Experiments on the stability, structure, and dynamics of flows in one- and two-sided lid-driven cavities

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Abstract Two-sided lid-driven cavities with three different cross-sectional aspect ratios ($\Gamma = 1.47$, $\Gamma = 1$ and $\Gamma = 0.76$) and a large span-wise aspect ratio ($\Lambda \approx 11$) are investigated experimentally. The one-sided driving with one of the walls at rest is given particular attention. Four different modes have been numerically predicted for the one-sided driving. The numerical calculations have been obtained for cuboidal cavities. Due to the experimental realization by cylinders, the moving walls of the cavity are slightly curved in the experimental setup. Quantitative experimental data of the quasi-two-dimensional basic flow show a good agreement in comparison to numerically computed data despite the slightly differing boundary conditions. Four modes are found for one-sided driving. Photographs and PIV measurements are presented. Critical parameters for the instability of the quasi-two-dimensional basic flow have been detected and compared to numerical data. The flow structures of the modes agree with the corresponding numerical predictions. One of the modes has been observed experimentally for the first time.

1. Introduction

Vortex dynamics in closed systems is relevant to fundamental fluid mechanics and technical applications. We investigate the incompressible and isothermal flow in a rectangular container called lid-driven cavity. The flow is induced by the tangential and independent motion of two facing walls. Thus different vortex configurations can be generated depending on the direction of the wall motion. Many important flow phenomena like corner eddies, quasi-two-dimensional (Q2D) vortices, multiplicity of flow states, a variety of flow instabilities, and transition to turbulence can be realized in this classical system. Due to its simple geometry and boundary conditions the lid-driven cavity is an ideal system for investigating the above phenomena. Besides, it is also used as a benchmark for numerical codes. Shankar and Deshpande (2000) have provided an overview on the lid-driven cavity problem. To date, the majority of investigations on the lid-driven cavity have been numerical, while experimental investigations are sparse. Albensoeder (2004) and Albensoeder and Kuhlmann (2003) solved the linear stability problem numerically. A review on the three-dimensional instabilities of the steady two-dimensional flow is due to Kuhlmann and Albensoeder (2005).

In the present experimental setup the cavity is formed in the gap between two coaxially rotating cylinders of radius large compared to the cross-sectional dimensions of the cavity. The cavity is therefore nearly rectangular. The flow is governed by the cross-sectional aspect ratio $\Gamma$ (width-to-height ratio), the span aspect ratio $\Lambda$ (span-to-height ratio) and the two Reynolds numbers ($Re_1,2$) associated with the circumferential velocities of the two cylinders and the cavity height. Here we investigate the stability of the Q2D flow for three aspect ratios ($\Gamma = 1.47$, $\Gamma = 1$ and $\Gamma = 0.76$) and consider the one-sided driving, in particular. To minimize end wall effects, the span aspect ratio $\Lambda \approx 11$ was selected much larger than in most previous experimental investigations. This is important, because some modes can only be observed experimentally when the span aspect ratio is large enough. Flow visualization was accomplished by aluminum flakes and a thin halogen light sheet. Supplemental PIV measurements have been made to obtain quantitative data.
Albensoeder (2004) and Albensoeder et al. (2001) predict numerically the existence of four different three-dimensional modes for the one-sided driving and aspect ratios from $\Gamma = 0$ to $\Gamma = 4$. One of these modes called Taylor-Görtler-Mode is the well known (e.g. Koseff and Street 1985 for $Re \gg Re_c$, Albensoeder et al. 2001). Two other modes were observed experimentally by Siegmann-Hegerfeld et al. (2008). At the considered aspect ratios we can investigate three of the modes experimentally when the cavity is driven one-sided. The fourth mode arises at $\Gamma = 1$ when the driving is two-sided. We provide photographs, PIV measurements and critical data of the four arising three-dimensional modes. This work is an extension of our previous measurements (Siegmann-Hegerfeld et al. 2008).

2. Experimental Setup

In Fig. 1 the geometry of the cavity and the coordinate system are shown. The dimensionless parameters are the aspect ratio $\Gamma$, the span aspect ratio $\Lambda$ and the two Reynolds numbers ($Re$).

$$\Gamma = \frac{d}{h}, \quad \Lambda = \frac{l}{h}, \quad Re_{1,2} = \frac{u_{1,2}d}{\nu}$$  \hspace{1cm} (1)$$

Where $d$ is the arithmetic mean of the two extremal cavity widths in $x$-direction (at $y = 0$, and $y \pm \frac{h}{2}$), $h$ is the height of the cavity, $l$ is the length, $u_{1,2}$ are the velocities of the two moving walls, and $\nu$ is the kinematic viscosity. We use the height as length scale, the viscous velocity scale $\nu/h$ and the viscous time scale $h^2/\nu$ for presenting the results in dimensionless units. When only one of the walls is moving and the other is at rest the driving mode is called one-sided. Parallel driving means that both walls move in the same direction. We use a single Reynolds number $Re^p_1 = -Re_1 = Re_2$ when the wall velocities are the same, i.e. for symmetric parallel driving. For the one sided driving we define $Re^e_1 = -Re_1$.

Two large cylinders located in an outer container are used to drive the flow in the cavity. They are parallel and rotate about their horizontal axes. Thus the cylinder’s surfaces form the two lateral sides of the cavity, whereas their distance defines the width $d$. To realize different aspect ratios the cylinders can be adjusted horizontally to vary $d$. A top cover, a bottom wall, and two side windows aligned with the curvature of the cylinders make the remaining rigid walls of the cavity. The top cover and the side windows are made of Perspex. The cavity and the outer container are filled with silicone oil (density $\rho = 0.956 \text{ g/cm}^3$). The experimental setup is described detailed in Siegmann-Hegerfeld et al. (2008).

To visualize the flow structures we use a halogen light sheet and non-isotropically shaped aluminium flakes (Merck) as tracer particles. A digital reflex camera (Nikon D70) is used to take...
photographs of the top \((x,z)\)-plane and the cross-section \((x,y)\)-plane of the cavity. The PIV measurements are conducted with a 2D-PIV System (DANTEC Dynamics) comprising a dual head ND:YAG Laser (200mJ/15Hz each cavity) and a CCD camera (2Mpix). We measure the \((x,y)\)-plane and the \((x,z)\)-plane illuminated with the laser-light sheet and the optical axis of the CCD camera normal to it. As seeding we used hollow white glass spheres (Potter Spherical 110P8CP00, nominal diameter 11µm, density \(\rho = 1.1 \text{ g/cm}^3\)).

2. Results

2.1. Boundary conditions

The numerical calculations used for comparison are obtained for exact cuboidal cavities and periodic boundary conditions. As the walls are slightly curved in the experimental setup and the span aspect ratio is finite, even though \(\Lambda \gg \Gamma\), it is important to know how these differing boundary conditions influence the results. The steady basic flow in the cavity is two dimensional in the absence of end walls. In the experimental setup the inevitable end walls induce a three-dimensional secondary flow. For that reason we call the experimental flow Q2D. The secondary flow is very weak in the cavity center \((z = 0)\) for a span aspect ratio \(\Lambda = 6.55\) (Blohm and Kuhlmann 2002 and Albensoeder et al. 2001). Since the span aspect ratio in the present experiment is even

Fig. 3 Detail of the PIV vector map Fig 2a, \(\Gamma = 1\), \(\text{Re}' = 400\), DSE
larger ($\Lambda \approx 11$) we can neglect the secondary flow induced by the end walls in a certain range around $z = 0$.

The curved walls as a potential reason for deviating results was suggested by Koseff and Street (1984b). They compared their experimental results for the one-sided driving with $\Gamma = 1$, $\Lambda = 1$ and $Re \geq 1000$ with the results of Pan and Acrivos (1967). While Koseff and Street used an endless copper belt to drive the flow in their experimental setup, so that the walls were plane, the experimental setup of Pan and Acrivos was realized by a large radius cylinder. The results were in qualitative agreement, but no agreement in the order of magnitude. Koseff and Street traced back this deviation to the influence of the curved wall. We similarly find deviations of the experimental data in comparison to the numerical ones which we ascribed to the influence of the curved wall (Siegmann-Hegerfeld et al. 2008). In the following we compare measurements of the Q2D flow to numerically calculated data to quantify these deviations.

Fig. 2 shows the steady basic flow in the cavity for the one-sided (a) and the parallel-driving case (b) for $\Gamma = 1$. The photograph and the PIV vector map display the experimental results. The streamlines in the middle are computed numerically. The representations show a good visual

![Fig. 2](image)

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![Fig. 5](image)

**Fig. 5** Q2D flow. Measured (symbols) and computed (lines) velocity profiles for $\Gamma = 1$, one-sided driving $Re' = 400$, $z = 0$, a $v(x)$, b $u(x)$, c $v(y)$, d $u(y)$. Positions and colors are displayed in Fig. 4
agreement, even thought the curvature of the walls is obvious in the streak-line photograph. The basic flow of the one-sided driven cavity consists of a main vortex and two secondary corner eddies. The downstream secondary eddy (DSE) is clearly visible on the streak-line photograph. Fig. 3 shows the DSE as a detail of the PIV vector map shown in Fig. 2a scaled differently. The upstream secondary eddy (USE) is not visible in the Fig. 2a (compare computed streamlines and photograph), because the exposure time was to short as it was adjusted to show the main vortex.

To compare the experimental data quantitatively with the numerical calculations we plotted in Fig. 5 and Fig. 6 the velocity profiles $u(x)$, $v(x)$, $u(y)$ and $v(y)$ of the PIV measurements for different positions $x = \text{const.}$ and $y = \text{const.}$ The measurement positions and the differing domains are shown in a true-to-scale sketch for $\Gamma = 1$ in Fig. 4. The PIV data are obtained by averaging 500 individual velocity records. Velocity profiles for one-sided driving ($Re^e = 400$) are displayed in Fig. 5. For most measurements the velocities agree qualitatively as well as quantitatively. As the streamlines have to follow the curved walls and the width of the cavity depends on the vertical coordinate $y$ in the experiment, deviations are expected at all measured position. Additional discrepancies can result from geometry imperfections, minute eccentricities of the rotating cylinders and other experimental uncertainties. As a result of the curved walls, the velocity vector is not parallel to the vertical, but has a small $x$-component at the walls. In fact, the velocity $u$ has to be $u = 34$ for $Re^e = 400$ at $(x,y) = (-0.5,-0.287)$ instead of $u = 0$ for plane walls. The corresponding velocity profile (Fig. 5b $u(x)$, light blue symbols) extrapolates to $u = -32.5\pm3$ at $x = -0.5$ consistent with the boundary condition. The velocity profiles $u(y)$ (Fig. 5d) show the largest deviations at all measured positions. Deviations near the driven wall arise for the velocity profiles $v(x)$ (Fig. 5a). In Fig. 6 velocity profiles for the parallel-driving case ($Re^p = 200$) are presented. The largest deviations at all measured positions can
be seen in the velocity profiles \( v(y) \) (Fig. 6c). The velocity profiles \( v(x) \) (Fig. 6a) deviate near the walls. Overall the quantitative data show a satisfactory agreement.

We conclude that a quantitative comparison between numeric and experiment is possible despite of the different domains. However, deviations due to the influence of the curved walls and depending on the measurement position, have to be taken into consideration.

### 2.1. Three dimensional flow structures

The one-sided driving at \( \Gamma = 1 \) represents the classical and widely studied cavity configuration, experimentally as well as numerically. We are interested in the transition of the steady two-dimensional flow to three-dimensional flow structures. Albensoeder (2004) and Albensoeder et al. (2001) calculated the linear-stability boundary of the two-dimensional flow numerically as a function of the aspect ratio and for infinite span. They found four different three-dimensional modes, two oscillatory and two stationary ones. All modes arise due to centrifugal instability mechanisms. The numerical investigation of Theofilis et al. (2004) for variable aspect ratio likewise yields four different modes, but three of them were oscillatory and one was stationary. The critical parameters agree for \( \Gamma = 1 \) but generally disagree for \( \Gamma \neq 1 \). So far, three of the numerically predicted modes have been observed experimentally. The experimental observations as well as the wavelengths and the oscillation frequencies agree, within close error bounds, with Albensoeder’s (2004) predictions. The so-called stationary Taylor-Görtler mode (TGM) was experimentally detected by Albensoeder et al. (2001). Two oscillatory modes have been reported in our former experimental investigations (Siegmann-Hegerfeld et al. 2008).

### Table. 1 Critical parameters

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( \Gamma = 0.76 )</th>
<th>( \Gamma = 1 )</th>
<th>( \Gamma = 1 )</th>
<th>( \Gamma = 1.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>( C'_c )</td>
<td>( C'_c )</td>
<td>( C'_c )</td>
<td>( C'_c )</td>
</tr>
<tr>
<td>( \text{Re}_{1c} )</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>-100 ± 1.3 (-100)</td>
<td>0 (0)</td>
</tr>
<tr>
<td>( \text{Re}_{2c} )</td>
<td>625 ± 15 (656 ± 1.5)</td>
<td>774 ± 10 (786.3 ± 6)</td>
<td>664 ± 10 (766)</td>
<td>350 ± 20 (495 ± 10)</td>
</tr>
<tr>
<td>( k_c )</td>
<td>7.45 ± 0.5 (7.19 ± 0.01)</td>
<td>15.36 ± 0.35 (15.43 ± 0.06)</td>
<td>8.8 ± 0.1 (7.37)</td>
<td>1.92 ± 0.1 (1.75 ± 0.02)</td>
</tr>
<tr>
<td>( \Omega_c )</td>
<td>465 ± 20 (480 ± 3)</td>
<td>0 (0)0</td>
<td>363 ± 10 (407)</td>
<td>0 (0)</td>
</tr>
</tbody>
</table>

Experimental and numerical data (in parentheses)
Albensoeder numerically predicts the fourth mode as a second stationary one with a long wavelength unlike Theofilis et al. (2004) who predicts another oscillatory mode. We find the fourth mode experimentally at an aspect ratio of $\Gamma = 1.47$ with a long wavelength and stationary as predicted by Albensoeder. As the modes arising experimentally agree regarding their properties with the numerical predictions of Albensoeder (2004), we use his notation. Fig. 7 represents a streak-line photograph of the oscillatory $C_1^e$-mode (a) and the stationary modes $C_2^e$ (also called TGM) (b), and $C_4^e$ (c). The mode $C_3^e$ is presented as a close-up in Fig. 9a. As the flow is time-dependent and weak the flow structure is only weakly recognizable in the photograph of the $C_3^e$-mode. The critical parameters for the onset of the modes are listed in Table 1. The data have been determined by visual investigations. The critical data of the modes $C_1^e$, $C_2^e$, and $C_3^e$ have been taken from Siegmann-Hegerfeld et al. 2008. The wave numbers ($k$) and oscillation frequencies ($\Omega$) agree well in all cases. But the critical Reynolds numbers deviate by $\Delta Re \approx 100 (\approx 13\%)$ for $C_3^e$ and $\Delta Re \approx 150 (\approx 30\%)$ for $C_4^e$. A minute eccentricity of the rotating cylinders could lead to the primary onset of the oscillatory mode, but we have no explanation for the deviation of the stationary $C_4^e$-mode so far. The premature onset might be caused by end walls effects. Usually the non-isotropic reflectivity of the aluminum flakes used for visualization immediately indicates any changes of the flow pattern. As the $C_4^e$-mode has a very long wave length and a small amplitude the critical point of this mode is more difficult to detect visually. To clarify the deviation we have to verify the critical Reynolds number and to measure the bifurcation by Laser Doppler Velocimetry in future investigations. In the following the modes are described in detail. Additional PIV vector maps show the three-dimensional spatial structure of the stationary modes.

2.1.1. $C_1^e$-mode

The oscillatory mode $C_1^e$ was experimentally observed as a standing wave at an aspect ratio of $\Gamma = 0.76$ according to the predictions of
Albensoeder (2004) and Albensoeder et al. (2001). Fig. 7a shows a streak-line photograph of the mode just beyond its onset. The three-dimensional pattern is visible over 50% of the cavity span around the centre of the cavity. We anticipate that the supercritical mode is suppressed by the end-walls-induced modification of the basic flow.

2.1.2. $C_2^e$-mode (TGM)

Koseff and Street (1984a) reported first on a Taylor-Görtler instability arising at the one-sided driving in a lid-driven cavity. Koseff and Street (1985) observed experimentally at Re > 1000 the appearance of three-dimensional vortices which showed a cell-like structure and sharply defined borders located between the primary vortex and the DSE. When the Reynolds number was further increased the vortices became time-dependent and started to meander. A further increase of the Reynolds number led to turbulent structures. The transition to turbulence is quoted with Re = 6000 - 8000. Other numerically investigations were carried out by Ramanan and Homsy (1984), Ding and Kawahara (1988), Aidum et al. (1991) and Theofilis et al. (2004) e.g.

Albensoeder (2004) and Albensoeder et al. (2001) predicted the critical Reynolds number for the stationary TGM and one-sided driving at $\Gamma = 1$. They obtained $Re^c_c = 786 \pm 2\%$. They report that a cellular flow appears, because the perturbation velocity $w$ vanishes periodically in $z$-direction. The pertubation flow forms vortices with opposite sense of rotation along the external stream lines of the basic flow. The vortices are pronounced near the corner where the USE is located in the basic flow up to the upstream rigid wall and show a nearly cylindrical cross section. However, they form no closed toroidal vortex, because the vortices are damped by the moving wall. According to the Taylor Couette system the vortices are called stationary Taylor-Görtler-Vortices (TGV). Albensoeder (2004) and Albensoeder et al. (2001) confirm their numerical predictions with their experimental investigations. They detected a critical Reynolds number of $Re^c_c = 810 \pm 15$ at $\Gamma = 1$ and $\Lambda = 6.55$ which is very close to their numerically calculated value. They reported that the TGV
Fig. 10 $C_1^e$-mode. Streak-line photograph (close up of Fig. 7c) and PIV vector maps, $(x,z)$-plane, $\Gamma = 1.47$, $Re^e = 600$, a) $y = 0.3$, b) $y = 0$, c) $y = -0.3$
are not very robust and strong influenced by boundary effects and they speculated that the TGV were only observable because the span-aspect ratio of $\Lambda = 6.55$ was just sufficient. Siegmann-Hegerfeld et al. 2008 confirm their results. The influence of the end walls in the experimental setup becomes manifest by a drift of the TGV. The symmetry breaking by the end wall secondary flow leads to a drift of the cells from the centre of the cavity in both $z$-directions towards the end walls. New cells are created near $z = 0$ and the cells are annihilated at $\Delta z \approx 2$ from the end walls. Therefore, we call the experimentally observed mode quasi-stationary. Fig. 7b shows a streak-line photograph of the TGM slightly above critical. The cells are suppressed near the end walls. A close-up of the cells and PIV vector maps at different $y$-positions near the wall where the USE is located in the basic flow are shown in Fig. 8. Owing to the drift, the phase is different in the vector maps. The velocity vector plots at different $y$ indicate the dependence of the flow structure on $y$. The TGV is most clearly visible in Fig 8c. The black frame in Fig. 8b indicates the source region where new TGV cells are being created in time.

2.1.3. $C_3^e$-mode

According to Albensoeder (2004) and Albensoeder et al. (2001) the oscillating mode $C_3^e$ is the critical mode for $1.163 < \Gamma \leq 1.207$ for the one-sided driving. Furthermore, the mode is predicted to be the most dangerous one for parallel unsymmetrical driving also for other aspect ratios. For $\Gamma = 1.2$ Albensoeder found the mode appearing as a standing wave. The main change of the flow structure appears analogously to the TGM in the region where the USE is located in the basic flow. Experimentally we observe the strongest change of the flow structure in the same region. But in the experiment a running wave appears as critical flow structure. Wave number and oscillation frequency show a good agreement with the numerical values, but not the critical Reynolds numbers. A streak-line photograph of the running wave is shown in Fig. 9a. The wave is to be seen quite weakly in the form of stripes at an angle from the upper left to the lower right in a distance of about 35mm. The photo of the cross-section displays no change of the flow structure. When the Reynolds number is increased by 30 the flow structure changes again and a standing wave appears in the experiment. The standing wave is displayed in Fig 9b. The change of the flow structure is now visible on the photograph of the cross section in the region where the USE is located in the basic flow.

2.1.4. $C_4^e$-mode

Kuhlmann et al. (1997) were the first who reported about the $C_4^e$-mode in their numerical investigations. Albensoeder (2004) and Albensoeder et al. (2001) pointed out the centrifugal character of the mode. According to their numerical predictions the mode should be stationary. The Q2D flow consists of a primary vortex adjacent to the moving wall. The critical mode deflects the primary vortex of the basic state periodically upward or downward. A wavy primary vortex is formed by this displacement. Experimentally we find a stationary mode in form of a wavy primary vortex corresponding to the numerical predictions shown in Fig. 7c. Fig. 10 shows a close-up of the supercritical flow structure and corresponding PIV vector maps at different $y$ positions. The vector map Fig. 10a show a flow structure equivalent to the photograph. The velocity field of the middle $y = 0$ of the cavity displays a more complex flow structure Fig. 10b. We suppose that the $C_4^e$-mode is only observable because we used a very large span-aspect ratio.

3. Summary and discussion

We investigated two-sided lid-driven cavities with three different aspect ratios ($\Gamma = 1.47, \Gamma = 1$ and
$\Gamma = 0.76$) and a large span aspect ratio ($\Lambda \approx 11$) experimentally. The driving is realized by two large cylinders. Hence, the moving walls of the cavity are slightly curved in the experimental setup. We compared our experimental data of the Q2D basic flow to numerical data which have been obtained for a cuboidal domain. Expected deviations due to the influence of the curved walls have been detected. As the data show a satisfactory agreement, we conclude that it is possible to compare quantitative results, despite of the different domains, when the deviations are considered. We investigated in particular the one-sided driving with one of the walls at rest. Four different modes have been observed and compared to the numerically predicted ones. Two of them are oscillatory and two are stationary as predicted by Albensoeder (2004) and Albensoeder et al. (2001). Photographs and PIV measurements of the four experimentally arising modes are presented. The properties of the modes, the critical wave numbers, and the oscillation frequencies agree good with the numerical data. However, the critical Reynolds numbers of two modes show larger deviations. The $C_4$-mode has been observed experimentally for the first time. We assume that the span-aspect ratio was just sufficient to observe this mode. The deviations of the critical Reynolds numbers raise questions for future investigations. Furthermore, the detection of the critical curve for $\Gamma=1.47$ would be of interest as well as other quantitative measurements, especially the bifurcations of the modes.

References