On the accuracy of one-point and two-point statistics measured via high-speed PIV

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Abstract
High speed particle image velocimetry (PIV) measurements with a sample frequency of 20 kHz are conducted in order to characterize the temporal and spatial response of such a system. The limits and drawbacks associated with the PIV measurement technique are investigated. Hot wire measurements are performed for comparison. In contrast to time resolved analog measurement techniques (e.g. hot wire anemometry) it is not possible to apply analog filters to the PIV signal in order to avoid aliasing effects. But the PIV processing itself acts like a filter in space and time. Here a special attention is turned towards the influence of the interrogation window size on the frequency spectrum and therewith the statistical quantities. Two types of flows are chosen for the study presented here. The first is grid turbulence. It provides nearly homogeneous and isotropic turbulence at a certain distance downstream of the grid. The second is the wake behind a cylinder, where the flow field has a more inhomogeneous character. It can be shown that the spatial averaging in the interrogation window results in a low pass filter for the frequency response and the transfer function is a modified sinc function.

Nomenclature

\( \delta_i \) Size of the interrogation window in the \( i \)’th direction
\( \Delta x_i \) Pixel shift in the direction \( x_i \)
\( d \) Cylinder diameter
\( E_{o,n} \) Outer noise factor
\( E_{i,n} \) Inner noise factor
\( f_s \) Sampling frequency
\( k_i \) Wave number in the \( i \)’th direction
\( \lambda_r \) Temporal Taylor integral scale
\( I_W \) Interrogation window
\( n \) Index of PIV snapshot
\( \nu \) Kinematic viscosity
\( N \) Number of snapshots by PIV
\( R \) Pixel resolution
\( \sigma \) RMS-value
\( S_t \) Strouhal number
\( t \) Time
\( \tau \) Time difference
\( U, V \) Averaged velocity comp. with respect to x,y
\( u, v \) Instantaneous velocity component with respect to x,y
\( u', v' \) Fluctuating velocity comp. with respect to x,y
\( U_\infty \) Free stream velocity
\( x, y \) Cartesian coordinates of the PIV plane

1 INTRODUCTION

Particle Image velocimetry is a non-intrusive technique to measure the flow velocity in an extended region instantaneously. Among others Westerweel (1997) describes the measurement principle and its limitations whereas Adrian (2005) gives an overview about the development of the method in the last two decades. With the introduction of high speed cameras and high-repetition-rate lasers the framing rate can be increased by several orders of magnitude compared...
to conventional PIV systems. For turbulent flows with moderate Reynolds numbers in air, the measurement frequency of such a PIV system can be as high as the Kolmogorov frequency in the investigated flow field. Since two-point as well as one-point correlation functions are directly related to the frequency spectra by the Wiener–Khinchin theorem, it is reasonable to take a closer look to the frequency response of the PIV-process chain. The therewith obtained results can be used as a basis for the understanding of varying time averaged statistical flow quantities determined from PIV measurements. Thus, the main goal of the present paper is to check the temporal frequency response of a high-repetition-rate PIV system whereas a special attention is paid on the influence of the interrogation window (IW) size used for the PIV evaluation. Two test cases are considered and the spectra obtained from PIV are compared to the ones measured via hotwire anemometry (HWA). The first is the flow field downstream of a square-mesh grid. The turbulence level of such a flow is usually low, typically in the range of $1 - 2\%$. Therewith the frozen turbulence hypothesis is a good approximation and it is a good test case for the capability of the PIV system to resolve the power spectrum of even small fluctuating velocities. The second example is the flow around a circular cylinder at a Reynolds number of $Re = 10,000$. A dominant oscillation pattern can be identified clearly in such a flow field resulting in a peak in the averaged power spectrum at the non-dimensional Strouhal frequency (Strouhal, 1878). The PIV measurement plane is perpendicular to the cylinder axis, so that the wake behind the cylinder is captured. It should be noted, that the setup in the present experiments was chosen to analyze the PIV method and to allow an easy comparison between the PIV and the hot-wire spectra. The setup is not optimized for the investigation of the respective flow field itself. For example Comte-Bellot and Corrsin (1966) stated, that in order to obtain real isotropy in grid turbulence the test section should have a contraction upstream of the grid position. This is not the case here.

2 THEORETICAL BACKGROUND

2.1 Tracer particles

The frequency response of the PIV measurement is limited by the ability of the used tracer particles to follow the instantaneous motion of the investigated flow field. If the particle-to-fluid density ratio $\rho$ is high, which is almost always the case for seeding in air, the particles will show a low-pass filtering behavior. Depending on their size, they will only follow the fluctuating velocity of the flow up to a certain cut-off frequency, which has to be specified by a particle-dynamic equation. Mei (1996) has given an extensive discussion of the particle response to isotropic turbulence. He suggests a formula for the estimation of the cut-off frequency $f_{\text{cut-off}}$, based on a 50\% energy response of the particle. It can be shown that for measurement frequencies in the range used in the present study ($f_s = 20 \, kHz$), the particle diameter should remain below $1\mu m$ (Henning and Ehrenfried, 2007).

High concentrations of particles with a mean diameter below $1\mu m$ can be generated with a proper designed seeding generator operated with an additional impactor (Kähler et al, 2002). By filling a large volume with particles and measuring the size distributions in certain time intervals, Kähler (2003) observed an increase of the mean particle diameter with time due to coagulation effects, reaching a fixed maximum of $3\mu m$ after 10 minutes. He concluded that this is an important result for flow investigations in closed loop wind tunnels, which would be crucial for the cylinder-wake experiment. But it was noticed during the experiment presented here that the particle concentration decreases rapidly when the seeding atomizer is turned off while the wind tunnel is running. Thus, it can be assumed that the vast majority of the particles do not remain for many cycles in the flow. This can be explained by the strong velocity gradients
the particles are undergoing while passing the driving fan. Thus, the wind tunnel itself has an impactor effect and it is assumed that the mean particle diameter remains below 1 $\mu m$.

2.2 PIV methodology

All PIV methods have in common that a resulting velocity vector represents a mean value over a specific volume and a specific time interval. The volume is defined in one direction by the thickness of the light sheet or the depth of view. In the two other directions the volume is determined by the interrogation window which is used for PIV evaluation. The time interval is given by the delay time $\Delta t$ between the two laser illuminations. Due to this averaging, PIV measurement results are always spatially and temporally filtered, additionally to any filter effects caused by the inertia of the tracer particles described above. Of course, a higher spatial resolution is obtained by the measurement of single particle displacements (Adrian, 1997). But the derivation of mean displacements averaged over a finite area, where many particles are present, increases the accuracy and robustness of the measurement, because inaccuracies in the displacement of individual particles partly cancel out each other. In order to benefit from this advantage of PIV, there always exists a lower bound for the choice of the interrogation window size in order to ensure an averaging over enough particles (Nobach and Bodenschatz, 2006). The temporal filter which is inherent in the PIV method due to the finite measurement interval corresponds to a moving average in time with constant weighting. Thus, the frequency spectrum of this filter is a sinc function. Accordingly, all temporal fluctuations are low-pass filtered and frequencies much higher than $1/\Delta t$ are significantly attenuated. If the physical area which corresponds to the interrogation window is rectangular and an averaging with a constant weighting is used, then in the two directions within the observation plane the spatial filter has also a sinc function like its wave-number spectrum. The cut-off frequency of the this spatial filter is determined by the size of the interrogation window. It has to be noted, that only in an ideal case the spatial averaging is performed over a rectangular area. In practice a distortion in the imaging or evaluation methods with window shift and window deformation can change this area in a complicated way. Of course, the characteristics of the spatial filter can also be changed by the use of a weighting function within the interrogation window. In the direction normal to the observation plane the filter characteristic is given by the structure and the thickness of the light sheet or by the focal depth. This depends on whether a thin light sheet is used or not. In any real flow field the temporal and spatial fluctuations are related to each other. Thus, a spatial filter can also affect the temporal response of a PIV system and vice versa. The spatial filter effect which is inherent in the PIV method due to the averaging over the interrogation window was experimentally investigated by Foucaut et al (2004). They considered the wave-number spectrum of a turbulent flow and evaluated how the spectrum depends on the size of the interrogation window. Using constant weighting within the interrogation window their results show, that indeed, as theoretically expected, the PIV processing results in a spatial low-pass filter with a sinc function as spectrum. Saarenrinne et al (2001) considered the effect of the spatial filter on averaged turbulent quantities like the Reynolds stress, the RMS of velocity fluctuations, or the turbulent kinetic energy. But their experimental results did not confirm the theoretically predicted dependency of the averaged turbulent quantities on the size of the interrogation window. They observed, that the attenuation of the turbulent quantities rather depends on the used image magnification and not, as expected, only on the physical size of the interrogation window. From that they concluded, that other measurement errors were critical for their observed results. It is well known, that the PIV procedure, apart from the described filter effects, brings a certain amount of extra noise into the measurement results, and that this noise has several causes and depends on many parameters. Huang et al (1997) give a
detailed description of the measurement noise due to the complete PIV-process chain. Part of the noise can be estimated by a measurement in a fluid at rest (Foucaut et al, 2004). Factors like camera’s dark-current noise, the non-uniformity of the illumination and digitization errors due to the limited fill factor of the camera chip can contribute to this part of the measurement error. For wind tunnel experiments, a measurement of the free flow can give an indication of the minimum noise floor to expect in the final measurement due to the free-stream turbulence level and peak-locking effects caused by the evaluation algorithm. Flow specific sources of error like improper particle seeding, strong velocity gradients and strong three-dimensionality of the flow motion also contribute to the measurement error but they cannot be specified by such measurements. They have to be minimized by the experimentalist by choosing a adequate seeding concentration, using advanced evaluation algorithms and adjusting the time delay between the two laser pulses in order to extract the maximum possible information from the moving particles (Adrian, 1997). It is not clear, how the averaging over finite volumes and finite time intervals effects the extra measurement noise. It is possible that part of this noise is attenuated by the spatial filtering like the fluctuations in the velocity field which belong to small flow structures. For example, the electronic noise in the camera generates small random fluctuations of the gray values in all pixels, which lead to small distortions of the particle images. These distortions correspond to random fluctuations in the interrogation window, which have a certain wave-number spectrum. Hence, one can expect that this part of the measurement noise is reduced by the averaging according to the characteristic of the spatial filter. Of course there may be also parts of the extra noise which cannot be identified with spatial fluctuations in the interrogation window. For example, certain inaccuracies and numerical errors in the peak fit procedure may not depend on the size of the interrogation window at all.

2.3 Spectral computations

In the present paper frequency spectra of high-speed PIV results are considered. It is clear, that there is no way to apply a proper anti-aliasing filter within the PIV process chain. It appears to be possible, that the filter properties of the PIV averaging, which were described above, can be used to suppress high frequency fluctuations in the velocity signal, so that no aliases of these fluctuations occur at lower frequencies in the spectrum. But to predict the influence of the PIV averaging on the frequency spectrum, it is necessary to understand the effect of the spatial filter on the temporal response of the system. This effect may depend on the type of flow which is considered. Additionally one has to be aware, that the extra measurement noise can contaminate the complete frequency spectrum. It is not obvious, how this extra noise effects the frequency spectrum and how this effect depends on parameters like the size of the interrogation window. For the grid turbulence experiment considered in the present paper $u' / U_\infty \ll 1$ holds, where $U_\infty$ is the mean velocity of the flow field. In this case Taylor's frozen turbulence hypothesis is a good approximation of the real flow. Then the time signal $u(t)$ at a fixed point corresponds to the velocity distribution $u(x/U_\infty)$ along a straight line in mean flow direction at a fixed time. Here it is assumed that the mean flow is parallel to the $x$-axis. Correspondingly, one can transform a frequency spectrum which is observed at a fixed point to a wave-number spectrum in $x$-direction. Each frequency $f$ corresponds to a wave number

$$k_x = \frac{2\pi f}{U_\infty}. \quad (1)$$

Rectangular windows with constant weighting are used as interrogation window, and the evaluation procedure uses window shifting and window deformation. The window shifting changes
the filter effect of the temporal averaging significantly. In the case with frozen turbulence and
\( u'/U_\infty \ll 1 \) the windows are shifted almost exactly in the mean flow direction. This means,
that the temporal filter operates now in a moving reference frame in which the flow field is
almost steady. The temporal filter is still effective, but it acts only on unsteady fluctuations in
this reference system. But there are only minor fluctuations present which are due to deviations
from the frozen turbulence assumption. In this special case, the main temporal fluctuations
which are recognized by an observer at a fixed position are not effected at all by the PIV in-
trum in the present case. Hence, the random spatial fluctuations appear as frequency dependent noise
in the measured frequency spectrum. This part is not affected by the spatial filter so that it becomes frequency
independent. Nevertheless, this part may also depend on the size of the interrogation window.
There are only minor fluctuations present which are due to deviations from the frozen turbulence assumption. In this special case, the main temporal fluctuations which are recognized by an observer at a fixed position are not effected at all by the PIV inherent temporal averaging. If the flow field is simply convected in x-direction only the spatial filter in this direction has a significant influence on the measured frequency spectrum, and the spectrum of this filter is given by the \( \text{sinc} \) function in the present case. This can be expressed by the approximate relation

\[
E_m(f) = E(f) \left[ \frac{\sin \left( k_x(f) \delta_x/2 \right)}{k_x(f) \delta_x/2} \right]^2 = E(f) \text{sinc}^2(k_x, \delta_x) \quad (2)
\]

between the true power spectrum \( E(f) \) of the velocity fluctuations \( u'(t) \) at a fixed point and
the measured spectrum \( E_m(f) \). Here \( \delta_x \) is the physical extension of the interrogation window
in x-direction, and the function \( k_x(f) \) is given by equation (1). Equation (2) is analog to the
relation which was used by Citriniti and George (1997) to describe the filtering effect of long
hotwire probes. So far extra measurement noise is not considered in equation (2). For the
present study this measurement noise is formally split in two parts. The first part is assumed
to consist of random fluctuations which are uniformly distributed in space and simply added to
the actual flow field. These fluctuations appear as white noise in the wave-number spectrum and
are attenuated by the PIV averaging like the fluctuations in the flow field, as explained in the
previous section. Hence, the random spatial fluctuations appear as frequency dependent noise
in the measured frequency spectrum. In the following, this first part of the noise is denoted as inner noise and described by the coefficient \( E^i_n(\delta_x, \delta_y) \), which is simply added to the real power spectrum. The second part of the noise is assumed to be additional white noise in the frequency spectrum. This part is not affected by the spatial filter so that it becomes frequency independent. Nevertheless, this part may also depend on the size of the interrogation window. The second part is denoted as outer noise and it is described by the coefficient \( E^o_n(\delta_x, \delta_y, \Delta t) \), which is added to the filtered power spectrum. Then the relation between the true and the measured power spectra is given by

\[
E_m(f) = \left\{ E(f) + E^o_n(\delta_x, \delta_y) \right\} \text{sinc}^2(k_x, \delta_x) + E^o_n(\delta_x, \delta_y, \Delta t) . \quad (3)
\]

Foucaut et al (2004) suggested a similar approximation in order to match a wave-number spec-
trum calculated from PIV-data to the one obtained via HW A. In contrast to the present study,
they considered the wave-number spectra and translated the frequency spectrum \( E_{HW A}(f) \)
measured by the hot-wire into a wave-number spectrum using Taylor’s frozen turbulence hy-
pothesis. Analog to their work here also hot-wire measurements are performed parallel to the
PIV, and relation (3) is used to calculate an estimated PIV spectrum from the hot-wire result.
But here the resulting frequency spectra are compared instead of wave-number spectra. There-
fore \( E_{HW A}(f) \) is taken as the real frequency spectrum \( E(f) \) and the noise coefficients \( E^i_n(\delta_x, \delta_y) \)
and \( E^o_n(\delta_x, \delta_y, \Delta t) \) are adjusted, so that the estimate \( E_m(f) \) fits best to the frequency spectrum
measured by high-speed PIV. It has to be noted, that equation (3) is a simplification in the
sense that the interdependencies between the spatial filters in different directions are neglected.
Theoretically an averaging in \( y \) or \( z \) direction can also have an effect on the wave-number spectrum in \( x \) direction. This depends on the type of flow and its statistical properties. In the
present tests square interrogation windows are mostly used and the thickness of the light-sheet is not large compared to the extension of the interrogation windows. It appears that under these circumstances at least for the considered case with frozen turbulence the influence on the $x$-spectrum of the averaging in $y$ and $z$ direction is rather small. Also Foucaut et al (2004) used this approach in their analysis and they obtained good results.

3 EXPERIMENTAL SETUP AND METHODS

3.1 PIV system

The PIV system consists of two diode pumped Nd:YAG pulse-lasers and a CMOS high-speed camera that records the light scattered by DEHS tracer particles. A seeding generator with 40 atomizer nozzles is used. In order to obtain particles with a narrow band distribution and a mean diameter below 1 $\mu m$ an impactor is installed at the exit of the generator (see (Kähler et al, 2002)). The camera (Type: Photron Ultima APX-RS) has an internal memory of 8 GB and a dynamic range of 10 bit per pixel. It has $d_s = 17.5 \mu m$ square pixels in a $1024 \times 1024$ px sensor. The maximum frame rate at full resolution is 3000 fps, while higher frame rates are available at reduced spatial resolution. The resolution in the study presented here is $320 \times 240$ px in case of the grid turbulence experiment and $256 \times 256$ px for the cylinder wake flow at a frame rate of $20\ kHz$. Limited by the internal memory this results in a maximum of 83558 and 98000 frames respectively for a single measurement with a duration over approx. 4 seconds. The frequency-doubled laser system (Type: Lee Laser, LDP-200MQG Dual) emits laser pulses with a maximum energy of $2.5 mJ$ per pulse/head at a frequency of $20\ kHz$ with a temporal width of $200\ ns$. The thickness of the light sheet was $1.5 mm$ in the investigation area. In order to eliminate errors due to the shape difference of the light sheet between two pulses, both lasers are fired simultaneously. This means that the so called frame straddling is not used in this experiment and the measurement frequency defines the time delay between two pulses.

![Figure 1: Schematically setup of the experiment. left Cylinder wake, right Grid turbulence.](image1.png)

![Figure 2: Sketch of the turbulence-generating grid.](image2.png)

3.2 PIV image processing

Different algorithms for the calculation of the correlation plane resulting from the image pairs are evaluated. A detailed description of the obtained results goes beyond the scope of this paper. In conclusion it can be pointed out that together with the multi-pass algorithm, the
sub-pixel image deformation reduces the noise of the calculated spectrum significantly compared to a standard algorithm. Other methods like for example the symmetric phase only filtering (Wernet, 2005) or the Nyquist frequency filtering have no effect or are even increasing the noise level.

High frequencies in turbulence are related to small scales of motion. In order to capture the maximum possible frequency range, the dynamic range and therewith the sub-pixel accuracy of the PIV processing is an important issue for the present investigation. In order to achieve this accuracy an appropriate estimator for the peak location in the correlation plane to a sub-pixel level has to be used. While it seems to be accepted in the PIV community that a Gaussian sub-pixel estimator is superior to centroid and quadratic fits (e.g. Westerweel (1997), Forliti et al (2000), Christensen (2004)), the results are not clear for the so called Whittaker reconstruction.

Previous results (Henning and Ehrenfried, 2007) have shown that the Whittaker algorithm is not the appropriate interpolation scheme for the present study. Therefore a least squares gauss-fit using a 3 × 3 pixel domain Gaussian fit is used for the present evaluations. The PIV data are processed using the PivView Software (see DLR contribution to Stanislas et al (2005)). After evaluating the performance of different PIV algorithms, a multi-pass algorithm (Raffel et al, 2007) with image deformation and sub-pixel interpolation (Scarano, 2002) is used for the investigation presented here. The size of the interrogation window is varied from 32 px × 32 px to 96 px × 96 px in steps of 16 px. In case of the grid turbulence experiment the flow is stationary and has a low turbulence level. Therefore both images used for the cross correlation are shifted relatively to each other prior to the PIV evaluation. The necessary magnitude of this integer pixel offset is determined by a PIV evaluation on a coarse grid for a small amount of image pairs. In order to identify and replace outlying data points in the velocity time series, a decision based Hampel filter is applied to the PIV data as proposed by Fore et al (2005). Only time steps exceeding the local median by a specific tolerance are modified by this filtering method. If the median absolute deviation (MAD) is used to set the tolerance, the governing equation reads:

\[
    u_i(n) = \begin{cases} 
        u_i(n) & \text{for } |u_i(n) - Z(n)| \leq t_h \text{MAD}(W(n)) \\
        Z(n) & \text{for } |u_i(n) - Z(n)| > t_h \text{MAD}(W(n)),
    \end{cases}
\]  

where \( u_i(n) \) is a velocity component in the i-th direction at the n-th time step, \( t_h \) a threshold parameter and \( Z \) the median of the moving window \( W(n) = [u_i(n - K), ..., u_i(n), ..., u_i(n + K)] \) where the width of the window is \( 2K + 1 \).

### 3.3 Hot wire probe

Hot wire measurements are performed using a DISA-probe model 55P11 with a 5 µm diameter, 1.25 mm long platinum-plated tungsten wire. It is operated with a commercial constant temperature circuit (DISA 55M10) with an overhead ratio of 0.7 and a cut-off frequency of 120 kHz. The output signal is digitalized with a 24 bit dynamic signal analyzer (Oros OR38) at 102.4 kHz for all measurements. A 50 kHz anti-alias filter is used before it is digitalized and stored. The number of samples was at least \( 5 \times 10^5 \) to ensure convergence. The probe wire is orientated normal to the orientation of the PIV light sheet plane for both experiments in order to obtain the same velocity components by both techniques. The probe position is adjusted to the middle of the field of view investigated by the PIV measurement. In order to obtain two-point statistical quantities a second probe is installed in case of the cylinder-wake experiment. It is located at the same x-z position as the first probe, shifted in y-direction via...
a traverse system. For the determination of the exact x-y position of both probes, images of the probes are recorded with the camera of the PIV system. In section 4 the hot-wire results are compared with the PIV measurements. For convenience the velocity results of the hotwire measurements are converted into pixel shifts using the relation

$$\Delta x_i = u_i \ast R \Delta t,$$

where $u_i$ is the measured i-th velocity component by the hotwire probe, $R$ the pixel resolution of the PIV measurement, $\Delta t$ the time delay between the two laser pulses and $\Delta x_i$ the resulting pixel displacement.

### 3.4 Grid turbulence

The tests take place in an open-return Eiffel-type wind tunnel at the DLR (German Aerospace Center) in Göttingen. The closed test section is 1 m long and has a cross sectional area of $0.4 \times 0.4 \, m^2$. A square-mesh grid (mesh width $M_g = 25 \, mm$) with a solidity of 0.36 is placed at the beginning of the test section (figure 2). The field of view is located at the center of the test section, parallel orientated to the side walls approx. $x/M_g = 40$ downstream of the grid (see figure 1). At this distance the flow is assumed to be homogeneous (Comte-Bellot and Corrsin, 1966). Parts of the test sections side wall and ceiling are replaced by glass plates in order to obtain optical accessibility. Measurements are conducted at a free stream velocity $U_\infty = 15 \, m/s$, whereas the wind tunnel speed is controlled by the revolution speed of the fan only. To minimize velocity deviation due to temperature, humidity and atmospheric pressure fluctuations, the PIV measurements where always carried out right after the hotwire measurements. The camera of the PIV system is equipped with a 100 $mm$ macro planar lens used for both a $R = 35.19 \, px/mm$ and $R = 11.47 \, px/mm$ measurement.

### 3.5 Cylinder wake flow

The cylinder wake experiment is carried out at the 1m wind tunnel test facility (1MG) at the DLR in Göttingen. It is a closed circuit type low speed wind tunnel with a open jet test section. The cross section of the nozzle is $1 \, m \times 0.7 \, m$. The distance between the nozzle and the collector is $1.3 \, m$ and the turbulence level is $0.15 \%$ at the center of the free flow. The Reynolds number is $Re = 10,000$ based on a cylinder diameter of $d = 20 \, mm$ and a free stream velocity $U_\infty = 7.5 \, m/s$. The laser light sheet of the PIV system is oriented perpendicular to the cylinder axis. The thickness of the light sheet is $1.5 \, mm$ in the investigation area whose center is located approx. $x = 4 \, d$ downstream of the cylinder vertically shifted to $y = 0.5 \, d$ in a coordinate system with the origin in the cylinder axis. The camera of the PIV system is equipped with a 100 $mm$ macro planar lens used for a spatial resolution of $R = 27 \, px/mm$.

### 3.6 Data processing

While PIV measures the two velocity components $u$ and $v$ separately, the velocity signal obtained by the HWA is $w_{hwa} = \sqrt{u^2 + v^2}$. Applying a Reynolds decomposition and rearranging we get:

$$w_{hwa}^2 = U^2 + 2Uu' + V^2 + 2Vv' + u'^2 + v'^2,$$

where capital letters are used to represent the mean values and a prime marks the fluctuating motion. For grid turbulence the turbulence level is low ($|v'|, |u'| \ll U$). Therefore the last two terms of the right side, which are of higher order, can be neglected. Choosing the coordinate
system so that \( V = 0 \), then the 3rd and 4th term are vanishing. It follows that \( w_{hwa} \approx u \), since \( u \) is always positive in the experiment presented here. Therefore, in case of the grid turbulence experiment, only the \( u \) component of the PIV results will be compared to the data obtained by the HWA. In case of the cylinder wake flow the turbulence level is high, approx. in the range of 25\%. The frozen-turbulence approximation is not valid and higher order terms of equation (6) can not be neglected. In order to compare the data obtained by both techniques, the magnitude of the vector product \( w = \sqrt{u^2 + v^2} \) is calculated from the PIV data, since it corresponds to the quantity measured by the hotwire.

### 3.6.1 Spectral computations

For the spectral computation of the grid turbulence data the hotwire signal is processed using 5120 point periodograms with an overlap of 50\%. Smaller segments of 1000 are used for the PIV data to keep the frequency bands approximately equal. In case of the cylinder-wake experiment 10240 point periodograms are used for the HWA data in order to resolve the expected Strouhal frequency properly (\( f_{St} = 75 \text{Hz} \) based on \( St = 0.2 \) for the configuration presented here), while the PIV frequency bands were kept again approximately equal by using segments of 2000. In all cases a Hanning window is applied to the time records.

### 3.6.2 One-point and two-point statistics

The temporal Taylor microscale \( \lambda_\tau \) together with the standard deviation \( \sigma_u \) are calculated and compared for the HWA and PIV measurements. While the first reflects the accuracy of time correlations, \( \sigma_u \) is the square root of the integral of the power spectrum. The microscale \( \lambda_\tau \) is calculated as:

\[
\lambda_\tau^2 = 2 \frac{\langle u'^2 \rangle}{\left( \frac{\partial u}{\partial t} \right)^2}.
\]  

With the above definition the microscale is the intercept of a parabola that matches the autocorrelation function at the origin (Tennekes and Lumley, 1972). Assuming that the frozen-turbulence approximation is valid in case of the grid turbulence experiment, \( u(x) \) can be identified as \( u(t/U_\infty) \) where \( u(t) \) is the velocity time-series measured at fixed point. Thus, the spatial microscale \( \lambda_x \) can be calculated from the HWA data and the Kolmogorov frequency \( f_K \) is calculated by the following relations:

\[
\varepsilon = 15 \nu/\lambda_x^2 , \quad \eta \equiv \frac{\nu^{(3/4)}}{\varepsilon^{(1/4)}} , \quad f_K \equiv \frac{U_\infty}{(2\pi\eta)},
\]  

Since the high-speed PIV measurement presented here results provide spatial and temporal resolved velocity data, two-point correlation functions are calculated as:

\[
R_{\Delta y}(\tau) = \frac{1}{N\sigma_{u_1}'\sigma_{u_2}'} \sum_{n=1}^{N} \left[ w_1'(x, y, z, t_n)w_2'(x, y + \Delta y, z, t_n + \tau) \right],
\]  

where \( N \) is the number of PIV samples and \( w \) is the velocity magnitude.
4 RESULTS AND DISCUSSION

4.1 Grid turbulence

One sample set of PIV data for each of the two used camera resolutions \( R \) will be presented and analyzes in this section.

For the Hampel filter (see section 2.3) the threshold \( t_h \) is chosen to be 3 and 4 while the size of \( W \) is 7 and 9 for the high and low magnification respectively. For the \( R = 35.19 \text{ px/mm} \) measurement, less than 1% are replaced in case of a IW size of \( 32 \times 32 \text{ px}^2 \). The rate decreases with increasing IW size.

4.1.1 Time averaged turbulence quantities

For the hotwire data, equation (7) together with the relations (8) gives \( \lambda_x = 4.9 \text{ mm} \), \( \varepsilon = 1.165 \text{ m}^2/\text{s}^3 \), \( \eta = 0.232 \text{ mm} \) and \( f_K = 10.29 \text{ kHz} \) in physical dimensions. The result agrees with findings in literature (e.g. (Comte-Bellot and Corrsin, 1971), (Zhou and Antonia, 2000)). Table 1 (left) lists \( \lambda_\tau \) together with \( \sigma_u \) in pixel dimensions for both the PIV and HWA measurement. For the PIV measurement \( \sigma_u \) decreases with increasing \( \delta_x \), while except for \( \delta_x = 32 \text{ px} \) in case of \( R = 35.19 \text{ px/mm} \), the value is attenuated compared to the HWA data. In case of the \( R = 11.47 \text{ px/mm} \) measurement, all values of \( \sigma_u \) are underestimated and decreasing with increasing \( \delta_x \). This shows that the attenuation of \( \sigma_u \) depends on the spatial resolution, rather than on \( \delta_x \) only. Since \( \sigma_u \) is the square root of the integral of the power spectrum, the spectral analysis in the following section will give a detailed explanation for erroneous estimations of the standard deviation. The integral scale \( \lambda_\tau \) is overestimated compared to the HWA result in case of \( \delta_x = 32 \text{ px} \) for both resolutions, while the value increases further with increasing IW sizes. This is an interesting result since the dependency of \( \lambda_x \) was expected to be similar to the one of \( \sigma_u \). But in contrary to that, the integral scale also seems to depend on the size of the IW in pixel units width itself and not only on the IW size in physical coordinates and therewith on the spatial resolution \( R \). The reason why \( \lambda_{\text{tau}} \) is overestimated in case of larger interrogation windows becomes clear by looking at the autocorrelation coefficient \( R_u(\tau) \) depicted in figure 3.

![Figure 3: Grid turbulence Autocorrelation coefficient \( R_u(\tau) \) and the osculating parabola from the time series obtained by PIV for different IW sizes compared to the hot wire result. left \( R = 35 \text{ px/mm} \), right \( R = 11.47 \text{ px/mm} \).](image-url)
The kurtosis of the correlation functions is decreasing with an increasing size of IW. Therefore the intercept of a osculating parabola (dashed lines) that matches the autocorrelation function at the origin is increasing with increasing IW-size. The overestimation of $\lambda_\tau$ can be explained in the following manner: The filter effect of the PIV processing causes a smoothing of $R_{u\tau}$ and therewith a broadening of the correlation function at the origin. Therefore $\lambda_\tau$, defined as the curvature of the autocorrelation function at the origin, is overestimated.

4.1.2 Spectral analysis

Figure 4 (left) shows the PIV spectra (colored lines) from the $R = 35.19 \text{ px/mm}$ measurement for different values of $\delta_x$ compared to the hotwire spectrum (black line). The HWA spectrum has a long region of constant exponential decay over the whole frequency range. The PIV spectra are decreasing more rapidly with increasing $\delta_x$ till they converge to certain values from whereon they remain relatively constant.

The low frequency range is very accurately resolved in all cases. No aliasing can be confirmed as observed by Wernet (2007) and predicted by Saarenrinne et al (2001). This can be explained by the low-pass filter effect of the interrogation window and the high measurement frequency of 20 kHz. In order to illustrate the fact that in case of the grid-turbulence experiment the IW-size in x-direction determines the observed filter effect, the resulting spectra of both, a PIV evaluation with a $32 \times 96 \text{ px}$ and $96 \times 32 \text{ px}$ window size are depicted in Figure 4 (left). In the $32 \times 96 \text{ px}$ case the spectrum is attenuated slightly at higher frequencies. In contrast to that the $96 \times 32 \text{ px}$ spectrum shows a stronger reduction in a wider frequency range. This shows that the filter effect due to the IW-size is mainly determined by $\delta_x$.

Figure 4 (right) depicts results for the $R = 11.47 \text{ px/mm}$ measurement. The same trend like in case of the higher magnification can be observed whereas the spectra have a shallow local minimum before they converge to a certain value. For $\delta_x \geq 48 \text{ px}$ this value corresponds to the one obtained from the $R = 35.19 \text{ px/mm}$ measurement for $\delta_x \geq 64 \text{ px}$. This shows that there exists a certain noise floor for the PIV measurement which is not possible to overcome by changing the magnification or parameters of the evaluation like the IW size. According to chapter 2.3 the averaging effect of the interrogation window can be expressed by
Figure 5: *left: Grid turbulence* Energy spectrum from the time series obtained by PIV (35.19 px/mm) for different IW sizes compared to the hot wire spectrum multiplied with a \( \text{sinc}^2 \) function and the corresponding result of equation (3).

A \( \text{sinc}^2 \) function in the frequency domain. Figure 5 shows the PIV-spectra compared to the HWA spectrum which is multiplied by the respective \( \text{sinc}^2(k_x, \delta_x) \) function (dotted lines) for the corresponding \( \delta_x \). Results for \( \delta_x = 48 \) px are not shown here for the purpose of clarity. The modified HWA spectra are reflecting well the PIV-spectra up to a certain frequency for all variations of \( \delta_x \). For higher frequencies the PIV spectra are showing the lift off at different levels.

In section 2.2 the measured noise is split into two parts \( E_n^i \) and \( E_n^o \) which are used for equation 3. The latter is estimated to be the plateau towards the spectra converge while the first is determined by fitting the curve to the hotwire spectrum. \( E_n^o \) decreases with increasing \( \delta_x \) up to a IW size of 64 x 64 px, whereas for this and higher values of \( \delta_x \) it remains approx. constant. The inner part of the noise \( E_n^i = 2.5 \times 10^{-5} \text{px}^2 \) is found to be constant for both resolutions, while for \( R = 11.47 \text{px/mm} \) the value is that low, that it can be neglected. The dashed lines in figure 5 are showing the modified spectra obtained by applying equation (3) to the HWA spectrum. The modified HWA spectra agree well with the PIV spectra for \( \delta_x \leq 64 \text{px} \). In case of \( \delta_x = 96 \text{px} \) the modified HWA spectrum shows a local minimum which is not present in the PIV spectrum. The same deviation is observed at the modified HWA spectra in case of \( R = 11.47 \text{px/mm} \) (not presented here). Therefore it can be concluded that equation (3) reflects well the spatial filter effect together with the measured noise up to a certain limit of the physical IW size.

### 4.2 Cylinder wake flow field

In case of the cylinder wake the parameter for the Hampel filter (see section 2.3) is chosen to be \( t_h = 3 \) while the width of \( W \) is 9. The filter is implemented in such a way, that both components, \( u(n) \) and \( v(n) \), are replaced if either of them exceeds the tolerance. Approx. 1.75% are replaced in case of a IW size of 32 x 32 px\(^2\). Like for the grid turbulence experiment, the rate
decreases with increasing IW size. While equation (3) seems to be a valid simplification for the conditions in the previously described experiment, it is disputable that the interdependency of the spatial filters in different directions can be neglected in this second test case. But in order to show the limits of the simplified model, the HWA spectrum is manipulated with equation (2) and (3), and the therewith obtained spectra are compared to the PIV results. In order to concentrate on the differences of the more three-dimensional and inhomogeneous cylinder-wake flow field compared to the grid turbulence, similar results will be described only briefly in the following.

4.2.1 Time mean turbulence quantities

Table 1 (right) lists \( \sigma_w \) in pixel dimensions and \( \lambda_\tau \) for both the PIV and HWA measurement. For the PIV data \( \sigma_w \) decreases with increasing \( \delta_x \), while except for \( \delta_x = 96 \) px, the value is overestimated compared to the HWA data. This general increase is caused by a deviation in the low frequency range, where most of the energy is contained. \( \lambda_\tau \) is in the order of the value obtained from the hotwire data in case of \( \delta_x = 32 \) px. With increasing \( \delta_x \) the magnitude of \( \lambda_\tau \) is more and more overestimated. This tendency agrees with the ones obtained for \( \lambda_x \) from the grid turbulence experiment. Figure 6 depicts the two-point correlation function \( R_{\Delta y}(\tau) \) for \( \Delta y = 0.17 d \) calculated from the PIV results for different IW-sizes compared to the HWA result.

![Figure 6: Cylinder Wake Two-point correlation function](image)

Figure 6: Cylinder Wake Two-point correlation function \( R_{\Delta y}(\tau) \) for \( \Delta y = 0.17 d \) calculated from the PIV results for different IW-sizes compared to the HWA result.

0.17 \( d \) calculated from the PIV results for different IW-sizes. The black line shows the result obtained from the data measured with two HWA-probes. In turbulent flows, and especially in the cylinder wake, large coherent structures mainly contribute to the cross-correlation coefficient \( R_{\Delta y}(\tau) \) for \( \Delta y = 0.17d \). \( R_{\Delta y}(\tau) \) is normalized by the standard deviation \( \sigma_w \). This is the square root of the integral of the power spectrum, where the high frequencies, which do not contribute to the correlation coefficient, are filtered by the PIV measurement technique as shown above. Therefore the resulting \( R_{\Delta y}(\tau) \) is increasing with increasing IW-size.
4.2.2 Spectral analysis

Figure 7 left shows the PIV spectra (colored lines) for different values of $\delta_x$ compared to the hotwire spectrum (black line). The dominant oscillation pattern can be identified by a resulting peak in the averaged power spectrum at $70 \text{ Hz}$ together with a second harmonic at $140 \text{ Hz}$ (barely visible in the figure 7). The normalized frequency $S_t = f d/U_{\infty}$ is 0.187 which is in good agreement with values found in literature (Williamson, 1996). The $\delta_x$ dependency of the PIV-spectrum is similar to the trend observed in case of the grid generated turbulence. The decay of the PIV spectrum becomes steeper with increasing $\delta_x$ till it converges to certain values from whereon it remains constant. In the same manner as for the experiment described previously, the HWA spectrum is manipulated with the $\text{sinc}^2$ function as well as with equation (3). The results are shown in figure 7 (right). For the low frequency range ($< 1600 \text{ Hz}$) the attenuation of the spectra is well described by the $\text{sinc}^2$ function. By inspecting the hotwire spectra (dased lines) manipulated according to equation (3), it becomes clear that the filter effect together with the noise of the PIV-processing chain is insufficiently described by that equation in case of the cylinder wake flow. A variation of the inner noise factor $E_n^i$ has no significant influence on the resulting spectra and therefore it was chosen to be zero.

5 CONCLUSIONS

Two types of flows, grid turbulence and a cylinder-wake, are investigated by means of high-speed PIV in order to characterize the temporal response of the measurement system. With an increasing window size, a spatial filter effect becomes more and more visible in the PIV spectra. It can be shown that the spatial averaging in the interrogation window results in a low pass filter for the frequency response. The transfer function can be expressed by a modified sinc function and the cut-off frequency depends on the chosen interrogation window size. This is true at least for moderate interrogation window sizes in case of the grid turbulence experiment, where the frozen-turbulence approximation is valid. It was shown how the described filter effect influences the result for one-point and two-point statistical quantities like $\sigma$, $\lambda$, and $R_{\Delta y}(\tau)$. It can be concluded that in order to obtain accurate measurement results with a high-speed PIV
system, the spatial resolution, the IW size and the measurement frequency have to be adjusted well by the experimentalist. A remarkable result is the absence of any aliasing in both the grid turbulence and the cylinder wake measurement.

In order to obtain the spectral response and the achievable signal to noise ratio of the whole PIV-process chain, the response function of the measurement system to a noise signal with a known amplitude and frequency distribution needs to be evaluated. Therefore future investigations will concentrate on the development of a system providing such a noise signal.

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Table 1: Summary of the statistical results for the velocity measurements for the used resolutions $R$ and interrogation-window size $IW$. Standard deviation $\sigma_u$, Taylor integral scale $\lambda_x$. All HWA results in pixels using equation 5. \textit{left} Grid turbulence, \textit{right} Cylinder wake.

References


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