ON THE INTERACTION OF TRAILING AND MACROINSTABILITY VORTICES IN THE VICINITY OF A STIRRED VESSEL IMPELLER

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\textbf{Abstract.} Ducci and Yianneskis (2007) have shown that mixing times of reactors stirred by radial impellers can be reduced by 20-30\% when feed insertion is made in the vortex core of precessional flow structures, denoted as macro-instabilities (MIs). The aim of the present work is to investigate the interaction between large scale flow structures, such as MIs and trailing vortices (TV), and assess to which extent local mixing might be affected by their combined activity. Proper Orthogonal Decomposition (POD) analysis was employed to identify and characterise the two different flow structures considered. It is shown that the combined presence of the trailing vortex and MI structures in the impeller vicinity results in energy levels that are substantially enhanced and thus locations showing further promise for feed insertion and mixing enhancement may be identified.

\textbf{Key words:} Mixing, Macro-Instability, Trailing Vortex, Proper Orthogonal Decomposition, Stirred Vessels.

1 Introduction

Mixing costs are estimated to be 0.5-3\% of the total turnover ($\approx$ 2-11 billion US dollars per annum) of process industries, with 50\% of chemical production being implemented in batch stirred vessels (Butcher and Eagles, 2002). As a consequence much effort has been invested to improve mixing efficiency of stirred vessels by investigating the fluid mechanics taking place in this type of reactor and to characterise the large scale structures responsible for transport phenomena and macro-mixing, as well as to gain better understanding of the dissipative length scales directly related to micro-mixing (Ducci and Yianneskis, 2005). In particular Ducci and Yianneskis (2007) have shown that the MI vortex can be used to reduce mixing time by 20-30\%, while Assirelli \textit{et al.} (2005) have found that micromixing efficiency can be enhanced with a seven fold reduction of product waste when a feeding pipe stationary with the impeller is used to realise the fed reactant in the region of maximum dissipation rate in the trailing vortices. The current work aims at investigating the interaction between macro-instabilities and trailing vortices which both have their origin at the impeller.
2 Flow configuration and experimental apparatus

A cylindrical mixing vessel of diameter $T = 294$ mm, equipped with four equi-spaced baffles was used for the PIV measurements. The experiments were carried out with a Rushton turbine of diameter $D = T/3$ and clearance $C = T/2$. The present experimental set-up was similar to those employed by Ducci and Yianneskis (2005) and Ducci et al. (2007). Refraction effects at the cylindrical surface were minimised by a trough filled with distilled water. Optical access from underneath was achieved by a glass window present in the bottom of the vessel and the PIV camera was focused on a horizontal plane located in proximity of the impeller ($z/T = 0.443$). The two flow structures under study have very different time scales ($T_{MI}/T_{TV} = 300$), and consequently two different data-sets were utilised to better characterise MIs and trailing vortices. The frame rate of the first set was 2 Hz and it was selected to capture the MI vortices, while the optimum frame rate for trailing vortex characterisation was 128 Hz. 2042 frames were collected for each data set, both for $Re = 33600$. The size of the interrogation area was approximately $1.071D \times 0.874D$, and the measurement spatial resolution was $\Delta x_i/D = 0.031$ and $\Delta y_i/D = 0.025$, which is sufficiently small to resolve flow structures with reference diameters of $0.2D$ (Yianneskis et al., 1987) and $0.2 - 0.4D$ (Ducci and Yianneskis, 2007) associated to TV and MI, respectively.

3 Proper orthogonal decomposition (POD)

POD was employed in this work to analyse the PIV data. It is a linear technique based on temporal and spatial correlation analysis, that allows to decompose a set of signals into a modal base, with modes ordered in terms of kinetic energy content. The first modes contain most of the energy of the flow and they are associated to large scale structures, while the last are the least energetic and represent small scale structures and turbulence. For an in-depth explanation of the methodology, please see Sirovich (1987), Berkooz et al. (1993) and Ducci et al. (2007). In equation (1) the POD analysis is applied to the fluctuating part of the velocity field, $\mathbf{u}'$:

$$\mathbf{u}'(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}) + \sum_{n=1}^{N} a_n(t) \Phi_n(\mathbf{x})$$

where $\mathbf{u}$ and $\mathbf{U}$ are the total and mean velocity flow fields and $\Phi_n$ and $a_n$ are the spatial eigenfunctions and the temporal eigenfunctions (often referred to as temporal coefficients) associated to the $n^{th}$ mode, respectively. POD offers an efficient tool to isolate the modes associated to the trailing vortices and MIs, and to investigate the interaction between them, by creating a low order model (LOM) obtained from the combination of the corresponding modes. Van Oudheusden et al. (2005) and Perrin et al. (2007) employed the POD technique to study the dynamics of vortex shedding in wake flows and concluded that large scale convective structures are captured by POD in pairs of modes representing orthogonal components of the periodic process investigated. In such flows a LOM can be
defined as follows:

\[
\vec{u}_{LOM}(\vec{x}, \phi) = \vec{U}(\vec{x}) + a_1(\phi) \Phi_1(\vec{x}) + a_2(\phi) \Phi_2(\vec{x})
\]  

(2)

where the periodic nature of the coherent structures associated to the first two modes is reflected in the sinusoidal variation of the coefficients \(a_1(\phi)\) and \(a_2(\phi)\) as shown in equation (3):

\[
a_1(\phi)/\sqrt{2} \lambda_1 = \sin(\phi), \quad a_2(\phi)/\sqrt{2} \lambda_2 = \cos(\phi)
\]  

(3)

It should be noted that in equation (3) the coefficient \(\lambda_i\) is directly proportional to the energy content of the \(i^{th}\) mode and the phase angle \(\phi\) is equal to \(2\pi ft\), where \(f\) is the characteristic frequency of the structure of interest.

4 Results and discussion

4.1 Phase resolved POD applied to PIV data

The first two highest energetic modes (1 and 2), which contain up to 30 % of the total fluctuating energy of the flow (i.e. with the mean motion excluded), can be associated to the trailing vortex structures. This is well reflected in figures 1 (a) and (b), where the velocity fields and the contours of the vorticity of the spatial eigenfunctions, \(\Phi_n\), of modes 1 and 2 are presented, respectively. The horizontal plane of measurements is not perpendicular to the trailing vortex axis and therefore only a minor component of the vorticity, perpendicular to the measurement plane can be obtained from the current data. This explains why the trailing vortex structures are not so clear when considering the vorticity contours of modes 1 and 2. On the contrary, the vector plots of figures 1 (a) and (b) provide a clearer indication of the trailing vortices with six distinct regions of velocity with higher magnitude vectors in both modes. It should be noted that the vorticity contours of \(\omega_z \Phi_n (\frac{\partial \Phi_n}{\partial x} - \frac{\partial \Phi_n}{\partial y})\) provide only a qualitative idea of size and shape of the flow structure associated to the \(n^{th}\) mode, and their intensity is dimensionally meaningful only when multiplied by the corresponding eigenfunction \(a_n(t)\).

Figure 1: Plots of the velocity field and contours of the vorticity associated to: (a) Mode 1; (b) Mode 2; \((Re=33600)\).

The non-dimensional phase-averaged coefficients \(a_1\) and \(a_2\) are shown in figure 2 (a).
It can be observed that the amplitude of the first two coefficients is similar, which implies that the energy contents of these two modes are comparable. The 90° phase delay between the coefficients $a_1$ and $a_2$ is also confirmed in figure 2 (b), where the non dimensional coefficient $a_1^* = a_1/\sqrt{2 \lambda_1}$ is plotted against $a_2^* = a_2/\sqrt{2 \lambda_2}$. The loci of points $a_1^*$ and $a_2^*$ form a circular shape, as they are scattered around the reference circle. According to van Oudheusden et al. (2005), this proves the orthogonality between the two coefficients and also satisfies equation (3). It should be noted that in this figure an increase of time, $t/T_{TV}$, corresponds to circles of hues varying from blue to red.

As mentioned earlier, the trailing vortex structures are not evident when the vorticity of the first pair of modes is considered (figures 1 (a) and (b)). An improved visualisation of the flow structures associated to the first two modes was obtained by estimating the kinetic energy, $E$, which is defined in equation (4):

$$E = \frac{1}{2} (u_1^2 + u_2^2) \quad (4)$$

where $u_1$ and $u_2$ are the directly measured velocity components in the $x$ and $y$ direction, as the third velocity component was not measured.

The vorticity contour and the vector plot of the mean flow is represented in figure 3 (a), showing a flow directed radially outwards. This was also reported by Ducci and Yianneskis (2007), for a horizontal plane located at $z/T = 0.433$ and for $Re = 27200$. It is worth noting that the intensity of the radial velocity is significantly higher than the tangential one, as the inertial forces due to the impeller jet lower circulation loop present below the impeller are dominant. The radially outward flow is also seen in figure 3 (b), where the kinetic energy contour, together with the velocity vector plot of the mean flow are shown.

The interaction of the first two modes and the mean flow is shown in figure 4 (a)-(d), where the kinetic energy contours together with the corresponding flow patterns of the $LOM_{TV}$ are presented for four positions of increasing $\phi_{TV}$. Six high energetic regions can be clearly distinguished in each plot of figure 4, with a clockwise movement in the tangential direction, which is well reflected from the reference traces denoted by
the black arrows. The inner tip of each blade is responsible for the six high energetic traces, as the inner points of each one are approximately positioned at a radius \( r/T = 0.1 \). The hypothetical centreline of each trace is slight curved backwards, against the direction of movement. Schäfer et al. (1997) carried out phase resolved analysis of velocity measurements in the impeller region, and also identified similar vortices clinging to the blades, with six individual regions of high kinetic energy, one behind each impeller blade.

### 4.1.1 MI characterisation

The velocity fields and the contours of the vorticity of the second two spatial eigenfunctions related to modes 3 and 4 are presented in figures 5 (a) and (b). In this case, the MI vortex axis is perpendicular to the measurement plane, and as a result the MI flow patterns of modes 3 and 4 can be characterised well from the vorticity contours. Two distinct regions of negative and positive vorticity can be observed, with a \( 90^\circ \) shift between them in the tangential direction \( \theta \). Ducci and Yianneskis (2005) and Ducci et al. (2007) also showed the same behaviour at a horizontal plane \( z/T = 0.25 \) and \( Re = 27200 \), with the same kind of flow pattern.

The non-dimensional frequency, \( f'_{MI} \), of modes 3 and 4 can be obtained from an FFT analysis of the temporal eigenfunctions \( a_3 \) and \( a_4 \) (figure 6 (a)) and its value is 0.0175, very close to the characteristic MI frequency 0.02 for high \( Re \) flows. From figure 6 (b), where the autocorrelation and cross-correlation coefficients associated to modes 3 and 4 are presented, is also evident that \( a_3 \) and \( a_4 \) (i.e. modes 3 and 4) oscillate with a non-dimensional frequency of 0.0175. The fluctuation period of the autocorrelation coefficients, \( R_{a_3a_3} \) and \( R_{a_4a_4} \), is \( 58 T_{imp} \) (i.e. \( f'_{MI} = 0.0175 \)), slightly higher than the one reported by Ducci and Yianneskis (2005) and Ducci et al. (2007) (50 \( T_{imp} \)), at a measurement plane of \( z/T = 0.25 \) and \( Re = 27200 \). The cross-correlation coefficient, \( R_{a_3a_4} \) of \( a_3 \) and \( a_4 \) denoted with the black line in figure 6 (b), shows a minimum value for \( \Delta T/T_{imp} = 14.5 \) which is a fourth of the period of fluctuation. Similarly to the considerations already made for the coefficients \( a_1 \) and \( a_2 \) in figure 2, the phase delay between the coefficients \( a_3 \) and \( a_4 \) is also \( 90^\circ \).

The phase resolved coefficients \( a_3 \) and \( a_4 \), associated to modes 3 and 4, are shown in
Figure 4: Plots of the superimposed LOM_{TV} and mean velocity fields and contours of the total dimensionless kinetic energy for four increasing values of φ_{TV}: (a) φ_{TV} = 0°; (b) φ_{TV} = 90°; (c) φ_{TV} = 180°; (d) φ_{TV} = 270°; (Re=33600).

Figure 5: Plots of the velocity field and contours of the vorticity associated to: (a) Mode 3; (b) Mode 4 (Re=33600).

It can be observed that the coefficients a_{3} and a_{4} exhibit similar magnitude to those of modes 1 and 2 (a_{1} and a_{2}, figure 2 (i)), associated to the trailing vortices. This implies that at this plane MIIs contain a significant part of the flow energy content at this
Figure 6: (a) FFTs of the temporal eigenfunctions $a_3$ and $a_4$; (b) Autocorrelation and cross-correlation coefficients of the temporal eigenfunctions associated to modes 3-4, ($Re = 33600$).

plane, almost equal to that of the trailing vortices.

Figure 7: Variation of the phase-resolved temporal coefficients $a_3/V_{tip}$ and $a_4/V_{tip}$ with $t/T_{MI}$.

The flow patterns and the non-dimensional kinetic energy of the LOM$_{MI}$ calculated from the combination of modes 3 and 4 for $\phi_{MI} = 0^\circ$ and $45^\circ$, are shown in figures 8 (a) and (b), respectively. Two regions of low kinetic energy can be identified in the cores of the two counter rotating circulation zones present in the outer parts of the plane investigated, while a high kinetic energy region is clearly present in the centre of the interrogation area where the two rotating structures merge to form a strong radial stream aligned along a diagonal of the plot, figure 8 (a). As the MI precesses around the the vessel axis (see figure 8 b), the circulation cores move clockwise and the central radial stream determined by the merging of these two structures becomes horizontal. To better understand the interaction between the mean flow and the MIs, the energy content obtained from the combination of the corresponding flow fields is presented in figures 9 (a) and (b) for $\phi_{MI} = 0^\circ$ and $45^\circ$, respectively. Direct comparison of figures 3 (b), 8 and 9, shows that the kinetic energy is higher in regions where the local mean flow is directed as the central radial stream associated to the MIs structures, and lower in regions where the local mean flow is directed against it. In other words the MI central stream enhances or suppresses
the flow motion associated to the mean flow, depending on whether they are locally co-
or counter-directed. As expected the region of highest kinetic energy precesses around
the impeller axis (see figures 9 (a) and (b)) as the central radial stream associated to the
MI changes orientation (see figures 8 (a) and (b)).

Figure 8: Plot of the LOM$_{MI}$ velocity fields and contours of the associated dimensionless kinetic
energy for (a) $\phi_{MI} = 0^\circ$ and (b) $\phi_{MI} = 45^\circ$ of the MIs.

Figure 9: Vector plot of the superimposed LOM$_{MI}$ and mean flow pattern and contours of the
total dimensionless kinetic energy for (a) $\phi_{MI} = 0^\circ$ and (b) $\phi_{MI} = 45^\circ$.

4.1.2 Interaction between MIs and trailing vortices

The aim of this work is to determine to which extent MI and trailing vortices interact
with each other and affect the local mean flow and to assess the dynamics of the flow in
proximity of the impeller region. The trailing vortex characteristic time scale is signifi-
cantly smaller than the one related to MI ($T_{MI} >> T_{TV}$), and consequently MIs will be
considered stationary in the remaining of the present analysis. Following this assumption,
a LOM of the flow composed by the trailing vortices, MIs and the mean flow is presented
in the plots shown in figures 10 (a)-(d), for four increasing $\phi_{TV}$ ($\phi_{MI} = 0^\circ$). The flow
pattern obtained shows a region of high $E$ in the left-bottom side of the area investigated. This behaviour is due to the highly energetic region of the MIs at this side of the flow, which is very well reflected in figure 8 (a). Similarly to the previous considerations made for figure 4, the reference trailing vortex trace, denoted with a black arrow, moves of 60° during a full $\phi_{TV}$ cycle.

Figure 10: Plots of the superimposed LOM$_{TV}$ and LOM$_{MI}$ and mean velocity fields, and contours of the total dimensionless kinetic energy for four increasing values of $\phi_{TV}$ and $\phi_{MI} = 0^\circ$ (MI stationary): (a) $\phi_{TV} = 0^\circ$; (b) $\phi_{TV} = 90^\circ$; (c) $\phi_{TV} = 180^\circ$; (d) $\phi_{TV} = 270^\circ$; ($Re=33600$).

### 4.1.3 A comparison between POD technique and phase resolved analysis

The previous analysis was based on the POD technique, but to produce reliable conclusions, kinetic energy profiles were also obtained with a phase resolved analysis, and a comparison between the results of the two techniques is made.

The two dimensionless kinetic energy estimations obtained from the phase resolved analysis and from the POD technique, with combinations of modes 1 and 2 with the mean flow, are shown in figures 11 (a) and (b), respectively. The two plots show very good agreement in the structure pattern, and the kinetic energy levels are quite similar. It can be concluded that the POD technique is very effective and capable to identify the trailing vortex structures as well as the MIs. The standard deviation of the difference
between the two estimates is $0.07/V_{tip}^2$. This is also confirmed by the two lines shown in figure 12, where the profiles of $E$ at $r/D = 0.34$, obtained from the phase resolved analysis ($E_{\text{phase}}$, black dashed line) and the POD technique ($E_{\text{POD}}$, red line), are presented. The estimations of $E$ are in good agreement, with in phase sinusoidal trends of comparable magnitude.

![Figure 11: Plots of the superimposed LOM$_{TV}$ and mean velocity fields and contours of the total dimensionless kinetic energy for $\phi_{TV} = 0^\circ$ obtained (a) from the phase resolved analysis and (b) the POD technique, ($Re = 33600$).](image1)

Figure 11: Plots of the superimposed LOM$_{TV}$ and mean velocity fields and contours of the total dimensionless kinetic energy for $\phi_{TV} = 0^\circ$ obtained (a) from the phase resolved analysis and (b) the POD technique, ($Re = 33600$).

![Figure 12: Vector plot of the mean flow pattern and contour of the dimensionless kinetic energy associated to the mean motion.](image2)

Figure 12: Vector plot of the mean flow pattern and contour of the dimensionless kinetic energy associated to the mean motion.

5 Conclusions

POD was employed to quantify and analyse the two major large scale structures, namely the trailing vortices and macro-instabilities, occurring at a plane below a Rushton impeller. The maximum normalised energy levels are around 0.5 and 0.45 for the trailing
vortex and macro-instability, respectively and 0.9 for their combined activity. As a consequence the mixing enhancement through carefully selected feed insertion locations, as identified by Ducci and Yianneskis (2007) for insertion into the MI alone, should be achievable and further testing is called for.

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Nomenclature

Abbreviations
LOM$_{MI}$ Low Order Model associated with the MIs;
LOM$_{TV}$ Low Order Model associated with the trailing vortex;
MI Macro Instability;
POD Proper Orthogonal Decomposition;
PIV Particle Image Velocimeter;

Greek Symbols
$\Phi_n$ $n_{th}$ POD spatial eigenfunction, -;
$\omega$ Dimensional vorticity in the vertical plane, s$^{-1}$;
$\phi_{MI}$ Phase angle of the MI precession, °;
$\phi_{TV}$ Phase angle of the trailing vortex precession, °;

Roman Symbols
$a_n$ $n_{th}$ POD temporal eigenfunction, ms$^{-1}$;
$C$ Impeller clearance, m;
$D$ Impeller diameter, m;
$E$ Energy content, m$^2$s$^{-2}$;
$N$ Number of POD modes, -;
$Re$ Reynolds number;
$T$ Vessel diameter, m;
$T_{imp}$ Period of impeller, s;
$T_{MI}$ Period of MI precession, s;
$T_{TV}$ Period of trailing vortex precession, s;
$\vec{u}$ Fluctuating velocity field, ms$^{-1}$;
$\vec{U}$ Mean velocity field, ms$^{-1}$;
$\vec{u}$ Total velocity field, ms$^{-1}$;
$V_{tip}$ Velocity of the tip of the blade, ms$^{-1}$;

References


