Generalized displacement estimation for non-stationary flow averages

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Abstract In data sets with relatively poor signal-to-noise ratio, averaged velocity fields are often the only achievable result. These averaged results are often determined using correlation averaging. We show that for instationary flows the use of correlation averaging can lead to unreliable results: summation of individual correlation peaks from an instationary flow creates a broadened peak. The location of the maximum of this peak generally does not coincide with the true mean displacement. We propose to use the centroid of the correlation result as a better estimator. This method is demonstrated with synthetic and experimental data, showing that it gives more reliable results, at the price of a small increase in noise level. For relatively small displacements, where the conventional method is not biased, the method is less suitable due to this increase in noise. Therefore a straightforward hybrid method optimizes the displacement estimation for optimal results.

1. Introduction

At first sight, obtaining average flow information in periodic flows, or instationary flows in general, using particle image velocimetry (PIV) seems straightforward: image pairs are recorded, vector fields are obtained using cross-correlation, and these vector fields are averaged to obtain the desired result. However, in some cases the signal-to-noise ratio of data is insufficient to obtain reliable instantaneous results. For instance, due to difficult imaging conditions or limitations on the amount of seeding material a high percentage of spurious data can occur. In these cases, events with relatively small displacement and gradients will be over-represented, so that the resulting average will not be representative of the time-averaged flow. The presence of ‘gaps’ in vector fields at locations of large displacements (and gradients) will bias the statistics. A further complication arises from the difficulty of validating vector fields with more than 5%-10% outliers in a reliable way (Westerweel 1994).

A common processing method for such noisy data is to perform the averaging step in the correlation domain (Delnoij et al. 1999, Meinhart et al. 2000). In this case, the local cross-correlation is computed for each image pair and then summed over all image pairs. Only after the processing of all data the vector field is obtained from the location of the displacement peaks in these correlation sums. For stationary flows (or phase-locked experiments in the case of a periodic flow), this leads to a significant increase in the signal-to-noise ratio: for instance, Vennemann et al. (2006) showed that for one particular application the number of outliers (which could be seen as a measure of the quality of the result), decreased from nearly 50% to below 5% by averaging over 10 image pairs instead of a single image pair. Here we demonstrate that straightforward correlation averaging can lead to unreliable results for instationary flows. We also present a relatively simple modification that significantly improves the outcome for the estimation of the average flow field.
2. Illustration of the problem

To demonstrate the problem, a synthetic data set based on a simple oscillating flow is generated. We describe this flow in terms of a displacement ($\Delta x$) in pixel units, rather than velocities in physical units; similarly, the locations are expressed in pixels. This greatly clarifies the discussion in the remainder of this section. The displacement field, which has non-zero components only in the $x$-direction, has the form

$$\Delta x(y, t) = C \, y \cos\left(\frac{2\pi t}{T}\right)$$  \hspace{1cm} (1)

In this expression, $C$ is a constant, and $T$ the cycle duration. The flow field described by (1) describes a linear gradient in the $y$-direction ($Cy$), with a magnitude that oscillates with time ($t$). Averaged over time, the mean displacement field is zero. A series of 512 synthetic image pairs of 128×128 pixels is generated with an average of 512 particles with a particle image diameter of 2 pixels (defined by the range for which the Gaussian intensity distribution is higher than $1/e^2$ of the central value). The value of $C$ is chosen so that the maximum displacement is 5 pixels. Out-of-plane loss and intensity variation effects are not simulated, since they do not influence the arguments put forward in this study.

The data series is processed using a correlation-averaging PIV algorithm implemented in MatLab (Poelma et al. 2008). A straightforward single pass with 32×32 pixel interrogation areas with 50% overlap is used here (more complex processing using multiple iterations and window deformation leads to comparable results). The PIV analysis yields an 8×8 vector field containing the mean displacement. Results for the linear oscillation case using synthetic data are shown in Figure 1. Here the displacement $\Delta x$ in the $x$-direction is shown as a function of $y$-position (data from all eight $x$-positions are shown at each $y$-location).

![Fig. 1 Mean displacement as a function of position for the linear oscillation (synthetic data set). The red lines indicate the true mean and maximum positive displacement profiles. Symbols indicate correlation-averaged data set (multiple $x$ positions per $y$-location shown).](image)

The expected mean is zero for each $y$-position. However, this result is obtained only in regions with relatively small displacements. For larger values (in the figure for $y > 50$ px), the observed mean
appears to be non-zero. This behavior can be explained by looking at the cross-correlation result at two y-locations (Figure 2). For small displacements ($y = 32$ px, $o$), the cross-correlation consists of a single peak, originating in the convolution of the particle image and the harmonically-oscillating displacement distribution. For large displacements (e.g. data at $y = 112$ px, $\triangle$), the peak actually splits into a bimodal distribution, as the displacement distribution increases in size. A standard PIV algorithm will pick either one of the peaks and use that to determine the displacement. In this case the initial conditions led to a very small asymmetry so that the ‘positive’ peak is chosen, but before convergence is reached it jumps back and forth between positive and negative values. Note that the displacement appears to follow the maximum displacement, but there is no trivial way of predicting what value of the flow it represents. The splitting of a single displacement peak into distinct peaks is similar to what is observed when spatial gradients are present within an interrogation area (Westerweel 2008).

![Fig. 2](image.png)

**Fig. 2** Two profiles from the correlation results of the linear oscillation data set, taken in a region with relatively low ($o$) and relatively high ($\triangle$) displacements.

The transition between the correct (zero) and incorrect (non-zero) estimation of the mean displacement occurs around a maximum displacement of twice the particle size (also checked by evaluation with other particle sizes; data not shown). So, for larger particles, more temporal variation is acceptable in an averaging process.

A second example is shown in Figure 3. Here, synthetic data is generated of a pulsating Poiseuille-like flow: a parabolic flow profile with a centerline velocity determined by the following equations:

$$\Phi = \frac{i}{T} \mod 1$$

$$\Delta x(r, t) = V_0(1 - r^2/R^2)e^{(-10\Phi^2)\Phi/0.136}$$

in which $\Phi$ represents the phase. The parameters in (3) are chosen so that again the maximum displacement during the cycle is 5 pixels. Figure 3 (left) schematically shows the cycle of the flow field and centerline velocity. The results from a correlation-averaged PIV algorithm are shown in the (middle) figure. Especially in the core region, the correlation averaging method ($\times$, labeled ‘conventional’ in the graph) significantly under-predicts the mean flow (dashed line). Again, the
reason can be understood by looking at the cross-correlation function from the region at the centerline (Figure 3 right): the summation of correlation results leads to a broad, asymmetric peak. The maximum of this peak (indicated by the vertical dashed line labeled ‘peak fit’) does not coincide with the true time-average.

![Fig. 3](Left) A pulsating Poiseuille-like flow, top figure shows the flow at 7 subsequent time steps, bottom graph shows centerline velocity during cycle. (Middle) Mean velocity profile (dashed: theoretical reference, crosses: conventional correlation averaging PIV, circles: centroid method). (Right) Average correlation function at centerline.

### 3. Displacement estimation using the centroid

To improve the accuracy of correlation averaging PIV, we need to generalize the displacement estimation method. In conventional PIV, the estimation of the displacement is done by locating a single distinct Gaussian-shaped peak (within sub-pixel accuracy) - this follows from the assumption that the displacement within an interrogation area is uniform. Such a uniform displacement will appear as a δ-function in the cross-correlation result, slightly broadened by the finite size of the particle images. The latter assumption no longer holds when the displacement is non-uniform, either in space or in time (see also Westerweel (1998) for a theoretical analysis). To better describe the shape of the resulting correlation function in these cases, we suggest using the centroid (or first order moment) as the displacement estimator:

\[
\Delta x_c = \sum_{\delta x} \sum_{\delta y} F(\delta x, \delta y) \delta x / \sum_{\delta x} \sum_{\delta y} F(\delta x, \delta y)
\]

In this definition, \(\Delta x_c\) is the centroid (and thus displacement estimation) in the x-direction. \(F(x,y)\) is the correlation function, expressed as function of the shifts \(\delta x\) and \(\delta y\) (cf. the horizontal axis in Figure 2). The summation over these shifts range from \(-N/2\) to \(N/2 - 1\) for an interrogation area of size \(N\) (for the simple case without zero-padding). The expression for the estimation of the displacement in the y-direction is equivalent.

Note that in early PIV applications the centroid was already used for estimating the displacement, see e.g. Keane and Adrian (1990). However, it is hardly used any more due to the severe pixel locking that it introduces (bias toward integer pixel displacements), especially for compact Gaussian peaks (Prasad et al. 1992). For the present cases, it is nevertheless very suited as an estimator. Pixel locking is not expected due to the broad shape of the averaged correlation peaks. While the definition of the centroid is straightforward, implementation has to be done with some care. Generally, as part of the calculation of the cross-correlation the mean component is subtracted of both interrogation areas. For lack of the true mean (of the entire ensemble of images), usually the
mean of each window is used. This result in a correlation peak that is surrounded by a slightly negative ‘valley’ (Westerweel 1997). As the displacement increases and the displacement peak moves off-centre, the contribution of this negative valley can bias the results, resulting from the finite window size. This can be solved by using only values of the correlation result above a certain threshold (zero or a fraction of the maximum peak height).

Applying this idea to the data sets improves the displacement estimation dramatically. For the case shown in Figure 1 the centroid is located near the origin, due to the symmetry of the correlation function (data not shown). For the case of the pulsating flow, the results for the mean flow agree with the expected values (Figure 3 (middle), open circles).

4. Application to blood flow measurements

To test the proposed method on experimental data, we use a data set taken from the study by Poelma et al. (2008). This data set was taken in the extra-embryonic blood vessels of a chicken embryo (the so-called vitelline circulation) using micro-PIV. Based on an elaborate phase-sorting method, the flow field during the cardiac cycle has been reconstructed in that study. The time-average of that data set will serve as the reference case for the present study.

The image pairs (not phase-locked) are processed using a correlation averaging PIV algorithm (24×24 pixel window size, 50% overlap). The displacement is obtained both by conventional three-point Gaussian peak fitting, as well as using the centroid method. In Figure 4 the result for the flow field using the centroid method is shown (the colors represent velocity magnitude). The result from Gaussian peak fitting was less noisy than the centroid-based result. However, when the average flow field is compared to the reference values, the advantage of the centroid method becomes clear: a cross-section of the flow profile (indicated by the green line in the flow field) shows that the conventional method underestimates the mean flow by nearly a third (see Figure 5).

A more systematic comparison between the conventional method and the centroid method is shown in Figure 6. This figure shows a scatter plot of the velocities at all locations as a function of the reference velocity (from the phase-locked results). The line $y = x$ indicates a perfect agreement with the reference case. Clearly, the conventional method starts to systematically underestimate the velocity for displacements larger than 1.5 pixels - the approximate size of the tracer particle images. Without any a priori knowledge about the temporal behavior of the flow, this underestimation cannot be modeled (and thus possibly corrected).

The results for the centroid method appear to remain unbiased for larger displacements. However, as will be clear from the graph, the noise level is significantly larger than the conventional method. Note that a part of the scatter can be attributed to noise in the reference signal, which leads to uncertainty in the ‘horizontal’ location of the markers. For accurate estimates of e.g. the blood flow rate from a fit to a velocity profile as shown in Figure 5, this increased noise level is preferable over a biased result. Note that, unlike the synthetic case shown in figure 3, it is nearly impossible to tell that the bias effect is present in the conventional analysis: the radial flow profiles appear to have smooth Poiseuille-like shapes. However, the interpretation of e.g. the maximum velocity of these profiles has hardly any physical (or physiological) meaning.

The centroid is outperformed by the conventional method for small displacements (less than 1.5 pixels): it has a lower noise level and no significant bias. Naturally, we can combine the conventional and centroid method: first, an analysis is performed using the centroid method. For locations with a low displacement, the displacement estimation is switched to the conventional
Gaussian peak fit. This does not require significant additional computational effort, as both use the same cross-correlation results.

**Fig. 4** The average flow field (magnitude) measured in blood vessels of the vitelline circulation of an embryonic chicken (see Poelma et al., 2008) for details. The velocity magnitude was determined using the centroid method.

**Fig. 5** A velocity profile along the cross section indicated by the green line in Figure 4, comparing the conventional three-point Gaussian fit (‘direct’, ◦) and new centroid method (◊) to the reference velocity field (∗).
5. Conclusions

We have shown that the conventional peak fit method is not suitable for correlation-averaging PIV of transient flows. Variations in time lead to a broadened peak, whose maximum location will generally not coincide with the mean displacement. In particular, this occurs when variations in the displacement become larger than twice the particle image diameter.

By generalizing the displacement estimation - for instance by using the centroid as estimator - more reliable results can be obtained. The noise level increases with this method, but is shown that it less biased than the conventional method for larger displacements. For the optimal result, a hybrid method is the best solution: only for large displacement regions of the image, the centroid method is used; for other regions the conventional peak fit can be used.

References


