Proper Orthogonal Decomposition application on the aerodynamic study of controlled annular jets

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Abstract

Proper Orthogonal Decomposition (P.O.D.) is a technique used for analysis of vortex structures in a turbulent flow. In this study, complex shear flows are observed by P.I.V. measurements (Particle Image Velocimetry) of a large diameter ratio annular jet. These jets present a lot of strong instabilities which become a real handicap on industrial processes (burners, glass fibers process, cooling system...). The fundamental interest of the P.O.D. technique is to observe each structure building the flow in order to understand what structure is responsible for what instability, and how we can apply an active forcing or a passive control to change the flow and its time evolution. P.I.V. measurements allow us to have a high spatial resolution and P.O.D. can complete these pieces of information with a temporal reconstruction.

We focus our study on the initial zone of the annular jet. This zone extends from the outlet of the nozzle to the end of the recirculation zone which is formed downstream an obstacle in the middle of a round jet flow. At the end of the recirculation zone, the stagnation point is submitted to periodic fluctuations around the jet axis. We try to apply an active control using acoustic waves with different frequencies (fundamental, first harmonic...). Measurements are conducted with a Reynolds number $Re_{Do}=10^7 800$ for which the stagnation point fluctuates with a frequency of 150Hz (the associated Strouhal number $St_{Do}$ is equal to 0.27). This study will permit us essentially to have a better knowledge of annular jets and to meet manufacturer’s needs.

The P.O.D. analysis applied on a natural annular jet and an excited annular jet enables us to see the significance of the triggering of the acoustic wave with the stagnation point motion. An active control is therefore necessary to use acoustic excitation to reduce instabilities in the initial zone of these turbulent jets. The position of the excitation affects the efficiency of the acoustic forcing. Then this study indicates that an excitation using an acoustic wave with a frequency equal to the inherent frequency of the stagnation point fluctuations reduces these instabilities while a double harmonic excitation frequency seems to have the inverse results.

1. Introduction

1.1 Generalities about annular jets

Annular jets are turbulent jets made of a cylindrical nozzle with an outer diameter called $D_o$ and an obstacle for the flow, located in the center of the nozzle. This obstacle may have different shapes, but in common cases, this obstacle is a disk with an inner diameter $D_i$. Annular jets are defined by the diameter ratio $r=D_i/D_o$. In most applications in the industrial domain these jets have large diameter ratios (superior to 0.7). Davies and Beer \cite{1} suggest in 1971 a first description of the flow of a basic annular jet with a diameter ratio varying from 0.33 to 0.73. The flow may be divided into three zones. The description of annular jets with a diameter ratio included from 0.45 to 0.71 is completed with the studies of Durao and Whitelaw \cite{2} and Ko and Chan \cite{3}. An annular jet is composed of three different zones in which the flow has a particular behavior (figure 1). In the first one called initial zone, a recirculation zone appears downstream the central obstacle because of a lack of any air supply which creates a pressure gradient in this area. The stationary surrounding air is carried away by the jet from the nozzle and a shear appears. This shear gives rise to instabilities...
which form periodic vortices (Kelvin-Helmholtz vortical structures). At the boundary of the recirculation zone, there is the stagnation point located on the jet axis. This point is characterized by a longitudinal velocity equal to zero and a transversal velocity nil in average. The position of the stagnation point is independent of the velocity of the injected air but depends on the nozzle geometry.

![Shape of the flow of a basic annular jet with a large diameter ratio](image)

**Fig. 1** Shape of the flow of a basic annular jet with a large diameter ratio

### 1.2 An efficient technique to observe flow structures

Turbulent flows have particular structures with a spatial coherent behavior. These structures appear in the flow in a periodic way and develop. Each structure is characterized by a spatial and a temporal frequency. A passing in the frequency space enables us to identify these vortical structures. This transformation coupled with a cut in the frequency spectrum of the flow allows a visualization of each of them independently with a new passing in the physical space. The difficulty to make this technique efficient is that frequencies associated to flow structures that we want to study have first to be known. This way is rarely conceivable in turbulence. There are different analysis techniques to decompose turbulent flows with applying arbitrary frequency filter. Proper Orthogonal Decomposition (P.O.D.) brings new opportunities because this post-processing extracts spatial or temporal structures using a rigorous mathematical decomposition basis which breaks free from all arbitrary steps.

P.O.D. is a technique that allows most probabilistic structures in an energetic point of view to be extracted from a statistical series of signals. This decomposition builds a modal base which needs only few modes to describe all principal aspects of the signal. P.O.D. processing is proposed for the first time by Lumley in 1967 and used in 1981 for the study of turbulent phenomena [4] [5]. He suggests that large scale structures or coherent structures are identifiable in a series of velocity fields. These structures contribute for a large part to the total kinetic energy of a velocity field. Adrian [6] applies this technique to develop PIV post-processing in order to bring out the inner driving mechanisms of the flow.
1.3 Acoustic forcing: a step to establish an active control

There are several methods to force a turbulent jet. Active methods are easier than passive methods to set up on industrial processes. An advantage of this way to control jet instabilities is the fact that it is an intermittent technique which can be switch off if it is necessary or could be repaired without acting on the jet facility. All these reasons have brought us to the choice of an acoustic forcing to establish a control of stagnation point instabilities of an annular jet.

The majority of works concerning the active control of annular jets are carried out with measurements by hot wire anemometry or visualizations. The study of Nakazano et al. [7] in 1991 talks about basic annular jets with diameter ratio ranging between 0.2 and 0.8. They use two loudspeakers, placed inside the nozzle, in a transversal way of the flow. The used frequencies correspond to inherent frequencies and they observe that they can influence the recirculation zone and decrease its length by exciting the flow. An annular jet with a diameter ratio equal to 0.95 submitted to the excitation of only one loudspeaker placed inside the nozzle (longitudinal excitation of the flow) is studied by Travnicek and Tesar [8]. Visualizations of this flow are carried out for different frequencies as the preferred mode frequency (St_{D_o}=0.38), the frequency of column jet mode (St_{D_o}=0.55) and frequencies corresponding to frequencies of formation of the vortices (St_{D_o}=2.47). This forcing enable them to influence the initial zone and they note that the harmonic and double harmonic excitation cause the increase or the suppression of the vortices appearance.

All these studies present pieces of information about temporal evolution of the annular jet response to an acoustic forcing but the spatial resolution is quite low. We have therefore decided to use P.I.V. measurements coupled with a P.O.D. post-processing to better know the influence of an acoustic forcing on the development of the flow.

In contrary to these previous studies, we have used a transversal excitation of the flow with a loudspeaker located outside the nozzle, at the outlet of the nozzle, because of disadvantages that an inner excitation presents to set-up on industrial processes.

2. Experimental set-up

2.1 Annular jet

The nozzle used for this study was designed in the CORIA laboratory. Its diameter ratio $r$ is equal to 0.91. The outer and inner diameters are, respectively, $D_o=53.88\text{mm}$ and $D_i=48.75\text{mm}$. The contraction area ratio of the nozzle is equal to 102. This nozzle is made of many components which reduce air flow rate fluctuations inside the nozzle (figure 2) in order to obtain a top-hat velocity profile at the nozzle outlet with a fluidic bed, silk filter layers, a bee nest and a convergent.
2.2 P.I.V. measurements

P.I.V. measurements are carried out on the annular jet without and with acoustic forcing. The flow is seeded with particles of olive oil which are about 1µm in diameter. These particles are perfectly suitable for this experimental device because they are small enough to follow all fluctuations of the flow but also to be sensitive to disturbances fathered by pressures and depressions of air created by the way of the acoustic wave, in the range of frequencies used in the acoustic forcing case for the study of the active control. The seeded flow is illuminated by a laser sheet created by a 532nm Nd:YAG laser with two cavities which gives two pulses of 5 ns and 75 mJ. These pulses are separated by a time delay $\Delta t=10\mu s$. The pulses are synchronized with the opening of the camera by the software “Davis 6.2” (Data Acquisition and Visualization Software) from Lavision. This system creates a longitudinal laser sheet which passes throw the center of the nozzle and illuminates the initial zone of the annular jet (along a distance of 2Do from the nozzle outlet).

A CCD camera 12bits Lavision FlowMaster 3S with a resolution equal to (1280x1024) pixels$^2$ collects the Mie diffusion signal given out by particles. The physical size of pixels is (6.7x6.7)µm$^2$. Each recorded image is divided into small interrogation windows which have a size equal to (32x32)pixels$^2$. We use an overlap of 50% on windows for a standard cross-correlation calculation of particles velocity.

2.3 Acoustic forcing device

As the time delay between two couples of images depends on the camera frequency (8Hz), if we want to have access at all acts of the acoustic wave on the jet, we have to use a P.I.V. device which could be connected to the acoustic forcing. Then we use a TTL casing in order to control the triggering of the camera according to the acoustic wave (figure 3). When the acoustic wave exceeds a threshold fixed by the casing features (13mV), the casing triggers the camera and the laser. A delay can be added between the receiving of the signal issued from the TTL casing and the triggering of the acquisition. Measurements are carried out with different delays between the acoustic wave and the acquisition. We scan 16 phases of the wave and for each phase (phases from 0 to 15) 100 images are recorded.
The loudspeaker used for this set-up is a boomer-medium of 130mm (AUDAX HM130CO). The range of frequencies of this device extends from 100Hz to 10kHz. Its sensitivity is equal to 90dB and its nominal power is 50W. We use only one loudspeaker placed in two different positions: face to the camera (position 1) and perpendicular to the camera (position 2), as it is schematized on the figure 3.

The other parameters studied are the frequencies of the excitation (fundamental and double harmonic) and the amplitude (0.9V and 3.62V).

The active control is observed for a basic annular jet developed with a Reynolds number $Re_{Do}$ based on the outer diameter equal to 107 760 (for a velocity at the outlet of the nozzle $U_0=30 \text{ m.s}^{-1}$). With this Reynolds number, the frequency associated to the periodic beat of the stagnation point is 150Hz. It corresponds to a Strouhal number $St_{Do}$ equal to 0.27. As we want to know jet responses to excitations of frequency equal to the fundamental frequency of the stagnation point beat but also equal to the double harmonic frequency, the forcing frequencies applied on the jet are respectively 148Hz or 152Hz ($St_{Do}$=0.27) and 298Hz or 302Hz ($St_{Do}$=0.54).

3. Proper Orthogonal Decomposition

3.1 P.O.D. analysis used as an efficient post-processing of P.I.V. measurements

P.O.D. analysis is used for the study of many signals but this technique represents a powerful tool to highlight coherent structures and to observe their features and evolution in a flow by applying P.O.D. on two-dimensional velocity fields. This kind of set of data generates therefore an optimal set of basis functions in modal base. In this context P.O.D. analysis uses physical functions with finite kinetic energy equivalent to the square integrable function. As coherent structures should be the structures that have the largest mean square projection, the method implies to find the functions $\Phi$ the most “similar” to the set of fields of velocity $U$. It boils down to find the field that maximizes the inner product with the velocity field:

$$\frac{\langle (U\Phi)^2 \rangle}{\langle (U^2)^2 \rangle} = \max_{\Psi} \frac{\langle (U\Psi)^2 \rangle}{\langle (U^2)^2 \rangle}$$

Where $\langle \rangle$ represents an ensemble average. After developing calculations, we obtain the following
eigenvalue problem:

\[ \int_D \left\langle U(x)U^*(x') \right\rangle \Phi(x') dx' = \lambda \Phi(x) \quad (2) \]

It could also be written in another way:

\[ \int_D R_{ij}(x,x') \Phi_j^{(n)}(x') dx' = \lambda^{(n)} \Phi_i^{(n)}(x) \quad (3) \]

D defines the two-dimensional domain of the velocity fields and \( R \) is the averaged two-point correlation tensor defined by:

\[ R_{ij}(x,x') = \left\langle U_i(x)U_j(x') \right\rangle \quad (4) \]

As the eigenfunctions \( \Phi \) build a complete base, each velocity field \( U_i \) could be written as following:

\[ U_i = \sum_{k=0}^{N-1} a_{k,i} \Phi_k \quad (5) \]

The form of the eigenfunctions is therefore:

\[ \Phi = \sum_{k=0}^{M-1} A_k U^{(k)} \quad (6) \]

With the decomposition of each velocity field as a linear combination of proper mode \( \phi \), we can calculate the eigenvalue \( \lambda_k \) corresponding to the energy contained in the mode \( k \). These eigenvalues are order by \( \lambda_i < \lambda_{i+1} \). To resume, each eigenfunction corresponds to a proper mode skeleton which has a weight in the flow described by the eigenvalue.

\[ u(x,y) = \sum_{k=0}^{N-1} a_k \phi(x,y) \quad \text{and} \quad \delta_{0,\lambda} = \langle a_0 a^* \rangle \quad (7) \]

### 3.2 Snapshot P.O.D. method

In the case of analysis of velocity fields, classic P.O.D. method meets a real difficulty. The size of the correlations tensor \( R \) is directly linked with the number of points contained in the analysis domain. If we consider a set of \( N \) velocity fields solved on a grid described by \( P \times P \) points (with \( P > N \)), the size of \( R \) will be \( (2P \times P) \times (2P \times P) \). P.O.D. decomposition will then be very long in term of calculation time. To take into account the turbulent phenomenon a large number of scales is necessary and the memory of usual calculators may be too low to resolve this study. Sirovich [10] in 1987 suggests a solution by resolving the system on the set of velocity fields (called snapshots) instead of points grid. The new correlation tensor used for this method in our case will be then of a dimension of \( N \times N \) which reduces considerably the calculation time.

### 3.3 Parameters of P.O.D. analysis of experimental P.I.V. results

P.O.D. analysis is used to study the flow of a natural annular jet of air without any forcing then it is applied on this annular jet excited by an acoustic wave created by only one loudspeaker at the nozzle exit. To study an excited annular jet, the acquisition of 100 velocity fields is triggered for 16 different time delays (phases) between the start of the acoustic wave and the illumination of the seeding flow.

The number of snapshots used is chosen equal to the number of velocity fields. In this way, any particular field is used to select snapshots. P.O.D. calculation results would be then totally objective since any structure is privileged.
4. Results of the acoustic forcing of the stagnation point

4.1 Aerodynamic characteristics of a natural annular jet

First, we have to know the features of the unexcited annular jet flow. We perform P.I.V. measurements on a natural annular jet to begin the study of the active forcing by acoustic excitations. 1000 velocity fields enable us to see the behaviour of the initial zone (from the outlet of the nozzle to a 3D₀ distance) on the figure 4.

![Fig.4 P.I.V. results for a natural annular jet flow (calculated with 1000 instantaneous fields): (a) instantaneous flow image, (b) average velocity field (ReDo=107 760), (c) radial velocity fluctuations, (d) longitudinal velocity fluctuations.](image)

A recirculation zone appears in the initial zone (figure 4 (b)) as it was described previously (figure 1). At the boundary of this recirculation zone, the stagnation point is submitted to considerable fluctuations (at the distance x=0.5D₀ from the nozzle outlet) which are about 30%. In the external and internal mixing layers, on the lip of the nozzle, some important fluctuations draw Kelvin-Helmholtz vortices location.

4.2 P.O.D. analysis of a natural annular jet

P.O.D. analysis was applied on a natural annular jet (figure 5) and results were discussed in a previous study in order to put the stress on the significance of the stagnation point oscillations [10]. After using P.O.D. calculation, each instantaneous velocity field has been reconstructed by choosing P.O.D modes. After this step, the Reynolds decomposition radial fluctuations were estimated. This method allows the evaluation of the influence of each mode on an instantaneous field. Mode 0 is the most energetic mode (it represents about 70% of the total kinetic energy of the flow) and is similar to the average field. When we combine modes with their associated weight in the flow (their projection value of the instantaneous field), we can extract the mode responsible for particular instabilities. In this way it is shown on the figure 6 that the mode 1 is the major actor for stagnation point radial fluctuations.
As we are interesting for the reduction of radial fluctuations of the stagnation point, we focus our study on the first modes (from the mode 0 to the mode 5) and more particularly on the mode 1. The next results have for objective to see what kind of response could be observed for these modes in the case of an annular jet forcing by acoustic excitations using different frequencies, excitation positions and different time delays between the forcing and the observation instant.

**4.3 P.O.D. analysis of an excited annular jet**

**4.3.1 Influence of the phase between the forcing and the analysis of the flow**

The board of the figure 7 shows results obtained for a scanning of 16 phases over one period of an acoustic wave of frequency 150Hz (equal to the natural frequency of the stagnation point
fluctuations around the jet axis), with an amplitude of 3.62V for the loudspeaker placed in front of the camera (position 1).

The first result that we can see with this figure is that fields of Reynolds decomposition radial velocities are different from a phase to another one in an irregular way. Results are different according to the studied phase because of the fact that if the acoustic wave comes on the stagnation point to vibrate in an opposite way to this point, then the acoustic wave pushes back the jet in order to immobilize transversally the stagnation point. On the other hand, if the wave comes on the stagnation point in order to carry away the jet with it, the wave accentuates fluctuations of the stagnation point which comes in resonance with the acoustic wave. The phase difference between the beat of the jet and the acoustic wave is then a significant parameter to take into account in order to establish a control of stagnation point fluctuations.

This result can be confirmed with the P.O.D. analysis. The temporal evolution of the stagnation point is transcribed again by the reconstruction coefficient $a$ of the mode 1. If we look at the square root of this coefficient, we have some information about the amplitude of the stagnation point motion (if this square root of $a$ is higher, the fluctuations of the stagnation point are greater). The figure 8 shows that the value of this parameter depends on the phase when the flow is excited by an acoustic wave with a frequency equal to 150Hz and amplitude of 3.62V.

Fig. 7  P.I.V. results of Reynolds decomposition radial velocities for a scanning of 16 phases over one period of an acoustic wave of frequency 148Hz, with amplitude of 3.62V
Fig. 8 Study of the square root of the reconstruction coefficient $a$ for the mode 1 according to phases, in the case of an annular jet excited by a 150Hz and 3.62V acoustic wave.

4.3.2 Influence of the excitation frequency

To check that this change of amplitude of the jet beat is induced by the features of the acoustic wave, we compare for each phase the influence of the amplitude and the frequency of the wave on Reynolds decomposition radial fluctuations (figure 9). The stagnation point transversal fluctuations are considerably reduced when the frequency of the acoustic wave is equal to 150Hz (natural stagnation point motion frequency) but are, on the other hand, greater for a frequency of 300Hz (double harmonic frequency). This observation has been checked on the whole 16 phases. We see on figure 9, that the jet is insensitive to the variation of the amplitude: transversal fluctuations are almost identical.

Fig. 9 Influence of the amplitude and the frequency of the acoustic wave (Reynolds decomposition radial and longitudinal fluctuations for the phase 3).

The energy distribution in modes is described on the figure 10 (a). We can see that the ten first modes are sufficient to represent a major part of the kinetic energy of the flow. The first important result given by this eigenvalues spectrum is that in the mode 0, which is associated to the mean field of flow, the energy increases with the acoustic excitation for any frequency of this acoustic wave. This result is the same for each phase. The frequency seems to have an influence only on the littlest modes (modes from the mode 2). The energy of the mode 1 is not clearly affected by the value of the frequency used but the mode 2 (which corresponds to the instabilities of the jet located on the edges of the flow as Kelvin-Helmholtz instabilities [10]) presents some differences between the different excitation frequencies. We can notice that the natural annular jet has less energy on the mode 0 but more energy on modes from the mode 2 to the littlest modes. The acoustic wave
provides therefore more energy to the greatest structures of the flow at the expense of the littlest ones.

The figure 10 (b) shows that the acoustic excitation reduces the square root of reconstruction coefficient for all modes and for each excitation frequency studied and more especially for the mode 1. This effect is partly in agreement with previous observations: an acoustic wave could reduce stagnation point transversal fluctuations. The effect of the frequency value is less clear than this result because of the dependence on phases. In fact, the response is different from a phase to another one. We have to find a way to trig the acoustic wave with the stagnation point motion to use all benefits of the acoustic forcing. It is the reason why an active control could be considered in order to apply the forcing according to the flow response obtained and to correlate these two actors.

4.3.3 Influence of the excitation position

In contrary to previous results for the study of the change of frequency or amplitude, the response of the annular jet excited by an acoustic wave with a frequency equal to 150Hz and an amplitude equal to 3.62V is totally independent of the phase when the loudspeaker position changes (figure 11). We notice that transversal fluctuations are less localized when the loudspeaker is placed in position 2 (perpendicular to the camera). On the rims of the Reynolds decomposition transversal fluctuations, a considerable zone of transversal fluctuations is induced directly by the acoustic wave because of the succession of pressures and depressions of the surrounding air of the jet, generated by this wave. We can all he same observe an increasing of transversal stagnation point fluctuations. We can also notice that these numerous areas of great fluctuations are on the Reynolds decomposition longitudinal fluctuations field. The approximation of a plane acoustic wave is valid only from a distance of one meter of the sound source but in our experimental set-up, the loudspeaker is placed at 30cm so we have to consider that this wave is spherical and induces therefore transversal but also longitudinal fluctuations. The location of the excitation is a decisive parameter. For an experimental set-up such as the one that we have used, the excitation seems to be more efficient for a reduction of the stagnation point beat in a two-dimensional section when this excitation is transversal to the studied section.

The P.O.D. analysis provides a validation of the first remark relative to the independent effect of the phase for an excitation acoustic wave with 150Hz, for any location of the loudspeaker. The figure 12 (a) shows that the acoustic forcing reduces the square root of the reconstruction coefficients for all modes and this effect is stressed when the loudspeaker is in the position 2 (perpendicular to the
camera). The mode 1 is subjected to this effect so a perpendicular excitation seems to be more efficient to reduce stagnation point radial fluctuations. The same study has been conducted for an annular jet forcing by an acoustic wave with a frequency of 300Hz (double harmonic of the natural stagnation point frequency) and an amplitude of 3.62V (figure 12 (b)). In this case, the effect is inverse to the previous case: for the mode 1, the stagnation point is more destabilized with a perpendicular excitation with a double harmonic frequency of the natural beat frequency.

![Fields of standard deviation of transversal velocities](image1)

![Fields of standard deviation of longitudinal velocities](image2)

**Fig. 11** Influence of the loudspeaker position (Reynolds decomposition of radial and longitudinal fluctuations for the phase 3).

![Influence of the forcing position](image3)

![Influence of the forcing position](image4)

**Fig. 12** P.O.D. results of square reconstruction coefficients for all modes:
(a) forcing acoustic wave with a frequency equal to 150Hz (natural frequency),
(b) forcing acoustic wave with a frequency equal to 300Hz (double harmonic frequency).

5. Conclusions

Annular jets are turbulent jets used in many industrial applications. Instabilities of the initial zone and more particularly the stagnation point instabilities are a real handicap for manufacturers. In this study, we have presented P.I.V. measurements of the effect of an acoustic forcing induced by one loudspeaker with different frequencies, different excitation positions (downstream the outlet of the nozzle) and different time delays between the start of the excitation and the start of the acquisition.
A P.O.D. analysis has helped us to identify the effect of this forcing on the flow structure in the initial zone. Eigenvalues spectrum and reconstruction coefficient standard deviation have given information about the change of the energy distribution and the amplitude of the stagnation point fluctuations around the jet axis. Mode 1 represents stagnation point instabilities so our study has been focus on first modes in order to understand how does the acoustic forcing act.

All results enable us to notice that the triggering of the excitation acoustic wave must be related to the stagnation point motion. This first remark compels us to consider another experimental set-up with an acoustic control supported by the modification of the forcing contingent on the observed jet response. The excitation position plays a role on the forcing. It seems to be more efficient when the acoustic wave (transverse from the jet axis) propagates in the measuring plan: in this case, a wave with a frequency equal to the natural frequency of the stagnation point reduces transversal fluctuations. An axisymmetric annular excitation in the nozzle exit, transversally to the flow would be imaginable in order to obtain a more efficient control.

Conclusions are still not definitely established because of numerous parameters at issue in this acoustic forcing but P.O.D. analysis gives promising results and is an efficient technique to observe effects on the flow structure in terms of energy, shape and instabilities.

References


