Cross-correlation analysis of aeroacoustic sound and flow field using time-resolved PIV

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Abstract We constructed a system for the simultaneous measurement of flow field by time-resolved PIV and sound pressure by a microphone in a small-scale wind tunnel; further, we investigated the cross-correlation between the two. In the case of the PIV measurement, a dual-plane PIV system is realized using a polarizing technique to measure flow fields of two cross-sections simultaneously and evaluate the coherence and phase structure of sound sources in the axial direction of a cylinder. The analysis of single cross-section PIV measurement showed a strong cross-correlation between the PIV-predicted sound pressure and the actual sound pressure at Strouhal number $St = 0.2$, which corresponds with the frequency of the Aeolian tone of a cylinder. In addition, downstream a cylindrical specimen, a well-ordered structure of sound sources was observed; this structure was related with the distribution of the Kármán vortex, whereas the sound source around the separation points was independent of $St$. We also reported that both power spectra and coherence distribution are necessary in estimating the aeroacoustic sound at the far field through a PIV measurement. We also experimentally demonstrated the cancellation of sound waves except at $St = 0.2$ and the doubling of the sound pressure at $St = 0.2$ as a dipole sound around the two separation points. Through the newly developed dual-plane PIV measurement, the linearity of the coherence and phase between two cross-sections with respect to the separation distance is clarified. Finally, the integration in a longitudinal direction is carried out by applying the obtained phase difference and the difference in the predicted aeroacoustic sound at the far field is shown by considering the phase.

1. Introduction

The noise from a high-speed train such as the Shinkansen mainly consists of rolling noise and the aeroacoustic noise originating from the turbulence of air flow around the train. It is particularly known that the power of the aeroacoustic sound is proportional to the sixth power of the velocity of the train, and hence, reduction of the aeroacoustic noise is increasingly required. Thus far, an array microphone or a directional microphone has been utilized in one of the methods for the estimation and identification of sound sources. In this method, the aeroacoustic sound generated from a pantograph or the head of a train was reduced step by step. It is important to capture the “flow field” and “vortex” for further reduction of aeroacoustic noise. In case of the mechanism of the generation of sound waves, Lighthill (1952) suggested a theory using the acoustic analogy in the 1950s. Curle (1955) reported that aeroacoustic sound can be estimated from the measurement of the pressure fluctuation on the body by considering the scattering effect of sound waves on the solid surface. In addition, Powell (1964) and Howe (1975) expressed the source term in Lighthill’s equation using vorticity in the flow field and clarified that aeroacoustic sound is generated by the unsteady behavior of vorticity. The “vortex theory” proposed by Howe implies that aeroacoustic sound generated from the flow field is mainly due to the scattering of the vortex sound on the body; the effect of this scattering can be calculated using the boundary-fitted Green function. The source sound term in the “vortex theory” should be spatially integrated considering the phase characteristics of the sound source structure. Experimental studies were conducted to investigate the relationship between the unsteady behavior of vorticity in the flow field and the generated sound wave (Iida et al. 1999).

In this study, the temporal evolution of the flow field around a cylinder is determined using a time-resolved PIV technique (Someya et al. 2007); these data are then applied to Howe’s vortex theory.
subject to the dipole sound in order to estimate the aeroacoustic sound in the far field. For this purpose, we did not use previous techniques such as holographic or scanning PIV (Brücker 1997, Zhang et al. 1997, Wieneke et al. 2006); instead we developed an apparatus for the simultaneous dual-plane measurement of two cross-sections, and extracted a coherent structure that was dominant to the aeroacoustic sound.

2. Prediction of aeroacoustic sound at the far field

We assume that the Mach number is low and that the source region is small as compared to the wavelength of the emitted sound; in other words, we assume that the source region is “acoustically compact”. In order to relate the flow field with the radiated sound, the equation of Howe’s vortex theory, which is the theory developed using Lighthill’s acoustic analogy, is applied and transformed in consideration of the limited analysis region (Takaishi et al. 2004) as follows.

\[
p(x, t) = \frac{\rho_0 x}{4\pi c_0|x|^2} \int \frac{\partial}{\partial t} (\omega \times u) \left( y, t - \frac{|x|}{c_0} \right) \cdot \nabla \varphi_i (y) d^2 y
\]

Where, \( p \) is the estimated sound pressure; \( \rho_0 \), the density of air; \( x \), the position vector of the observer; \( y \), the position of sound sources; \( c_0 \), the speed of sound; \( \omega \), the vorticity vector; \( u \), the velocity vector; and \( \varphi_i \), the acoustic effect of the body.

Next we define the sound source term in equation (1) as follows.

\[
I(y, t) = \frac{\partial}{\partial t} \left( (\omega \times u)(y, t) \cdot \nabla \varphi_i (y) \right)
\]

The velocity field obtained from the PIV measurement is limited to the \( x \)-\( y \) plane, and the flow around the cylinder is supposed to be two-dimensional. We treat only the aeroacoustic sound emitted in the \( y^2 \)-direction. Therefore, equation (2) is expressed as follows.

\[
I(y, t + |x|/c_0) = \frac{\partial}{\partial t} \left( (\omega \times u)(y, t) \cdot \nabla \varphi_2 \right)
\]

Where, the suffixes 1, 2, and 3 correspond to the \( x \)-, \( y \)-, and \( z \)- axes, respectively. The delay time owing to the sound propagation is also considered.

From equation (1), it can be observed that the spatial distribution of velocity and vorticity around a cylinder and their respective time derivatives should be determined in order to predict the sound pressure at the far field. The acoustic effect of the body, \( \varphi_{2c} \), is calculated analytically for a cylinder using equation (4) below. This acoustic effect is shown in Fig.1.

\[
\frac{\partial \varphi_2}{\partial y_1} = \left( \frac{D}{2} \right)^2 \frac{2y_1y_2}{(y_1^2 + y_2^2)^{3/2}}, \quad \frac{\partial \varphi_2}{\partial y_2} = \left( \frac{D}{2} \right)^2 \frac{-y_1^2 + y_2^2}{(y_1^2 + y_2^2)^{3/2}}
\]

Fig.1  Spatial distribution of \( \frac{\partial \varphi_2}{\partial y_1} \) (left) and \( \frac{\partial \varphi_2}{\partial y_2} \) (right)
3. Experimental apparatus

Fig. 2 shows a schematic of the experimental apparatus. The test section of the wind tunnel is of the open type. The maximum wind velocity is 43 m/s, and the turbulent intensity is 0.7%. A glass cylinder with a diameter of 6.0 mm is used as the specimen in this experiment; this cylinder is bounded on both sides by end plates made of acrylic.

The radiated sound is measured using an omnidirectional microphone (B&K: Type4951) placed 300 mm above the cylinder. The main objective of using this microphone is to compare the actual radiated sound with the sound predicted using the time-resolved PIV.

3.1 PIV measurement (Dual-plane PIV system)

A high-repetition-rate Nd:YLF laser (NewWave Pegasus-PIV, 10 mJ at 1 kHz), which provides irradiation in the double-pulsed mode, is used as a light source; the scattered light is recorded using two high-speed cameras (Photoron APX-RS, SA5). The interval of the double-pulsed laser is 9 μs, and the repetition rate per one cavity is set to 5 kHz. The high-speed cameras with telescopic and macro lenses (Sigma 150 mm f3.5 and Nikon AF Micro Nikkor 105 mm f2.8, respectively) are set lateral to the anechoic room and pointed at the measurement cross-section of the cylinder. These cameras have a spatial resolution of 896 × 336 pixel at a sampling rate of 10 kHz; one of the calibration values is 21.37 pixel/mm. The sampling ratio of the velocity vectors becomes 5 kHz because we apply the frame straddling method to acquire camera frames. The region of interest is $x: -0.7D \sim 5.5D$, $y: -1.2D \sim 1.2D$, where $D$ denotes the diameter of the cylinder, and the system of coordinates is comprised of $x$-axis toward the downstream, $y$-axis along the vertical line, and $z$-axis along the cylinder; in this coordinate system, the origin is set at the point at which the centerlines of the wind tunnel and the cylinder intersect.

In this PIV measurement, tracer particles are introduced through seeding rakes into the wind tunnel before the contraction; these seeding rakes are adjusted to aim at the two cross-sections when dual-plane PIV is carried out.

3.2 Optical system

Fig. 3 shows the light pass and the arrangement of the optical equipment. The emitted laser beam is introduced into an anechoic room after being transformed into circular polarized light by a birefringent element and split into two beams by a polarizing beam splitter. Then the two beams, which are normally polarized to each other (p-polarized and s-polarized), pass through the cylindrical lenses and diverge to form laser light sheets; these light sheets are appropriate for PIV measurement. The shadowed area below the cylinder is illuminated again by two reflecting mirrors because the transmitted laser light sheets of the cylinder are not sufficient. In order to correct the divergence of laser light sheets in the direction of thickness, the optical system including these reflecting mirrors is constructed together with a cylindrical lens.

Because both p-polarized and s-polarized lights simultaneously illuminate the seeding particle, polarizing filters are placed just before each high-speed camera in order to selectively eliminate the unwanted polarized light. This enables us to measure two separate cross-sections around the entire field of the cylinder. Although several patterns for selecting the two cross-sections can be adopted, we adopt the following measurement pattern, considering the subsequent statistical procedure.

- (A) Fixed cross-section: At the center of the cylinder along $z$-direction
- (B) Variable cross-section: Can be varied from $z = -2.0D$ to $4.0D$.

We denote the separate distance between the two cross-sections as $\Delta z$. 
Fig. 2  Schematic figure of the experimental setup

Fig. 3  Schematic figure of the optical system
4. Results of single cross-section measurement

4.1 PIV calculation condition

The calculation conditions of PIV are as follows. The step size of the interrogation window is \(12 \times 12\) pixel, a multi-grid correlation method is used, and 3-point Gauss fitting is used for the sub-pixel peak fitting. The total number of obtained velocity vectors is \(73 \times 27 = 1971\). 9500 pairs of PIV raw images are captured in one time measurement and FFT analysis for these data produces a result that is averaged 147 times; the overlap ratio is set to 50% and the data length is 128.

4.2 Prediction of sound sources through PIV measurement

A cylinder of with a 6 mm diameter is used as a specimen for this single cross-section measurement. The wind velocity \(U_{\infty}\) is 15.0 m/s. The term \(I\) in equation (2) is called the “sound source” here; it is calculated at each of the PIV calculation grid points.

Fig.4(a) shows a comparison between the spectral analysis results of the sound pressure predicted using PIV and that measured by the microphone. A Strouhal number \((St = fD/U_{\infty})\) is a dimensionless number representing frequency; \(St = 0.2\) corresponds with the peak frequency of the Aeolian tone of a cylinder. The peak \(St\) agrees well with the result of the microphone at \(St = 0.2\); therefore, the radiated sound generated by a strong vortex fluctuation can be well captured by our experimental method. Both the coherence and phase difference between the sound pressure predicted by PIV and the actual one measured by the microphone are shown in Fig.4(b) as functions of \(St\). Error bars for the phase are also plotted. The coherence value is prominent at \(St = 0.2\), and a significant cross-correlation is observed between the PIV-predicted and the measured sound pressure. Moreover, the phase difference at \(St = 0.2\) becomes almost 0°, indicating that the PIV-predicted sound pressure and the measured sound pressure are in phase. It should noted, however, that this phase difference has a margin of error of about +/-20° at \(St = 0.2\), which is estimated from the sampling frequency of PIV.

Fig.5 shows the color map and isoline of the power spectra and the phase of the integrand expressed in equation (1). The phase is calculated from the FFT output; its basis is at \(t = 0\). Large sound sources near the two separation points of the cylinder are observed for (a), (b), and (c). Downstream, at \(x/D > 1.5\), a more distinguished structure of sound sources is observed for (a), and the spatial distribution corresponds well with the dynamic structure of the Kármán vortex.

![Fig.4](attachment:image.png)

(a) Power spectrum                (b) Coherence and phase difference

Fig.4  Comparison between the sound pressure level predicted using PIV and the actual one measured using a microphone.
Fig. 5   Power spectra (left) and phase (right) distribution of sound sources at three Strouhal numbers. An isoline is drawn at every 4 dB.

(a) Top: $St = 0.2$,  (b) Middle : $St = 0.3$,  (c) Bottom: $St = 0.4$

Fig. 6   Coherence (left) and phase (right) between the sound pressure predicted using PIV and the actual sound pressure measured by the microphone.

(a) Top: $St = 0.2$, (b) Middle : $St = 0.3$, (c) Bottom: $St = 0.4$
Fig.6 shows the coherence and phase distribution between the actual sound pressure fluctuation measured by the microphone and the sound sources predicted using PIV according to our analysis method. For the sake of comparison, the results for $St = 0.3$ and 0.4 are presented in addition to the result for $St = 0.2$. Considering the dependency of coherence on the Strouhal number, it is observed that the coherence value is small except for $St = 0.2$, although a slight increase is observed in the coherence at $St = 0.4$; this increase corresponds with the harmonic of the Aeolian tone. In addition to the coherence, the phase distribution at $St = 0.2$ reveals that the phase structure is well-ordered behind the cylinder and even in the immediate vicinity of the cylinder. These results confirm the validity of our prediction method based on equation (1) and that the experimental accuracy is sufficient for the evaluation of the radiated aeroacoustic sound radiated at the far field.

Next we compare Fig.5 and Fig.6 and discuss the location of the sound sources at $St = 0.2$. From Fig.6(a), we observe that there are two dominant sound source areas: Area 1: $x/D = 1.5 – 2.0$, $y/D = +/- 0.3 – 0.6$ and Area 2: $x/D > 2.5$, $-0.5 < y/D < 0.5$. However, as indicated in Fig.5, however, the strongest sound sources are located around the two separation points of the cylinder. This discrepancy arises from the fact that the cancellation of sound wave is not considered in Fig.5 and Fig.6, and therefore the power spectra of Fig.5, which indicates the possibility of the presence of sound sources, does not necessarily agree with Fig.6. In the case of a cylinder, it is supposed that the sound sources present around the two separation points of the cylinder cancel out at the far field except in the case of $St = 0.2$. To prove this, we spatially divide the measurement area into five compartments (A0, A1, B0, B1, and C), and predict the aeroacoustic sound at the far field for each compartment. Fig.7 implies that the power spectrum density of areas B0 and B1, in which the coherence value is kept small, peaks at $St = 0.2$ as a result of the spatial integration; these areas redouble each other as a so-called “dipole sound.”, resulting in a 6 dB increase of the sound pressure level.

Above discussion is summarized as follows.

1. Except at $St = 0.2$, frequency components are largely set-off as a result of the spatial integration, and the sound pressure at the far field reduces although the neighborhood of separation points has a broadband component.

2. At $St = 0.2$, which is the frequency of the Aeolian tone, a coherent structure is included at around the two separation points of the cylinder.

![Fig.7 Effect of the integration area around the cylinder](image-url)
5. Results of dual-plane PIV measurement

We simultaneously measured the flow fields of two cross-sections; these flow fields can be used to obtain the flow and sound source structures in a longitudinal direction after processing a statistical analysis. In this section, we discuss the coherence and phase structure of sound sources based on the same analysis that was described in the previous section.

5.1 Raw images illuminated by polarized light

Fig. 8 shows instantaneous raw images under the condition that the fixed and variable cross-sections coincide with each other (that is, $\Delta z = 0.0D$). As mentioned in section 2, the polarizing filters just before each camera are rotated to eliminate the unwanted laser sheet. In order to verify the effectiveness of the polarizing filters in the elimination of the unwanted laser sheet, the filter for the fixed cross-section in case (c) is rotated 90° and the laser sheet is not fired for the variable cross-section. No clear seeding particle is detected, indicating the validity of this polarization method.

5.2 Coherence and phase in a longitudinal direction

Fig. 9 shows the coherence and phase distribution of the two cross-sections separated at a distance of 1.0D at three different Strouhal numbers. It should be noted that the coherence and phase distribution of the two cross-sections are correlated, unlike the results of the single cross-section discussed in Section 4. The coherence distribution in the longitudinal direction increases at $St = 0.2$ and most notably in the region of $2.5 < x/D < 3.5$ and $-0.25 < y/D < 0.25$. On the other hand, the phase distribution does not make so much difference in Strouhal number and is almost in phase in the wake of the cylinder. It is supposed that the sound structure around the Karman vortex region has a stronger cross-correlation than that around the two separation points of the cylinder in the longitudinal direction of the cylinder, and the prominent vortex structure is formed with a matched phase. In addition, the experimental accuracy of flow field in this PIV measurement should also be considered; this accuracy might be lower around the separation points of the cylinder than that in the wake of the cylinder.

Fig. 8 Instantaneous raw images acquired by PIV measurement. The fixed cross-section and the variable one are adjusted to coincide with each other.
Fig. 10 shows a comparison of the coherence or phase structure against the change of $\Delta z$ at $St = 0.2$, which corresponds to the frequency of the Aeolian tone. In terms of the coherence distribution, the peak of the coherent value decreases with increase in the distance between the two cross-sections. Moreover, when $\Delta z = 4.0D$, the peak value of the coherence decreases to 0.5. For phase difference, the phase is uniformly distributed at 0° in the wake of the cylinder at $\Delta z = 0.25D$, and it is gradually increases with the increase in $\Delta z$, until eventually a 30° difference is observed at $\Delta z = 4.0D$. From these results, we expect that the sound sources have a phase structure in which the $z$-plane on the central (fixed) cross-section is prior to that on the outer one.

The left part of Fig. 11 shows the dependence of a coherence and phase change on $\Delta z$ at $x/D = 3.0$ and $y/D = 0.0$, where the peak value of coherence is detected. The coherence and phase changes linearly with $\Delta z$, and the coherence is observed to be symmetrical in $\Delta z$. The right part of Fig. 11 shows the aeroacoustic sound experimentally estimated on the basis of the phase property obtained in the left part of the figure and the aeroacoustic sound estimated using the single cross-section measurement in which the phase property in $z$-direction is assumed to be perfectly identical. We only consider $St = 0.2$ because we apply the phase difference in $z$-direction at $St = 0.2$ to the whole frequency span. The peak of the sound pressure level calculated using dual-plane PIV decreases by 2.2 dB. The difference of the sound pressure level would be more clarified when specimens that have more 3-dimensional flow structures are investigated. At the same time, a more highly advanced data handling method for coherence and phase should be used in the future.
(a) $\Delta z = 0.25D$

(b) $\Delta z = 1.0D$

(c) $\Delta z = 2.0D$

(d) $\Delta z = 4.0D$

Fig. 10 The distance $\Delta z$ is varied from $0.25D$ to $4.0D$ at $St = 0.2$. Left: Coherence map of sound source between the two cross-sections Right: Phase map of sound source between the two cross-sections

Fig. 11 Left: Coherence and phase distribution depending on $\Delta z$ at $x/D = 3.0, y/D = 0.0$, and $St = 0.2$. Right: Comparison between the SPL predicted by single-plane PIV and that predicted by Dual-plane PIV (The phase property in a longitudinal direction is considered)
6. Conclusion

We constructed a system for the simultaneous measurement of a flow field by time-resolved PIV and sound pressure with a microphone in a small-scale wind tunnel; further, we investigated the cross-correlation between the two. In the case of the PIV measurement, a dual-plane PIV system is realized using a polarizing technique to measure two cross-sections simultaneously and evaluate the coherence and phase structure of sound sources in the axial direction of a cylinder. First, the analysis of a single cross-section PIV measurement showed a strong cross-correlation between the PIV-predicted sound pressure and the actual sound pressure at $St = 0.2$, which corresponds with the frequency of the Aeolian tone of a cylinder. Moreover, downstream the cylindrical specimen, a well-ordered structure of sound sources was observed; this structure was related with the distribution of the Kármán vortex, whereas the sound source around the separation points was independent of $St$. We then clarified that both the power spectra and coherence distribution are necessary in estimating the aeroacoustic sound at the far field through a PIV measurement. We also experimentally demonstrated the cancellation of sound waves except at $St = 0.2$ and the doubling at $St = 0.2$ as a dipole sound around the two separation points. Second, through the newly developed dual-plane PIV measurement, we found that the coherence and phase between two cross-sections linearly vary with the separation distance. Finally, we carried out the integration in a longitudinal direction by applying the obtained phase difference and showed the difference in the predicted aeroacoustic sound at the far field by considering the phase.

References


