Free surface measurements by stereo-correlation

Sébastien Jarny\(^1\), Markus Jehle\(^2\), Laurent David\(^3\)

\(^1\): LEA, University of Poitiers, Poitiers, France, sebastien.jarny@univ-poitiers.fr
\(^2\): LEA, University of Poitiers, Poitiers, France, markus.jehle@lea.univ-poitiers.fr
\(^3\): LEA, University of Poitiers, Poitiers, France, laurent.david@univ-poitiers.fr

Abstract Rheological parameters have been extracted from the measurement of the thickness and velocity of a bentonite suspension flow. The flow is induced by the opening of a small tank on an inclined wall. The yield stress is in good agreement with conventional rheometrical tests made using a rheometer and the kinetic energy is close to previous results measured for similar flows.

1. Introduction

Many materials, such as paints, clay suspensions, concrete or inks, are complex fluids which present non-Newtonian properties. To optimize their utilization or to predict their flow, it is necessary to determine their laws of behaviour or at least some typical parameters. In general, we realize viscometric flows (i.e. constant history of the strain rate) by confining a fluid within different geometries under specific boundary conditions (torque, rotation velocity). In this case, there are two relations: one between the torque and the shear stress \(\tau\) and the other between the rotation velocity and the shear rate \(\dot{\gamma}\) leading to the behaviour law. To validate the continuum assumption, the size of the gap must be at least ten times larger than the largest particle. So, for the coarse suspensions (concrete, debris mud…), we have to develop specific devices with very large gaps to reach this hypothesis. But there is a simple way to determine the rheological properties of pasty materials: the inclined plane flows. Indeed uniform, steady flows over an inclined plane are viscometric flows where the thickness is directly linked to the shear stress, and the surface velocity is related to the shear rate. So, this technique is particularly appropriate for coarse suspensions, as soon as we are able to measure the evolution of the free surface and the surface velocity during the flow. Moreover some natural flows, such as debris flows, avalanche or lavas flows, can be modeled using this way.

Pasty materials are considered as yield stress fluids, i.e. they begin to flow when they are submitted to a shear stress greater than a critical value, the yield stress. Above this value they behave like a solid. Using the inclined plane geometry, the critical thickness \(h_c\) for fluid stoppage or incipient flow is directly related to the fluid yield stress \(\tau_c\) through \(\tau_c = \rho gh_c \sin \beta\), where \(\rho\) is the fluid density, \(g\) the gravity, and \(\beta\) the plane slope. This technique is particularly appropriate for determining the yield stress of coarse concentrations [Liu and Mei (1989), De Kee et al. (1990), Coussot and Boyer (1995), Coussot et al. (1996), Sofrā and Boger (2002)], as one may arbitrarily fit the boundary conditions (slope angle) in order to get a critical thickness much larger that the largest particle size for the continuum assumption to be valid. Additionally, for the simple relation between the yield stress and the flow characteristics to be valid, the sample thickness must be much smaller than its spreading extent over the plane. As a consequence, the sample volume to be used is often rather large; which can be a disadvantage of the technique. The usual procedure consists in pouring a given volume of paste and measuring the final thickness in the central region of the layer after
fluid stoppage. Uhlherr et al. (1984) have developed another technique based on the determination of the critical angle of the flow start for a uniform thickness of fluid. More recently, Jarny et al. (2008) have applied this technique to the flow of a white cement paste on an inclined plane. By measuring in the same time the evolution of the thickness and the surface velocity, the complete time dependent behaviour laws have been determined. To this aim, the flows were filmed with a high resolution numeric movie camera (24 pictures per second, 1024×1280 pixels). They observed lines of a laser grid projected onto the free surface with a small angle incidence. With the help of geometric considerations the position of these lines in time can be used to deduce the thickness of the fluid layer. The surface velocity was determined with the help of a PIV (Particle Image Velocimetry) technique cross-correlating the information of two successive pictures. Chanson et al. (2006) have applied the same technique to determine the flow dynamic of a dam break wave using non-Newtonian time dependent fluid. Nevertheless, due to the small number of lines (5) they only obtained limited information about the fluid thickness. Cochard and Ancey (2008) or Robin and Valle (2004) have developed a more accurate method by projecting patterns onto the fluid surface. Moreover, their technique takes into account shadow zones that could appear during the flow but they do not have access to the surface velocity. Recently, developments of free surface measurements have been proposed to extract both the surface location and its three-dimensional displacement (Adam et al., 2005, David and al., 2005, Jehle et al, 2008) by stereo-correlation.

In this paper, the results are focused on the extraction of the rheological properties of a bentonite suspension flows on an inclined plane and the potentiality of method. In a first part, we present the principles of the measurement techniques for 3D free surface and 3D surface velocity. Then, we focus on the preparation procedure of the material and the description of the setup. We describe the results obtained for the flow with and without an obstacle. The first case allows us to check the efficiency of the technique of measurement. The second one is used for analyzing the 3D surface and 3D velocity surface results and for determining the rheological behaviour of the bentonite suspension. Finally, these results are compared to others obtained using conventional rheometers and to numerical calculations.

2. 3D surface measurements

As the paper is more centered on the potentiality of the technique for rheological property measurements, the StrainMaster3D software from Lavision has been used. The measurement of the 3D displacement of free deforming surfaces with PIV is achieved by correlation of images obtained by a stereoscopic camera setup (Fig. 1). The 3D procedure consists of three steps. Firstly a 3D volume calibration, secondly a static measurement of the surface shape, and finally a dynamic measurement of 3D surface deformation and 3D displacement of surface elements. At first, a 3D volume calibration is done by translating a calibration plate with a translation stage and recording the different images for the different locations. It provides a mapping function M (defined from a camera pinhole model) for each camera and links the real reference with the camera references. Using M the surface height H(x, y) at position (x, y) is calculated by the offset of particles in the stereo images at time t (static measurement). At the second step, for each stereo image at time (t, t+Δt, t+2Δt, etc.), the surface height calculation using M provides a scalar field describing the actual surface of the object S (x, y, H(x, y)) which is a prerequisite for 3D displacement calculation. By PIV correlation methods the virtual shift (parallax offset) of identical surface elements (patterns) is calculated (Fig.1). For each image location the corresponding surface height can be calculated from the virtual shift with the mapping function M. Finally, 3D displacement calculation leads to a vector field describing the displacement of every surface point. The 3D surface displacement V (x, y, H(x, y)) is calculated by cross-correlation from successive stereo images (t(t+Δt)) in the same way as described for 2D PIV with V (x, y, H=0). With the surface height information and the mapping function M, for each vector position (x, y) the 2D displacement vector is calculated for both stereo
cameras separately. Finally, the 3D displacement vector \( V(x, y, H(x, y)) \) is calculated by triangulation (Fig.2) from the two 2D displacements of both images together with the corresponding surface height.

Fig. 1: Iterative process of the surface height location (from LaVision, 2006)

![Fig. 1: Iterative process of the surface height location (from LaVision, 2006)](image1)

Fig. 2: 3D surface displacement evaluation by stereo-correlation (from LaVision, 2006)

![Fig. 2: 3D surface displacement evaluation by stereo-correlation (from LaVision, 2006)](image2)

3. Materials and experimental setup

3.1 Preparation procedure

The clay suspension, with a pseudo mass concentration of 8% (i.e. 8 g of powder for 100 g of water), is prepared with deionized water and a natural bentonite powder (Wyoming). First, the clay powder is gently poured into the water under agitation with a homogenizer IKA T25 at a rotation speed of 11000 rpm. Then, the suspension is mixed at the same rotation speed during 30 min to ensure the good dispersion of the clay particles. We added a red dye (Congo Red, Prolabo) to enhance the contrast with the white particles of polystyrene. The suspension stays at rest for two weeks before the beginning of the tests in order to let enough time for the complete hydration of the clay.
3.2 Experimental setup

The experimental device is composed of an inclined channel in PMMA, width 35 cm and length 69 cm, resting on a fixed frame making it possible to regulate its angle of inclination. In our case the angle was fixed to 10° for all the experiments carried out. The higher part of the channel constitutes the fluid tank (Fig.3). It is closed by a plate which can be removed for the opening; the flow is then generated. This flow is recorded using 3 cameras CCD Flowmaster Intense of LaVision with objectives of 50 mm focal distance. They are placed around this device in the same plane of the channel, i.e. at 10° from the horizontal plane (Fig.4). They are focalized on the same area situated at the center of the inclined channel with an angle of 30° from the vertical plane. Once the flow has begun, fine polystyrene particles, with a diameter range between 2 to 20 µm, are powdered to create the particle distribution being used for the correlation. Two configurations have been measured: a flow with a short cylinder for validating the measurements (height and velocity), another one without cylinder to access the rheological properties.

4. Results

4.1 Flow with an obstacle

We consider the flow of the red bentonite suspension on the channel where a solid cylinder is placed on the center of the plane (Fig.5). This configuration allows us to verify and to validate the method used here to obtain the 3D surface and the 3D velocity surface. Indeed, during the flow the mud is separated on the two sides of the obstacle. Thus there are different levels of thickness ranging from the origin wall (h=0 mm) to the thickness of the cylinder (h=16 mm). Using these two surface references we will be able to check the validity of our measurements.

The results presented on the figure 6 seem to give a good approximation of the 3D surface. The reference wall (h=0 mm) and the top surface of the cylinder (h=16 mm) are perfectly calculated. Moreover the general 3D thickness of the fluid is well depicted and is situated between these two reference heights. Nevertheless some defects are present on the front of the flow and on the vertical walls of the cylinder. The first ones are related to the position of the two cameras and certainly a lack of markers on the front and the second ones correspond to a shadow area. In fact, the adjustments of the CCD cameras have been made to ensure a good correlation of the thickness on the top of the flow because this area is the most importance for rheological measurements. The light source position has not been optimized for this configuration because of the constraints of the setup. To enhance the quality of the results on these typical zones, camera and light source position could be optimized.
From the 3D surface we are able to calculate the 3D surface velocity (Fig.7). We observe the same defects as for the 3D surface. On the front area, the velocity is not well defined because of the high slope of the surface. Moreover, it probably confirms the lack of markers on this zone. The same remark is true on the interface between the vertical walls of the cylinder and the fluid. But we obtain a good velocity field on the top surface of the flow, which is the principal zone of interest for rheological measurements.

4.2 Flow without an obstacle

We consider now the flow without the cylinder (Fig.8). In this configuration, we have to determine two parameters to measure the rheological behaviour of the fluid,: the shear stress ($\tau$) and the shear rate ($\dot{\gamma}$). So we assume that the flow is uniform and the edge effects are negligible (i.e. the layer thickness is much smaller than its longitudinal and lateral extent). Let us consider the case of steady state flow. From the momentum equation we find the wall shear stress ($\tau_w$) variation with the distance from the wall ($h$):

$$\tau_w(h) = \rho gh \sin \beta$$

where $\rho$ is the fluid density, $g$ the gravity and $\beta$ the plane slope.
In our case we have $\rho = 1052 \text{ kg.m}^{-3}$ and $\beta = 10^\circ$. Note that the velocity of the fluid along the free surface is simply found by integration of the shear rate between the plane and the free surface:

$$V_{\text{surf}} = \int_0^h \dot{\gamma}(y) dy .$$

Differentiating this equation we find a simple relation between the shear rate along the wall and the variation of the surface velocity with the layer thickness under constant slope:

$$\dot{\gamma}(\tau_w) = \left( \frac{dV_{\text{surf}}}{dh} \right) \beta$$

(2)

where $V_{\text{surf}}$ is the surface velocity and $h$ the thickness of the fluid layer. This equation makes it possible to determine the flow curve from a set of $(h, V_{\text{surf}})$ data in a sufficiently wide range.

Fig.8 Bentonite suspension flow without the cylinder

First for evaluating the wall shear stress we use the 3D surface results where we observe the defects described before (Fig.9). So for respecting the hypothesis we exclude the data from the front of the flow. The shear rate is calculated using the thickness and the surface velocity. To ensure that the edges effects do not affect the velocity, we choose a window situated on the centre of the flow for the surface velocity calculation (Fig.10). Moreover, because the velocity evolves as a function of the length in the flow direction, this area is then reduced to a square window ($25 \times 25$ mm) based on the centre of the length direction. Then all the results are then averaged on this window and we follow the evolution of the thickness and the surface velocity as a function of time.

Fig.9 3D surface height.

Fig.10 3D surface velocity on a central area
From these results we build the flow curve (i.e. the shear stress versus the shear rate) of the material which permits us to determine the behaviour law of the material. This curve is compared with the rheogram obtained from a conventional coaxial cylinders rheometer (Fig.11). Conventional rheometrical tests were also carried out on the same bentonite suspension with a Bohlin Gemini rheometer equipped with a Couette geometry (inner radius $r_i = 16.8 \text{ mm}$ , outer radius $r_o = 18.3 \text{ mm}$). Before the measurement, a pre-shearing phase at $500 \text{ s}^{-1}$ was applied to each sample for $200 \text{ s}$ following with a rest period for $600 \text{ s}$ corresponding to those applied for the inclined plane flow. Then, the imposed shear stress (imposed torque) on the sample was varied logarithmically every $40 \text{ s}$ from $10 \text{ Pa}$ to $30 \text{ Pa}$ and then from $30 \text{ Pa}$ to $10 \text{ Pa}$ . The most important difference resides on the shear rate scale only. Indeed, we are not able to obtain a number of shear rate decades with a flow test on an inclined plane. Several tests with different slope inclinations are necessary to achieve a greater shear rate. Nevertheless, the experimental results from the inclined channel are coherent with conventional test results. We obtain a yield stress ($\tau_c = 16.2 \text{ Pa}$ ) for conventional data by applying the Herschel-Bulkley model, and it is well correlated with the inclined plane result ($\tau_c = 17 \text{ Pa}$ ).

Coussot et al. (2005) have established a classification of the flow regime from physical observations of bentonite suspension flow on an inclined channel. They distinguished four main flow types:

- regime I: the fluid was submitted to a rapid acceleration at the gate opening and flowed rapidly downstream;
- regime II: the fluid initially flowed rapidly then its velocity decreased abruptly after some distance; the layer of material flowed slowly during some time then completely stopped;
- regime III: the whole material started to flow, then separated into a tail which remained attached to the initial dam and stopped flowing, and a front which went on flowing downstream over some distance then stopped;
- regime IV: the material slightly deformed but never flowed downstream even after several days.

All these regimes are dependent of the mass concentration of the clay suspensions and of the time of rest before the gate opening. So we can assume that our case is well described by the regime II definition. To validate the new behaviour law they have developed, Coussot et al. (2005) completed their study with 2D numerical calculations. The four regimes were observed and can be easily identified by the variation of the total kinetic energy (Fig.12). To compare with these results we calculate the surface kinetic energy from the $(25 \times 25 \text{ mm})$ square window and we suppose that the surface velocity is constant in a $500 \mu\text{m}$ fluid layer (Fig.13). The comparison of these two curves reveals that the timescales are not the same order of magnitude because of the length difference of the two channels. Except this point, the surface kinetic energy directly obtained from our
5. Conclusion

Surface height and velocity measurements have been carried out for evaluating rheological properties of a mud flow. The optical measurement technique has been applied to flows on a slope plane with and without cylinder. The first flow has been used to validate the technique and to verify its potentiality. The second has allowed the rheological parameter measurements. The results have been compared with classical measurements from a rheometer and with the literature and are in good agreement.

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References


