Analysis of a pulsed jet in cross flow by multi-plane snapshot POD

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Abstract In this work, snapshot Proper Orthogonal Decomposition (POD) is used to study a pulsed jet in cross flow. The velocity fields are extracted from Stereoscopic Particle Image Velocimetry (SPIV) results. The studied pulsed jet is characterized by a frequency $f = 1$Hz, a Reynolds number $Re_j = 500$ based on the mean jet velocity $U_j = 1.67$ cm/s and a mean velocity ratio of $R = 1$. POD results of instantaneous, phase-averaged and fluctuating velocity fields are presented and compared in this paper. Snapshot POD applied on one plane allows to obtain an organisation of the first spatial eigenmodes. A distinction between the “natural modes” and the “pulsed modes” is carried out. Secondly, the correlation tensor is established with four parallel planes (multi-plane snapshot POD). This technique allows to obtain volume spatial modes. These obtained modes are interpolated and the volume velocity field is reconstructed with minimal number modes for all the times of the period of pulsation. These reconstructions are compared to orthogonal planes to the transverse jet in order to valid the obtained three-dimensional velocity fields. Briefly, another technique of multi-plane snapshot POD is tested consisting of polynomial interpolation of obtained spatial coefficient $a_i(z)$. Finally, our approach consists in using POD for the 3D flow field reconstruction from experimental data issued from parallel planes to the flow. This technique is capable to extract relevant information of a complex three-dimensional flow.

1. Introduction

Continuous jets in uniform transverse flow have a three-dimensional structure which has been investigated with analytical, experimental and numerical approaches in the last few years. They have many applications in combustion, propulsion, pollutant dispersion and they are very interesting from a fundamental perspective. This research aims to understand and to clarify the complicated structure of the flow. The work on the jet is based on the analysis of the jet trajectory, structure and mixing in cross flow. Currently, scientists seek to increase the performance of the jet. In order to improve the mixing and penetration of the jet in cross flow, one approach consists in pulsing the jet. The different parameters are the frequency of the jet, the duty cycle, the amplitude of the pulsation, the time of injection and the shape of the signal applied to the jet. Johari (2006) carried out a review of the pulsed jet. To analyze these jets, one needs data which can be obtained by numerical simulation or experimentally. This paper deals with experimental results.

They can be obtained using several techniques, but in order to understand the complex flow and its dynamical evolution over time, measurements of the three-dimensional velocity field are essential. Several technical measurements have been presented over the last decade based on the scanning method, the defocusing method, or the multi-plane illumination method. They used a multi-plane SPIV technique in order to obtain the three-dimensional structure of turbulence in a boundary layer flow. Kähler (2004) uses SPIV technique with four pulsed lasers and four cameras. Velocity fields are obtained in two close parallel planes. In the same way, Liberzon (2005) links up the defocus, stereoscopic and multi-plane illumination concepts to have information in three parallel planes (XPIV). Another solution which requires the determination of the flow structure is the fast scanning of the light sheet with SPIV measurements (SSPIV, Hori 2004). The three-dimensional structure of
the flow involves the extension of a common planar PIV technique towards a volumetric method. Holographic PIV allows the instantaneous measurements within a complete volume (Herrmann 2004). In the same way, the tomographic PIV technique enables to measure the velocity field on a volume illumination and simultaneous imaging of the light (Elsinga 2006). On the same manipulation, David (2007) carries out tomographic PIV on a continuous jet in cross flow. These different techniques are limited (restricted illuminated volume, difficulty in carrying out measurements). These data contains a huge amount of information, hence some analysis must be carried out to extract the most interesting feature of the flow.

One of the tools classically used is POD (Proper Orthogonal Decomposition) which can extract the most energetic elements of the flow. The flow is decomposed on an orthogonal base which is optimal on an energetic point of view. The POD mathematical principle is explained in part 3. Braud (2004) analyses the wake-mixing-layer interaction using multiple plane PIV. Its approach allows to apply a 3D-POD with parallel planes. Recently, Druault (2007) has presented a new POD application for the 3D reconstruction of the mean velocity field from velocity fields obtained in selected parallel and orthogonal planes of a turbulent flow.

In our paper, a new technique is employed to analyze the pulsed jet linking up the SPIV technique and the multi-plane snapshot POD. Data is used at different locations in the flow in order to reconstruct this one. This flow structure is three dimensional. Velocity fields with three components are extracted from Stereoscopic Particle Image Velocimetry (SPIV) in six planes transverse and orthogonal to the jet. In a first part, experimental device and SPIV technique is detailed. In a second part, snapshot POD has been applied to instantaneous, fluctuating and phase-averaged velocity fields in order to identify the different first modes. Finally, it is presented how three-dimensional flow reconstructions have been obtained from measurements in different planes.

2. Experimental device

The experimental device is a closed-loop horizontal hydrodynamic water channel. Within the channel, a flat floor is placed to generate a 20 mm thick boundary layer close to the jet nozzle. The square jet of \( L = 30 \) mm width flows from bottom to top (Fig 1-a). The pulsed jet is generated by a first pump which produces a steady flow. At the same time, a sinusoidal signal (\( f = 1 \) Hz) is applied to the second pump in order to obtain a flow rate variation between zero and twice the mean flow rate. The flow dimensionless numbers are the Reynolds number \( Re_j \) and the ratio \( R \) between the velocity jet \( U_j \) and the velocity cross flow \( U_{cf} \), as shown in by the following relations:

\[
Re_j = \frac{U_j L}{v} = 500 \quad R = \frac{U_j}{U_{cf}} = 1
\]

The SPIV system, as shown in Fig 1-b, is composed of a double pulsed Nd:YAG laser (30 mJ/pulse) operating at 532 nm. Time separation between the two laser pulses is adjusted for the velocity values and it is equal to 30 ms for the longitudinal planes (x-z plane) and 20 ms for the orthogonal planes (y-z plane). The CCD camera has a 12 bit dynamic range, a resolution of 1374 x 1040 pixels\(^2\), and a double frame per second recording rate. The cameras are located behind the pulsed jet in backward configuration at 45 degree. Each of them is placed on a one-axis Scheimpflug adapter in order to adjust the angle between the image, object, and main-plane of the lens. 45 degrees prisms also allow astigmatism corrections. The calibration procedure and stereo PIV cross-correlation analysis are performed using Davis 7.2 software with multi-pass processing with correction of misalignment. The first pass is realized with 64 x 64 pixels interrogation areas.
and 75 % overlapping. Finally, the three last passes are realized with 32 x 32 pixels interrogation areas and 50 % overlapping.

![Image of SPIV system](image)

Fig 1: (a) - Manipulation scheme and (b) - SPIV system of the pulsed jet in cross flow

The SPIV measurements are carried out in four parallel longitudinal planes (x-z plane) and two planes (y-z plane) orthogonal to the transverse flow. The data is recorded in phase (200 instantaneous velocity fields at 20 points distributed uniformly along the sinusoidal signal period). After processing, the velocity fields are adjusted between them to minimize errors of measure and to correct the discrepancies of the laser-sheet location. The velocity profiles between two orthogonal planes are correlated to determine the best match. A sub-cell technique allows to improve the positioning of the plane. The result of this correlation is shown in Fig 2 where the mean velocity magnitude of the seven SPIV measurements is presented.

![Image of velocity magnitude](image)

Fig 2: Mean velocity magnitude of the six SPIV measurements

3. Proper Orthogonal Decomposition method

Mathematically, the basic concept of classical POD, as introduced by Lumley (1967), is to extract the most energetic modes from a fluctuating velocity field. In this work, the application of classical POD has a lower convergence than snapshot POD because we have more available spatial data than temporal data. The snapshot POD introduced by Sirovich (1987) has therefore been used to obtain
the POD modes. This decomposition is based on the solving of the following integral problem:

$$\int_{\mathcal{T}} C(t,t') a_n(t') \, dt' = \lambda_n a_n(t)$$  \hspace{1cm} (1)$$

where $\lambda_n$ is the $n$-th eigenvalue, $a_n(t)$ the eigenmodes and $C(t,t')$ is the two-point time correlation tensor, as shown in the following equation:

$$C(t,t') = \frac{1}{N} \int_{\Sigma} u_i(\tilde{x},t) u_i(\tilde{x},t') \, dx$$  \hspace{1cm} (2)$$

where $u_i$ is the velocity field used to calculate the correlation tensor, $N$ is the number of snapshot POD and $\Sigma$ is the full integral domain. After determination of the spatial modes $\Phi_i^{(n)}$, a velocity field can be projected on to the POD basis in the following way:

$$u_i(\tilde{x},t) = \sum_{n=1}^{N} a_n(t) \Phi_i^{(n)}(\tilde{x})$$  \hspace{1cm} (3)$$

4. Results

4.1 Notation

The notation used for the triple decomposition of the instantaneous velocity fields $u_{pn}$ is given in the following equation:

$$u_{pn} = \tilde{u} + u_{pn}' \quad \quad u_{pn}' = \tilde{u}_p + u_{pn}''$$

$u_{pn}$ : the instantaneous velocity field (p represents the phase and n the field’s index in the phase)

$\tilde{u}$ : the mean velocity field

$u_{pn}'$ : the fluctuating velocity field is divided into two parts: $\tilde{u}_p$ the phase averaged fluctuating velocity field and $u_{pn}''$ the remaining amount of the fluctuating velocity field

4.2 Snapshot POD one plane

First, snapshot POD is applied to one longitudinal plane only ($y/L = 0$). The correlation tensor is established with two velocity components for three cases: instantaneous, phase-averaged and fluctuating velocity fields. In these cases, the spatial modes issued from several snapshots POD are compared. Then, the aim is to distinguish an organization of the first modes. A correlation coefficient allows the comparison of two spatial modes. When this method is applied to instantaneous velocity fields and to phase-averaged velocity fields, the first mode obtained is very similar to the mean velocity field. As a result, the proposed snapshots POD are obtained with the first spatial mode.

In Fig 3, the five first spatial eigenmodes obtained by snapshot POD applied to phase-averaged fields (on the left) are presented. The spatial mode vector magnitude is represented in these figures. Furthermore, its application to phase-averaged velocity fields allows to obtain modes which depend on the frequency of the pulsed jet. The phase-averaged fields represent averages conditioned by the position on the phase. In this decomposition, the frequency of the pulsed jet is very important.
Fig 3: Five first spatial eigenmodes obtained by snapshot POD applied to phase averaged (on the left) and instantaneous velocity fields (on the right).
Moreover, snapshot POD is applied to instantaneous velocity fields in Fig 3 (on the right). It can be observed that apparently, modes 1 and 2 are very similar. Mode 3 on the left is similar to mode 5 on the right. Modes 3 and 4 on the right correspond to propagation. It is possible to check more precisely these impressions.

The eigenvalues of the temporal correlation matrix associated with these modes are shown in Fig 4-a. Decrease of energy is shown for the phase-averaged fields (square symbols) and for instantaneous fields (gradient symbols). In this figure, the first mode has an identical energetic level in both cases (mode 1). Modes 2-3 obtained by phase-averaged fields have a similar energy to the modes 2-5 obtained by instantaneous fields. Mode 3-4 obtained by instantaneous fields have the same energy.

The Fig 4-b represents the correlation coefficient between the modes of the two cases. The first mode of each case is identical. The mode 2-3 obtained by phase-averaged fields correspond to mode 2-5 obtained by instantaneous fields (they correspond to approximately 99% of the correlation coefficient). These results confirm the observation previously obtained. Instantaneous modes 3-4, which doesn’t exist within phase-averaged fields, are called natural modes while the other modes (modes 2 and 5) are called forced modes. In this case, the distinction between natural and forced modes is possible because the forcing frequency is far from the natural frequency of the jet which is around 0.3 Hz.

![Fig 4](image)

**Fig 4:** (a) - Eigenvalue of spatial modes issued from phase averaged (square symbols) and instantaneous velocity fields (gradient symbols) and (b) - Correlation coefficient between the both cases mode number

However, if the same procedure is applied to instantaneous velocity fields from the same phase, no forced spatial mode corresponds and the spatial modes 3-4 of the decomposition with instantaneous velocity fields are found. This confirms that the forced spatial modes are described by the phase-averaged velocity fields.

POD analysis is a powerful tool to investigate the influence of the forcing on the natural flow. In the case presented in the paper, a distinction has been clearly made between modes only due to the forcing and natural modes. Moreover, it has been proven that phase averaging gives the forced part of the flow.
An example of reconstruction from four modes of the forced velocity fields is presented in Fig 5 for the y=0 plane. A zoom is performed on the relevant part of the jet (no information of interest exists in the remaining domain). At t = 50 ms, the period of the pulsed jet starts, while at t = 250 ms the expulsion of the fluid reaches its peak. In this figure, spatial influence of the forcing is very limited.

![Fig 5: Mean velocity magnitude and velocity fields at t = 50 ms and t = 250 ms respectively for y/L = 0 plane (pulsed jet at f = 1 Hz and Rej = 500).](image)

Then, the influences of the measurement planes and the addition of the third velocity component were studied. The organization of forced modes with three components is similar to the one obtained with two components but the spatial mode is different. The modification is very important on the longitudinal strong three-dimensional planes.

When snapshot POD is applied to orthogonal planes of the jet (y-z plane), forced modes do not appear because on these planes measurement is carried out when the pulsed jet is developed. The region where the pulsed modes, as seen before, is limited to a small region close to the jet outlet. Consequently, they don’t have any sensible influence on the transversal planes far from the outlet.

### 4.3 Snapshot POD multi-plane

Four parallel measurement planes have been obtained by SPIV. The idea is to build the correlation tensor with information taken from these different planes. The phase-averaged fields lead to a synchronization of the different planes; hence it is possible to reconstruct the phase evolution of the three-dimensional velocity fields. From POD analysis in one plane, it can be seen that the three-dimensional flow obtained corresponds to the forcing. A correlation tensor is built with the phase-averaged velocity fields. Snapshot POD allows establishing volume spatial modes (with different spatial resolution). As for snapshot POD one plane, the organization of modes is similar.

A reconstruction from four modes of the pulsed jet is presented in Fig 6-a. In this example, 99.9% of the energy is reached with four modes. In this figure, velocity magnitude is represented at t = 250 ms. The reconstructed velocity fields were compared to the initial data with a correlation coefficient as shown in Fig 6-b. In this figure, the importance of modes is put forward. To understand this figure, we need to know that the phase begins at instant 1 and the end of phase is instant 20. From the graph, it can be deduced that the mode 2 has a strong influence during the injection of fluid; hence this mode represents the effect of injection (instant 3 to 11 and 13 to 20). On the contrary, mode 3 dominates when mode 2 has no influence. So the forcing of the flow can be essentially described by a balance between mode 2 during injection and mode 3 during suction.
4.4 Data interpolation

The forcing flow field is reconstructed in four planes. The spatial resolution in the spanwise direction (axis y) is lower than the other two directions. It is interesting to check if the flow can be efficiently interpolated on a more refined grid. For example, it would be useful to study transport and mixing. Indeed, this interpolation is carried out on each POD mode by an Inverse Distance Weighting method. A homogeneous data volume is obtained for each mode. Then, measurement volume reconstruction is achieved. To confirm these results, the two orthogonal planes obtained by SPIV measurements are used. To illustrate this work, the reconstruction from four modes of the pulsed jet is done at $t = 0$ ms.

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**Fig 6:** (a) - Velocity magnitude fields at $t = 250$ ms (pulsed jet at $f = 1$ Hz and $Re_j = 500$) and (b) - Correlation coefficient between the reconstruction with n-modes and the initial data. This analysis is realized on the four parallel planes.

**Fig 7:** SPIV orthogonal plane ($x/L = 2.05$) at the top and reconstruction plane after interpolation at the bottom at $t = 0$ ms. Components $u$, $v$, $w$ respectively from left to right.
Three velocity components are presented in Fig 7 for SPIV orthogonal plane \((x/L = 2.05)\) and reconstruction plane after interpolation. Here, POD is used for interpolation for two reasons: modes are robust to noise so it is better for interpolation and POD compress information (with four modes and all temporal coefficients, it is possible to reconstruct all the data). These results on three components are very close for the two-plane components and some differences appear in the exterior part of the cross flow. It is more in relation to the poor agreement between the plane \(y/L = 0.41\) and \(x/L = 2.05\) for the \(u\) component than the employed spatial-reconstruction technique.

Another multi-plane snapshot POD technique has been used in order to reconstruct the jet structure. Instead of using a POD decomposition on spatial modes \(\Phi_i(x,y,z) a_i(t)\), it is possible to use a spatial-temporal decomposition \(\Phi_i(x,y,t) a_i(z)\). To reconstruct the jet, a 1D polynomial interpolation on each \(a_i(z)\) is performed. These results are very approximate because there are few planes and these planes are too much spaced.

5. Conclusion

A pulsed jet in cross flow characterized by a frequency \(f = 1\) Hz, a Reynolds number \(Re_j = 500\) has been studied by SPIV measurements and snapshot Proper Orthogonal Decomposition. A distinction between the “natural modes” and the “pulsed modes” has been carried out from the median plane analysis. This effect has been confirmed with the use of two or three components of velocity and for the other \(y\)-plane of measurement.

Multi-plane snapshot POD has been built to analyse the pulsed jet in cross flow in the whole volume. In this case also, it has been proven that the phase-averaging of the flow, for the frequency considered, leads to a forced flow. From these data along the \(y\)-planes, spatial modes have been interpolated to extend the data everywhere. Comparison with real measurement in the \(x\)-planes has confirmed the efficiency of the technique of reconstruction. This technique is capable to extract relevant information of a complex three-dimensional flow and allows studying the flow when forcing and natural flow are coupled.

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References