2 \right)^{1/2}}
\]  

Comparison of the resulting velocity profiles was based on a two-component two-dimensional (2C-2D) Fast Fourier Transform (FFT) based cross-correlation using 2 passes with an integer window shift, a maximum displacement limit and a normalized median vector validation. Interrogation windows were sized to maintain 5 to 8 particles per window with a 75% overlap as indicated in Table 1.

![Fig 2. Schematic of the simulation camera arrangement and calibration error (\( \varepsilon_{\text{calib}} \))](image-url)
Table 1. Simulation sub-volume and interrogation region dimensions

<table>
<thead>
<tr>
<th>Particle per pixel (ppp)</th>
<th>Sub-volume size (x, y, z)</th>
<th>Interrogation Region (x, y, z)</th>
<th>Particle per window (ppw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>250, 1, 5 pixels</td>
<td>1000, 1, 20 pixels</td>
<td>5</td>
</tr>
<tr>
<td>0.1</td>
<td>200, 1, 5 pixels</td>
<td>800, 1, 20 pixels</td>
<td>8</td>
</tr>
<tr>
<td>0.2</td>
<td>100, 1, 5 pixels</td>
<td>400, 1, 20 pixels</td>
<td>8</td>
</tr>
<tr>
<td>0.3</td>
<td>67, 1, 5 pixels</td>
<td>268, 1, 20 pixels</td>
<td>8</td>
</tr>
<tr>
<td>0.4</td>
<td>50, 1, 5 pixels</td>
<td>200, 1, 20 pixels</td>
<td>8</td>
</tr>
</tbody>
</table>

An example of the reconstruction coefficients against iteration number for a range of reconstruction methods with a seeding density of 0.2 ppp are shown in Fig. 3. The results from MLOS-SMART reconstruction approach those of 5 MART iterations after approximately 40 iterations and remain far more stable than those of MART as the number of iterations is further increased. The sharp rises associated with the AMLOS-SMART reconstructions represent the adaptive intensity corrections occurring after 10 and 40 SMART iterations, respectively. The binary searching methods provide some benefit over the standard MLOS-SMART reconstruction however this effect is far less pronounced than that obtained by the averaging. These particular results refer to a search radius of 1 pixel and a correction factor $\kappa = 0.1$, yet varying these had only a marginal effect on the reconstruction coefficient. Increasing the intensity of a particle after a successful search by dividing by $\kappa$ also provided little benefit. The improvement offered by the averaging is likely associated with the degree to which particle intensities are matched. For instance if a bright particle in volume A is matched to a bright particle in volume B then both are probably true particle locations and both will retain a high intensity after adaptation. If however a weak particle is matched with a strong particle then the search method will retain the relative strength of each particle, whereas an average will approach an equilibrium between the intensity in each exposure as would be expected if the same particle is observed in both exposures.

Fig 3. Effect of iterations and adaptive steps on the standard and adaptive reconstruction methods using averaged or searching intensity corrections. Simulated volume 1000x1x200 pixels, 4 cameras -30, -10, +10, +30 deg, seeding density of 0.2 ppp. Adaptive correction is applied every 10 or every 40 MLOS-SMART iterations.
Fig. 4 shows an example of instantaneous velocity fields produced after cross-correlation of the reconstructed particle intensity fields. In this case 5 MART iterations and 10 MLOS-SMART iterations provide similar velocity fields that follow the general shape of the simulated sinusoidal profile, however show the presence of noise near the far size of the volume. Results are shown for the AMLOS-SMART reconstructions stopped just before the 10th average intensity correction, which correspond to 99 iterations if corrections are performed every 10 iterations or 399 iterations if corrections are performed every 40. Reconstruction is stopped just before correction as beyond 3 corrections the correlation coefficient typically reduces slight after each subsequent correction, before being improved during following iterations (see Fig. 3). In both cases AMLOS-SMART reduces fluctuations in the instantaneous velocity fields and significantly improves the velocity in the z direction.

Fig 4. Velocity fields and contours of longitudinal velocity produced by alterative reconstruction methods for a simulated volume 1000x1x200 pixels, 4 cameras -30, -10, +10, +30 deg, seeding density of 0.2 ppp. (a) MART 5 iterations; (b) MLOS-SMART 10 iterations; (c) AMLOS-SMART just prior to the 10th average adaptive correction, correction every 10 iterations; (d) AMLOS-SMART just prior to the 10th average adaptive correction, correction every 40 iterations.
In order to examine the effect of seeding density on adaptive reconstruction, a series of images were constructed with seeding densities ranging from 0.05 ppp, which is identified as the optimal seeding density for Tomo-PIV, up to 0.4 ppp. In each case the reconstruction and interrogation regions sizes were altered in the homogeneous x direction in an attempt to maintain the same overlap and number of particles in each interrogation region, along with the same grid spacing through the volume thickness (z). From this point when we refer to the AMLOS-SMART reconstruction we will only consider the use of the average intensity correction.

![Fig 5. Reconstruction coefficients for AMLOS-SMART reconstructions using average intensity correction under varying image seeding densities. (a) Correction every 10 iterations; (b) Correction every 40 iterations.](image)

The effect of seeding density is shown in Fig. 5, where increasing the seeding density beyond 0.05 ppp results in a significant decrease in both the initial and final reconstruction coefficient. This can be explained by the presence of ghost particles, whose number and relative intensity is shown to be proportional to seeding density (Elsinga et al. 2006, Atkinson and Soria, 2007). As seeding density increases the number of adaptive corrections required before the solution approaches the tangent also increases, with around 9 corrections leading to roughly the optimal reconstruction coefficient in each case. For this reason we will consider AMLOS-SMART reconstructions just before the 10th correction where the solution is presumed to be at its most accurate. It is possible that in many applications 3 or 4 corrections would be sufficient.

A comparison of the reconstruction coefficients after 9 corrections under the influence of seeding density, calibration error and image noise, show the relative benefits of the AMLOS-SMART method when compared to MART and MLOS-SMART (see Fig. 6). It should be noted that all reconstructions, including MART, were performed using an in-house implementations, however MART results compare favorably with those presented by Elsinga et al. (2006) under similar errors and image noise for 0.05 ppp. In almost all cases AMLOS-SMART improves the reconstruction coefficient however the extent of this improvement is highly dependant on seeding density. In the absence of calibration error and image noise for a seeding density of 0.05 ppp, 9 adaptive corrections increase an already high MART reconstruction coefficient of 0.92 by only 0.04, however as seeding density increase to 0.4 ppp and the MART coefficient decreases to 0.46 the improvement becomes 0.2. Overall the quality is still decreased, yet the benefit of this method becomes significant. For low seeding density the AMLOS-SMART provides only small improvement under the influence of image noise and calibration errors. In all cases adaptive correction after 10 iterations provides a better performance as errors and noise increase, however this improvement is mostly noticeable at higher seeding densities where calibration errors are less than 0.8 pixels and image noise is less than 30% of the peak intensity.
Fig 6. Reconstruction coefficients for MART, MLOS-SMART and AMLOS-SMAR under varying seeding densities, calibration errors and image noise. (a) Calibration error for seeding 0.05 ppp; (b) Calibration error for seeding 0.2 ppp; (c) Image noise for seeding 0.05 ppp; (d) Image noise for seeding 0.2 ppp; (e) Seeding density.

The effect of the AMLOS-SMART methods on the velocity field was examined by comparing the random and bias errors between the imposed and the calculated velocity profiles. Fig. 7 shows the effect of seeding density where, as seen in the reconstruction coefficient, AMLOS-SMART has little benefit for seeding densities of 0.1 ppp or less, where for all methods the random errors and peak bias errors are approximately 0.12 and 0.16 pixels, respectively. At 0.4 ppp the adaptive correction reduces the random errors from 0.88 to 0.20 pixels, however this results in an increase in the mean bias error from -0.02 to 0.1 pixels. In this case this manifests as an underestimation of flow near the side of the volume and an overestimate in the center.
Fig 7. Random and bias errors for MART, MLOS-SMART and AMLOS-SMART methods under varying image seeding densities. (a) Random velocity error in $u$ velocity; (b) Random velocity error in $w$ velocity; (c) Mean bias error across velocity profile; (d) Magnitude of peak bias error.

Following the results of Novara et al. (2010) the use of an adaptive intensity correction is expected to improve the reconstruction quality and reduce the random velocity error associated with Tomo-PIV reconstructions from high seeding density images. One area that has not been examined is the effect of these corrections in the presence of calibration errors and image noise, particularly at standard Tomo-PIV seeding densities of 0.05 ppp, where the benefit of these correction are otherwise less noticeable.

Reconstruction quality ultimately depends on the accurate calibration of each camera in order to properly determine the location of each particle in the volume. Fig. 8 shows the effect of this calibration error on standard MART reconstruction and the adaptive AMLOS-SMART. Even at a seeding density of 0.05 ppp AMLOS-SMART reconstructions do offer an improvement over MART reconstructions for calibration errors greater than 0.4 pixels. For calibrations errors of 1 pixel AMLOS-SMART with adaptive correction every 10 iterations, random errors are reduced from approximately 0.4 to 0.2 pixels with negligible change in the mean bias error. At higher seeding density the relative improvement offered by such corrections increases, similar to that seen in Fig. 7, however the overall accuracy is still reduced by the calibration error. For lower seeding densities the correction offered by this technique is probably not worth the extra computational expense for calibration error less that 0.5 pixels and certainly wont be necessary if self-calibration is able to reduce the calibration error to less than 0.1 pixels as reported by Wienke (2008). However in cases of high image distortion where self-calibration may not be able to achieve such corrections, the extra computational time associated with AMLOS-SMART reconstruction may still be worthwhile.
Fig 8. Random and bias errors for MART, MLOS-SMART and AMLOS-SMART methods under varying calibration errors. (a) Random velocity error in $u$ velocity; (b) Random velocity error in $w$ velocity; (c) Mean bias error across velocity profile; (d) Magnitude of peak bias error.

In most Tomo-PIV applications experimental images are preprocessed to remove the background intensity and where possible remove the influence of image noise, however the extent to which this is achieved varies considerably depending on the quality of the illumination and optical arrangement that is available for a given experiment. The effect of residual image noise on each of these reconstruction methods is shown in Fig. 9. As expected, image noise increases the random velocity error just as it decreases the reconstruction coefficient. Unfortunately AMLOS-SMART has relatively little effect on the velocity errors associated with image noise. For instance when noise is 40% of the particle peak intensity, adaptive correction has negligible effect on the random error in the $u$ velocity and only reduces the error by 0.07 pixels in the $w$ velocity. At higher seeding densities AMLOS-SMART provides a far more significant improvement with noisy images, however this correction is almost entirely associated with reducing the error introduced by higher seeding density, rather than that introduced by image noise. In the case of zero calibration error, at low seeding density there appears to be almost no benefit to using the more computationally intensive AMLOS-SMART or similar tracking correction approaches. This reinforces the importance of proper image preprocessing in Tomo-PIV.
Fig 9. Random and bias errors for MART, MLOS-SMART and AMLOS-SMART methods under varying image noise. (a) Random velocity error in $u$ velocity; (b) Random velocity error in $w$ velocity; (c) Mean bias error across velocity profile; (d) Magnitude of peak bias error.

Conclusions

A method to adapt the efficient MLOS-SMART technique for improved reconstruction accuracy has been presented based on the use of an estimated displacement field and an adaptive intensity correction, which is applied to the previously identified potential particle locations. This approach redistributes the intensity between particles that follow the flow and those that do not, consequently increasing the intensity of true particles and decreasing the intensity and influence of ghost particles. Using this method reconstruction of a 1000x1000x200 pixel volume should be possible in under 2 hours per volume object compared to 10 hours for similarly corrected MART based reconstructions. The performance of this Adaptive MLOS-SMART (AMLOS-SMART) method has been evaluated using a series of simulated particle fields and camera images. Results indicate for seeding densities higher than 0.1 ppp AMLOS-SMART can provide a significant improvement in the reconstruction quality and a reduction in random velocity error, even in the presence of calibration error and image noise. At standard Tomo-PIV seeding densities around 0.05 ppp AMLOS-SMART was shown to maintain a random velocity error of less than 0.2 pixels despite calibration errors up to 1 pixel. The same benefits were not seen for low seeding densities in the presence of image noise, indicating that careful image preprocessing is still needed to ensure accurate velocity measurements.
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References