A Lagrangian algorithm for multiphase free surface flow

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Abstract A Lagrangian algorithm for the analysis of images related to a complex multiphase flow induced by the violent impact of a wave against a rigid wall, is proposed. Mathematical and numerical models are based on the Features tracking algorithm. Data are interpolated on a regular grid by adopting a non-Sibsonian Natural-Neighors scheme on a Voronoi tessellation. A detailed description of the flow features characterizing the interaction between the impulsive wave pressure gradient, the rigid wall and the free surface is given, together with a measure of the velocity of sound inside the flow. The accuracy of the method is exploited by using synthetic images based on the SPH numerical data of a free oscillating hemisphere, pounced on a rigid wall near the free surface.

1. Introduction

Physical investigation of the kinematical and dynamical flow field characterizing violent water free-surface motions under gravity is a challenge of the experimental fluid dynamic, and a key issue in several fields of the engineering. Multiphase flows, breaking waves, jet formation and air compressibility may occur, making difficult the measurement of the local kinematics and dynamics quantities.

In the context of coastal engineering, the impact of steep water waves can result in damage or collapse of structures. In particular failure of vertical breakwaters and coastal defenses has led to much attention been given to the pressure distribution which occurs when steep storm waves meet sea walls (Chan 1994, Chan & Melville 1998). A comprehensive review of research concerned with water wave impacts on walls is proposed in the review of Peregrine (2003). From the dynamical point of view, extremely large impact pressures can be measured which largely exceed those associated with the internal pressure of the waves. Recorded pressures are in the order of 10-100 times the hydrostatic pressure associated with the impacting wave height, depending on the impact conditions (Hattori et al. 1994, Cooker & Peregrine 1992). This clearly suggests that the largest pressures are essentially due to the flow inertia, gravity effects being negligible. The shape of the impacting wave has a significant effect on wave impact pressure exerted on vertical walls (Hull & Muller, 2002). Aeration and turbulence in the water also occurs from the breaking of previous waves and tend to be non-uniform.

In the naval context, the knowledge of the flow features occurring during the violent liquid motion inside confined spaces (Faltinsen et al., 2004), is a key issue for the safety of LNG (liquid natural gas) carriers. Since these ships have to operate in various filling conditions of their tanks, it is important to deeply understand the main features of the phenomena appearing with the tank almost completely (Rognebakke & Faltinsen, 2005), partially (Colagrossi et al., 2005) or barely filled (Boucassee et al., 2007). In particular violent free surface flows may appear when the energy spectrum of the ship motion is focused in the region close to the lowest natural tank mode. Then, large slamming loads (Faltinsen et al., 2004) may occur undermining even the integrity of the structure. In this condition a proper prediction of the impulsive loads and of its duration may matter for a suitable estimation of the hydroelastic effects.

A complex flow may characterize the evolution of a slamming phenomenon. For instance, when the initial impact angle is small, compressibility may matter as a consequence of the air entrapped. A
mutual interaction between gaseous and liquid phases may occur. Increasing the pressure inside the gaseous cavity may cause its oscillation and then its collapse with the formation of a mixture of gas bubbles and liquid. In this case the pressure inside the tank, i.e. the ullage pressure, strongly influences the appearance and the evolution of the gas bubbles during the impact.

Thus, a very complicated flow dominates the evolution of the hydrodynamic field of a steep wave impacting a wall. The most peculiar modality of impact is certainly the flip-through (Peregrine, 2003) in which a steepening wave approaches a vertical wall. Its presence delays the breaking of the wave inducing the rise of the wave trough at the wall. Its focusing with the advancing wave front causes a very large vertical acceleration of the flow turning it in the focusing area. Large pressures are then obtained as a consequence of the formation of a jet rapidly rising against the wall, at accelerations of the order of (100-1000)g, as measured in Lugni et al. (2006), by using the same Features Tracking (FT) algorithm proposed in the following.

In Miozzi (2005) the FT technique, was proposed for the direct measurement of vorticity. The main advantage of using the FT algorithm relies on the existence of a measure of the trackability of a feature. All tracking operations are performed where the solution exists, i.e. in points in which the image texture guarantees a convergence for the iterative scheme.

This approach permits to place the investigated feature in the middle of the interrogation window, maximizing the signal-to-noise ratio. By repeating the operation on a sequence of images, the complete and accurate evolution of the trajectory of the particle is gained and a Lagrangian analysis approach can be developed.

In this paper the same FT algorithm is extended to the estimation of the sound speed of an acoustic wave front. The propagation of acoustic wave fronts generated in the presence of a multiphase flow is analyzed. To this purpose a free-surface wave which impacts against a vertical wall is considered. In the following, after a detailed description of the FT algorithm (section 2), the comparison with the results of a SPH solver whose data are also used to generate synthetic images, is presented. Subsequently the experimental set-up (section 4) as well as the results of the analyzed flow field (section 5), are introduced.

2. The Feature Tracking and the Lagrangian approach

The aim of the FT algorithm is to solve, in a least-square approach, the minimization problem that arises from the optical flow equation, once a specific model for the motion of an interrogation window has been assumed. The model of motion can be described by the so-called warp function

\[
W(X, p) = [W_x(X, p) \ W_y(X, p)]^T,
\]

which belongs to the set of allowable operators mapping points from a template R, inside the frame I at time \( t_0 = 0 \) to the frame J at time \( t_1 = 1 \). The outcome of the mapping is a function of the point position \( X \) inside R and of the warp parameters vector \( p(x_0, u_0, u_1, \ldots) \). This vector has \( n \) components, where \( n \) is a function of the complexity of the warp. Explicit expressions can be found in Miozzi (2004) for the cases of pure translation and affine deformation. Here it only suffices to note that \( x_0 \) represents the centroid of the template R (also called feature in the rest of the paper), while \( u_0 \) represents the velocity of the centroid itself.

\( W \) transforms points \( X \) in the coordinate frame of the template R in points \( x = W(X, p) \) in the coordinate frame of the image I according to:

\[
x = x_0(t_1) + X(t_1) = W(X(t_1), p).
\]

In this expression, both \( x \) and \( X \) represent sub-pixel locations. With this background elements, it is now possible to take-up the so-called SSD (Sum of Squared Differences) minimization problem. This problem consists in evaluating the warp-parameters vector \( p \) that minimizes the intensity differences between the template R on frame I and its corresponding region on image J, warped back onto the coordinate frame of the template R:
The non-linear Eq. 2-1 can be linearized by means of a first-order Taylor expansion of \( J(W(X, p \pm \Delta p)) \). After some algebra the final expression for the FT system becomes:

\[
\Delta p = G^{-1} b \quad \left( [n\times 1] = [n\times n] [n\times 1] \right)
\]

where:

\[
G = \sum \left[ \nabla J \frac{\partial W}{\partial p} \right]^T \left[ \nabla J \frac{\partial W}{\partial p} \right] \quad \text{and} \quad b = \sum \left[ \nabla J \frac{\partial W}{\partial p} \right] \left[ I(X) - J(W(X, p)) \right] = -\sum \left[ \nabla J \frac{\partial W}{\partial p} \right] I_t,
\]

with \( I_t = \frac{\partial I}{\partial t} = J(W(X, p)) - I(X) \).

Eq 2-2 is iteratively solved assuming an initial value \( p_0 \) for \( p \), until \( \|\Delta p\|^2 < \epsilon \) or the number of iterations exceeds a chosen threshold: in the former case, the feature is validated; in the latter case the feature is rejected. A typical value for the error threshold is \( \epsilon = 0.0001 \).

When a feature is near a rigid wall, the interrogation window falls outside the flow domain. In this case, a logical mask is built to exclude out-of-flow elements of both the interrogation windows I and J from the evaluation of \( G \) and \( b \) (Miozzi, 2008). Particles near the free surface are tracked in the standard way.

For an image pair, velocity and velocity derivatives within the interrogation window can be approximated by the expressions:

\[
\tilde{p}_i = \frac{p_i - x_0}{t_1 - t_0} \quad i = 5, 6
\]

\[
\hat{p}_i = \frac{p_i}{t_1 - t_0} \quad i = 1, 2, 3, 4
\]

where \( p_i \) is the parameter set of the warp \( W \) (\( p_5, p_6 \) are the displacements, \( p_1, p_2, p_3, p_4 \) are the gradients in affine deformation), \( x_0 \) is the template centroid position at time \( t_0 \), \( t_1 - t_0 \) is the frame sampling time in seconds, i.e. the time lag between image I and image J and \( \tilde{p} \) is the set of warp parameters with lengths expressed in pixels and time expressed in seconds (velocity and its derivatives).

The solvability of the system in Eq. 2-2 is strictly related with the invertibility of the matrix \( G \), i.e. the solution exists if there exists an inverse \( G^{-1} \) matrix so that \( G^{-1} G = G G^{-1} = I \), where \( I \) is the unitary matrix. Let us consider Eq. 2-2 \( G \) matrix is invertible and well conditioned if its eigenvalues are both not null and do not differ too much. It can be observed that for a covariance matrix like \( G \) (symmetric, semi positive defined matrix of real numbers), the eigenvalues \( \lambda \) are always real and non-negative (Danielson, 1992).

If the solution of the Eq. 2-2 is searched at locations where the image gradients are not null (and sufficiently strong), both the eigenvalues of the \( G \) matrix will be large enough to guarantee that \( G \) can be inverted. In practice, the control on the eigenvalues is based on a minimum threshold value that takes into account the noise inside the image. As a consequence, the choice of the threshold value for the minimum eigenvalue is a critical step in the feature description. Moreover, the presence of noise requires both eigenvalues to be large enough, while the good conditioning of the matrix requires that the two eigenvalues do not differ too much. Those conditions can be simply matched by observing the minimum eigenvalue of correlation matrix of the intensity gradients \( G \) in small windows (3x3) all over the image: if this minimum eigenvalue is larger than the noise level, the matrix \( G \) is well conditioned. Points that are consistent with those conditions are called “good
features to track”: they are detected inside the first frame of a couple or sequence of images and tracked by the FT algorithm. In the present study, the typical value of the minimum threshold has been imposed equal to the 0.01% of the maximum among the minimum eigenvalues. The concept of good features to track has been introduced as a requirement of the tracking algorithm. The uniqueness of the FT in the PIV and PTV framework concerns the choice of a quantitative measure of the quality of the tracking results, the $G^{-1}$ matrix, providing that it is evaluated at points (the good features set) where the solution of tracking exists and is well conditioned.

Due to the Lagrangian nature of the FT, the vector $\tilde{\mathbf{p}}$ can be assigned to the middle point of the displacement between time $t_0$ and $t_1$. The forward differences scheme adopted in Eq. 2-1 is substituted with a central differences one, improving the accuracy of the tracking.

When dealing with image sequences, the arrival points of one tracking operation between images 0 and 1 can be used as starting points for a new tracking operation between images 1 and 2 and so on, until the end of the sequence. Lost features are replaced with new ones, in respect to a minimum distance parameter (new trajectories born). By applying the tracking to a whole image sequence, a complete set of trajectories is obtained.

A trajectory is uniquely defined in a label space (Kraichnan, 1968, Bennet, 2006) and exhibits arbitrary time of release $s$ (i.e. the time at which its first position has been tracked) and time of death (i.e. the time at which the tracking of its associated particle fails). It is described by its own label $a$, and, at a discrete time instant $t$, it is characterized by its position, its velocity and its velocity spatial derivatives, as evaluated from the FT in the Lagrangian framework. Lagrangian accelerations are evaluated by using the velocity data along the trajectory, i.e.:

$$\frac{\partial}{\partial t} \mathbf{u}(a,s|t)$$

With the aim to reduce the spiky noise in the acceleration data, a robust local regression fit (LOESS, locally weighted scatterplot smoothing) has been applied to the time series of the velocity (Cleveland and Loader, 1996). A simple second order differentiation scheme extracts the acceleration from the smoothed sequence.

All the results presented in the following are obtained in a Lagrangian fashion but, in order to produce comparable datasets, they are used to build a Voronoi tessellation over which an interpolation on a regular grid is performed. Interpolation scheme uses the Natural Neighbours support by applying a non-Sibsonian interpolation scheme.

3. Comparison with synthetic data

The use of synthetic images permits to compare the results of the image analysis with the previously known data (analytical or numerical) describing the synthetic velocity field, and to estimate the performances and the robustness of the image analysis algorithm. Such a comparison is usually carried out at a specified time instant on a regular grid. Performances of FT can be found in Miozzi (2004), Stanislas et al. (2008), Miozzi et al. (2008), where results in terms of bias and rms errors can be found, usually for viscous and rotational flows.

A fundamental application of the present FT method concerns the capability to measure the sound speed related to the compressibility of an air-water mixture in an almost potential flow. To this purpose, aiming to validate the algorithm as well as to estimate the accuracy of a measure involving an acoustic wave, numerical data coming from a Smoothed Particle Hydrodynamics solver (SPH, Colagrossi and Landrini 2003) have been used.

The SPH is a fully-Lagrangian mesh-less technique, originally developed to deal with astrophysical problems and successfully extended to a variety of fluid dynamics problems. The main feature of the SPH is its flexibility in handling complex flow fields and in including all fundamental physical
effects. In particular, the method is able to deal with free surface flows, even with breaking waves, multi-phase flows and then, as in the present case, bubble dynamics in water.

In the SPH solver, the fluid is represented as a collection of N particles that interact each other through suitable equations (see Colagrossi and Landrini, 2003, for more details) for the kinematical flow field and an equation of state for the pressure field:

\[ p(\rho) = P_0 \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \]

where \( P_0 \), \( \rho_0 \) and \( \gamma \) are chosen to keep the maximum density oscillations under a prescribed level. In practice, this is accomplished by choosing a suitable sound speed \( c_s = dp/d\rho \) (weakly-compressible hypothesis).

The SPH simulation here proposed describes the behavior of a gaseous half-cylinder bubble, pounced on a rigid wall close to the free surface. A multiphase flow field is then considered: an air phase for the bubble (with a suitable \( c_s \)) and a liquid phase with \( c_s = 70 \text{ m/s} \) for the surrounding water. By forcing the bubble with an initial isotropic compression, the air cavity evolves in an oscillatory motion, mainly governed by the compressibility effects which are related with the air sound speed. The consequent acoustic wave front propagates in water with the speed of sound of the liquid phase. Because the gravity is omitted in the simulation, i.e. no buoyancy forces act, the bubble oscillates around its mean position.

The main advantage of using such a simulation is related to the Lagrangian nature of both the SPH and FT algorithms: the acoustic wave is then captured in a Lagrangian fashion, i.e. when it collides with the fluid particle.

In the SPH solution, the particles are initially disposed in a regular way, imprinting the synthetic images with a strong texture character. Images have been generated by considering the label of a randomly chosen set of particles at time \( t=0 \) and following them at each time instant in the SPH simulation. The number of selected labels is around 23000, which gives, on a 1024x1024 pixel image, a particle density of 0.022 particle/pixel, i.e. a mean of 13.75 particles in the interrogation window (25x25 pixel). Particle diameters range from 2 to 3.5 pixels, while their brightness from 140 to 200.

![Figure 1: a detail of an SPH-based synthetic images](image1)

![Figure 2: pressure wave front from the SPH simulation, traveling away from the bubble. Free surface at Y=0.](image2)

Images have been analyzed by using a 25x25 interrogation window, with a Gaussian widow of \( \sigma = 1 \).
4. A method for the estimation of the sound speed

The analysis of the synthetic images obtained from the SPH data highlight the capability of the FT algorithm to capture the horizontal and vertical accelerations related with the acoustic wave front (see Figure 3 and Figure 4 for the vertical and horizontal acceleration, respectively, in the field at a fixed time instant). With the aim to estimate the speed of sound in water, we propose a simple geometric method, inspired to the “method of the characteristics”. A generic (horizontal or vertical) section of a single acceleration component is taken at each time instant and its values are used to build a new function in the space $x_i - t$ (see Figure 5 or Figure 6). The passage of the perturbation is then identified and a graphic measure of $dx_i/dt$ can be determined from its trace. With a suitable selection of an horizontal and vertical section, the sound speed can be estimated as:

$$V_s = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}.$$

![Figure 3: Vertical acceleration at time $t = 10$ (a.u.). The horizontal dashed line indicates the position of the section whose time history is shown in Figure 5. The Red filled points show the position of the wave front as evaluated from the horizontal section.](image1)

![Figure 4: Horizontal acceleration at time $t = 10$ (a.u.). The vertical dashed line indicates the position of the section whose time history is shown in Figure 6. The green filled points show the position of the wave front as evaluated from the vertical section.](image2)

![Figure 5: Time history of the vertical acceleration along the horizontal line shown in Figure 3.](image3)

![Figure 6: Time history of the horizontal acceleration along the vertical line shown in Figure 4.](image4)

An application of this technique is shown in Figure 3 to Figure 6. Four horizontal and three vertical sections have been chosen inside the fluid domain. In Figure 3 and Figure 4 vertical and horizontal sections are shown. Figure 5 and Figure 6 report the corresponding time histories where the passage
of the perturbation marks the space with an easily identifiable trace. The corresponding slope defines the vertical and horizontal component of the wave front velocity. The hypothesis of linear propagation of the perturbation seems to fail by observing Figure 5. Here the slope gently changes for small values of the horizontal position. This behavior is due to the presence, in the SPH domain, of a numerical beach used to damp out the reflection induced by the border of the numerical domain. In the present case, the numerical beach begins at a radial position corresponding to 100 pixels from the center of the bubble and increases its effects with the distance from the bubble. These grid cells are then omitted from the calculation of the slope of the trace. The results obtained can be used both to identify the position of the wave front and to evaluate the sound speed. For the former, a single horizontal or vertical section is required, for the latter both components are needed. Figure 3 and Figure 4 show a series of red and green filled points highlighting the position of the front as estimated by the horizontal and vertical sections, respectively. The sound speed has been calculated at the intersection points between horizontal and vertical sections, and at the corresponding times t= 9, 10 and 11. These results suggest a mean value of 71.5 m/s, with a standard deviation of 2.1 m/s. They are very close to the sound speed imposed to the liquid phase of SPH solver, i.e. 70 m/s (about 17 pixel/dt in image units).

5. Experimental setup

The present study, related to the development of an algorithm of analysis of Lagrangian data, is part of a widest research activity. The main aim is the experimental investigation of the effect of the ullage pressure on the evolution of the air-cavity entrapped during a wave impact against a vertical wall caused by a sloshing event in a rigid prismatic tank. To this purpose an ad hoc plexiglas tank, reinforced with steel and aluminium structure, has been built. A global view of the tank is shown in Figure 7 The same geometry of the tank used for the previous experiments at atmospheric pressure (Lugni et al. 2006) has been reproduced: L = 1m the length, H = 1m the height, b = 0.1m the width. Finally the same filling depth, i.e. d = 0.125m, has been used. The transversal aspect-ratio of the tank ensures an almost-2D flow in the middle vertical plane of the tank unless flow instabilities are excited. A mechanical system forces a pure-sway motion with a sinusoidal law, Asin(2\_t/T), being A the amplitude and T the period of the prescribed motion.

A suitabler vacuum pump was used to vary the ullage pressure inside the tank between 1bar, i.e. atmospheric pressure, down to 15mbar. In the arrangements used for the present experimental investigation, the tank was equipped with eight differential pressure probes along a vertical wall, with maximum range of linearity varying between 14kPa up to 40kPa. During the tests flow visualizations were performed through high-speed digital video cameras. A high-speed camera Photron Ultra was placed very close to the lateral wall of the tank, as shown in
the photo of figure 2, and focused to minimize perspective errors in the images. More in detail, the measurement area corresponding to 1024x512 pixels, was focused at the center of the image. A target of calibration has been used both to evaluate for each run the magnification factor (~7 pix/mm), and to check the deformation of the measurement region. A frame rate of 4000 fps was used to well capture the high velocities of the flow during the formation of the jet, as well as to capture the oscillation of the air-cavity entrapped during the impact event. To ensure synchronization, the trigger signal, used to start the camera, has been acquired by the acquisition system. In the following, to test the potentialities of the FT algorithm proposed, the case corresponding to a ullage pressure of 25 mbar has been considered.

6. Results

In Lugni et al. (2006) a detailed observation of the flow field characterizing the evolution of a wave impact event allowed for the identification of three different impact modalities in dependence of the different modes the wave approached the wall. A first mode, called mode ‘a’, identifies the impact of an incipient breaking wave and is characterized by the formation of a pure flip-through event. The second mode, i.e. mode ‘b’, is typical of the impact of a broken wave with no phase mixing. In this case the most important feature is the formation and the evolution of an air cavity at the wall. Finally the third one, mode ‘c’, is caused by the impact of a broken wave with air-water mixing. A turbulent flow induced by the fragmentation of the free surface before the impact identifies this type of impact. Hereinafter we focus our attention on the kinematical and dynamical evolution characterizing the flow field of the mode ‘b’ impact.

Although some experimental evidence of the evolution of the bubbly flow during the wave impact against a vertical wall is available, to our knowledge no detailed descriptions of the kinematical and dynamical evolution exists.

To highlight the potentialities of the algorithm proposed, such an algorithm has been applied to the comprehension of the mentioned physical phenomenon. The availability of the flow field in terms of Lagrangian velocities and accelerations allows for a very detailed description both of the kinematics and the dynamics features characterizing the phenomenon. Figure 9, Figure 10 and Figure 11 show the Lagrangian acceleration field close to the wall during the evolution of the wave impact. Experimental images of the event are also reported in background, as well as the lines of radiative pressure (i.e. the tangent at each point to the acceleration path), equivalent to the streamlines of the velocity field. More in detail, a spatial integration has been made by using the streamline MATLAB function and, as seeding points, a suitable dataset has been used to properly reproduce the behaviour of the whole field.

The measured quantities allow for identification of several stages characterizing the evolution of the bubble along the wall. In particular during the entire evolution (see Figure 9 and Figure 10) we can identify four main stages:

a) closure of the air cavity against the wall (top panels of Figure 9);
b) isotropic compression/expansion of the air cavity (bottom panels of Figure 9);
c) anisotropic compression/expansion (top panels of Figure 10)
d) rise of the air cavity along the wall (bottom panels of Figure 10).

Stage a) is governed by the air leakage from the forming air cavity. The compression/expansion of the gaseous phase, caused by the surrounding water fluid, increases the air momentum. Just after the closure of the air cavity, the impinging water-jet causes the occurrence of a flip-through-like event: a vertical jet is then formed. The last one is further energized by the air leakage forced by the compressibility effect (stage b). During this stage, he compression/expansion of the bubble is almost isotropic (bottom panels of Figure 9) and the leakage effect causes an irregular and almost turbulent vertical jet.
The bubble remains at the same height along the wall, i.e. the effect of air compressibility counteracts the effect of the wave field which would tend to rise the bubble along the wall. Successively, i.e. during stage c), the air cavity is stretched vertically along the wall. At these instants, we can identify two different flow regions inside the bubble dominated by the balancing between compressibility and wave field effects. In particular, in the bottom part the former counteracts the latter, i.e. the bottom portion of the bubble remains at the same height. Differently in the top portion the upward wave flow prevails inducing a stretching of the bubble (see top panels of Figure 10). Due to the air leakage, the acceleration field around the bubble is strongly reduced during the mentioned first three stages. As a consequence the energy associated with the bubble compression is strongly reduced. Then air-leakage stops and the bubble rises along the wall because of the advection induced by the external flow field in the water phase (see bottom panels of Figure 10).

Figure 11 helps better understand the nature of the anisotropic mode. The reflections of the acoustic wave front against the boundaries (rigid walls, bottom and free surface) cause a change of direction of the rarefaction-compression shock wave (see dashed red lines in Figure 11). Constructive/destructive interferences occur among several acoustic wave fronts: the one induced by the natural bubble oscillation (whose frequency is mainly dominated by the sound speed of gaseous phase) and the many reflected wave fronts traveling with the speed of sound of the liquid phase.

This global behavior is confirmed from the right panel of Figure 12, showing the time evolution of the vertical acceleration along a vertical section. After the impact event ($t=0$) the first wave front
travels downward (first blue area in right panel of Figure 12); it gets reflected by the bottom (red area at the time instant $t=0.003$). The reflected acoustic wave front hits the free-surface inducing a new reflection downward (time instant $t=0.006$). This phenomenon continues cyclically with a reduced energy because of the air leakage, of the radiation damping, and a partial energy redistribution on smaller scales induced by the free-surface.

Figure 10 Lagrangian acceleration field and radiative pressure lines in the flow field during the stage c) (top panels) and stage d) (bottom panels). The bold numbers indicate the position of the pressure transducers along the wall.

Figure 11 Lagrangian acceleration field and radiative pressure lines in the flow field during the stage c).
The data of Figure 12 can be also used to estimate the sound speed.

![Figure 12 Time evolution of the horizontal acceleration along an horizontal section (left). Time evolution of the vertical acceleration along a vertical section (right)](image)

Although the water is, usually, considered as an incompressible fluid, in the last decade it is affirmed the idea that when impact pressures are sufficiently high compressibility of water matters. Quantifying results show that even a small quantities of bubbles (<2%) generates a reduction of the sound speed from around 1500 m/s to less than 100 m/s (Van Wijngaarden, 1965, Watanabe and Prosperetti 1994, Cooker, 2002).

In accordance with the method described in section 4, two horizontal sections, corresponding to y = 3.8 and 18.9 mm, and two vertical sections, corresponding to x=11.4 and 18.9 mm, have been selected. The table below shows the estimate of the sound speed at the four points, obtained as intersections of the horizontal and vertical sections.

<table>
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<th>X (mm)</th>
<th>Y (mm)</th>
<th>V_s (m/s)</th>
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</thead>
<tbody>
<tr>
<td>11.4</td>
<td>3.8</td>
<td>165.5</td>
</tr>
<tr>
<td>11.4</td>
<td>18.9</td>
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<td>18.9</td>
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Table 1: Estimation of the sound of speed V_s

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