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Measurement of the position of rotor blade vortices generated by a helicopter in free-flight by means of the stereoscopic Background Oriented Schlieren Method (BOS)

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Abstract: A free-flight full-scale out-door test to localize blade tip vortices of a helicopter in space was performed by means of the stereoscopic Background Oriented Schlieren method (BOS) at the DLR site and research flight center in Braunschweig. The objective was a quantitative feasibility study of the BOS measurement technique for spatial vortex investigations. The results show that it is possible to detect and to measure the position of three-dimensional vortex structures of a free-flying MBB Bo 105 helicopter in space.

1. INTRODUCTION

With increasing use of civil helicopters, the problem of noise emission of helicopters has become increasingly important within the last decades. Blade vortex interactions (BVI) have been identified as major responsible for the acoustic nuisance which occurs especially during low speed descending flight conditions like landing approach or maneuvers. Moreover, the interactions affect blade loads and rotor performance. Therefore, many aeronautical research organizations develop and run numerical codes in order to predict the rotor wake (see e.g. Beaumier et al. 1994). These aerodynamic results can then be used for further numerical investigations of the acoustic near and far field of BVI. (e.g. Burley et al. 1991, Ehrenfried et al. 1991).

Theoretical and experimental studies were conducted in order to validate models for predicting BVI and to determine the model-to-full-scale acoustic scaling conditions (Schlinker and Amiet 1983, Splettstößer et al. 1984) and generally confirmed the necessity of wind tunnel tests. It has been found that high speed noise at the advancing blades can be quite well scaled whereas the scalability of BVI noise underlies certain restrictions. It is assumed that Reynolds number effects do not allow to scale the vortex dimensions easily. Therefore, aerodynamic rotor model investigations have been undertaken at large scales e.g. in the large low speed facility of the Dutch-German wind tunnel DNW-LLF (e.g. Splettstößer et al. 1997).

By reviewing the literature of the last decade, it can be seen, that particle based flow velocity measurements have most frequently been applied (e.g. Sullivan 1973, Biggers and Orloff 1974, Raffel et al. 1998). Most likely, because they are assumed to deliver a lot of quantitative information needed for BVI prediction.

Density measurements of the BVI phenomena on the other hand are also desirable, because the vortices under investigation are compressible and density gradient techniques can therefore be used in order visualize flow structures. Density measurements have been performed quite frequently, but mostly in two-dimensional basic investigation (e.g. Meier et al. 1998, Chandrasekhara 1994). Density gradient visualization by the classical schlieren method as e.g. conducted by Tangler (1977) and were reported more recently by Bagai and Leishman (1993). For large or full–scale investigations on rotating systems, the traditional quantitative density techniques like Mach-Zehnder or point diffraction interferometry seem to bee less feasible, since they require delicate optical systems. Due to the aforementioned reason of limited scalability and the well known imperfections of particle based optical techniques to resolve the vortex core in detail and their costs, additional large or full scale tests of an easy to use optical technique are desirable.

With the optical measurement technique BOS (Background Oriented Schlieren) density gradients can be visualized, even for applications of industrial interest (Raffel *et al*. 2000, Richard *et al.* 2000). The underlying principle of this technique is the deviation of light rays passing through objects with density gradients as integrated along their paths through the density object.

The promising results obtained at the first BOS experiments (visualization of blade tip vortices and heated flows) and the very simple experimental set up induced a rapid development of this new measurement technique: Knowing the position, where the deviation takes place, it is possible to determine the local density of a 2D flow field by means of BOS quantitatively (Klinge *et al.* 2002). 3D axis-symmetrical density objects can be investigated by means of BOS if their position in space is known (Kirmse 2003).

Further on, it has been shown that the position of a 3D axis-symmetrical density object with filament structure can be determined in space using stereo BOS and a dedicated algorithm for determination of its position (Klinge *et al.* 2004). Thus, this technique is of high interest for application at helicopter studies, where the position of the rotor blade vortices is to be investigated with respect to different flight conditions. Such tests should preferably be carried out at full-scale, as the behavior of such vortices may depend on Mach number and Reynolds number which limits the use of laboratory or wind tunnel tests with scaled models.

The objective of this paper is to show, that it is possible to determine the position of 3D density objects with certain features by means of the stereoscopic BOS method quantitatively even under the conditions of an industrial test performed out-doors. This has been done in an experiment which was carried out at DLR’s research flight center in Braunschweig during a flight test of a full-scale helicopter in order to determine the location of the blade tip vortices.

2. THEORY

The principle of the BOS-technique

The principle of the background oriented schlieren (BOS) technique, which we used for our experiments can best be compared with the density speckle photography as described by Debrus et al. (1972), Köpf (1972), and in an improved version by Wernekinck and Merzkirch (1987), but uses white light speckle instead of laser speckle.

The set up for stereo BOS measurements requires only two cameras to be focused on two backgrounds consisting either of randomly distributed dot patterns artificially applied by painting or structured natural background e.g. concrete. The size of the individual elements of the pattern should be optimized according to the magnification of the set up such that one structure element is of the size of 2-3 pixels when imaged by the CCD chip. For each camera at least two images have to be stored: a reference image without the density object being present and a measurement image, with the density object to be investigated being present. The principle set up is shown in **figure 1**. Without the density object, one element of the background pattern is imaged on the image plane as indicated by the black line. With the density object, the light rays from the background passing through the density object will be deviated when passing the density object, finally resulting in a deviation angle ε, and thus will be imaged on another position in the image plane, as indicated by the dashed red line. This means that the complete background pattern will be displaced when imaged through the density object, the displacements being different for each location in the image plane. This local distance between the two images of the background pattern in the image plane (Δx) can be detected by using standard cross-correlation algorithms, as have been developed for particle image velocimetry (PIV).

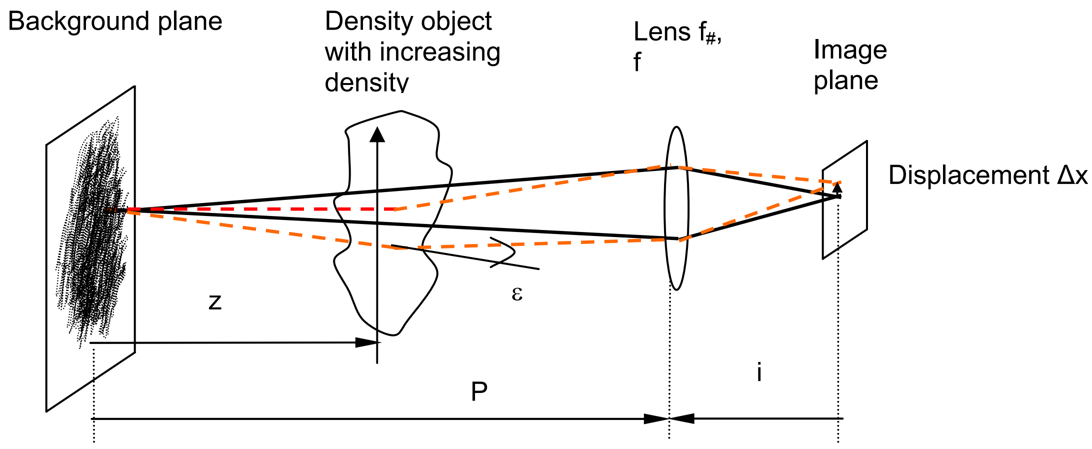

Figure 1: BOS set up: z: Distance between the background plane and density object;
P: Distance between the background plane and the lens;
i: Distance between the lens and the image plane

The importance of the density object position

A given density object leads to a certain light ray deviation. However, the displacement map of the background pattern does not uniquely define the density object. An important aspect influencing the displacement map is the distance between the background plane and the density object. This means that a density object giving rise to certain light ray deviation may lead to totally different displacement map of the background pattern if the distance between the background pattern and the density object changes. This fact underlines the need to know where the investigated density object is located in space in order to be able to reconstruct its density field. In some applications, this knowledge will be obviously given, e.g. if the density field of a nozzle is investigated. In other applications, it is not easy or even impossible to predict or measure the position of the density object. This is in particular the case for full-scale out-door measurements, where the density object may appear in principle at any position between the camera and the background. Knowing the distance between background and density gradient (z) is mandatory for every later quantitative determination of density.

The geometrical arrangement of background pattern and observation camera with respect to the density object must be carefully chosen. In order to be able to detect a certain displacement of the background pattern the density object has to have a certain distance to the background. However, if the density object is too close to the camera, the displacements of the background pattern will interfere and the information coming from different density objects may possibly overlap. Overlapping density objects may be difficult to separate and to quantify.

BOS set up optimization

As already mentioned, the most important aspect for a BOS set up is the position of the density object with respect to the position of the camera and the background. These are the distances z and P and the focal length f of the utilized objective (fig 1). Some further important parameters are the size of the local interrogation area for the cross correlation, when analyzing the displacements of the background pattern, the aperture of the lens, the sensitivity and the resolution of the system. As already shown by Klinge (2003) it is possible to optimize the parameters of the set up in a way that as much information as possible can be captured by the experiment. Further on it was shown that an increase in sensitivity means a loss in resolution of the system and vice versa. The optimization task for a BOS set up mainly consists in finding a good compromise between sensitivity and resolution. A good resolution and a sufficient sensitivity can only be achieved with a small investigation volume comprising the density object, which means the usage of a large focal length objective.

Summarizing these findings, the following rule of thumb can be given: If the distance z is smaller than 50% of the distance P, the results will show a good resolution and a sufficient sensitivity. The more the distance z is decreasing, the more the sensitivity of the system will decrease, too. The sensitivity of the system can be increased without changing P and z by increasing the focal length of the objective.

Another important issue is to optimize the overlapping volume of the investigation areas of the two cameras of the stereo set up and to keep the moving density object inside this volume while the measurement is performed. This issue includes a precise planning for the camera arrangement.

Density object position determination

Further evaluation of the displacement map of the background pattern is possible only, if the distance between the density object and the background is known. To be able to determine the deviation angle ε, the location of the position of the deviation of the light ray (density object) has to be known accurately.

Even if several cameras are imaging their backgrounds through an overlapping volume capturing the same density object, it is difficult to locate this density object in space. First of all the direction of light ray deviation changes with respect to different viewing angles, because only the density gradients perpendicular to the direction of the propagation of the light ray lead to a deviation of the light ray; secondly the same specific feature of the density object has to be reliably identified in both images. In particular, if axis-symmetrical density objects shall be investigated, this is difficult, because they normally do not exhibit significant features which can be identified from the different views reliably enough. A unique feature of the density object identified by each camera is needed to locate this feature in space by e.g. triangulation algorithms.

Of course tomographic methods could be applied as well. However, for industrial tests and due to costs the number of cameras which can be used for the BOS set up is limited. Thus, it is necessary to make use of further information about the structure of the density object. In this application use will be made of the fact, that only a few concentrated density objects exist, that they exhibit strong density gradients, and are clearly separated in space.

Such 3D axis-symmetrical density objects without any specific features along their main elongation are of great interest at industrial applications, because e.g. blade tip vortices of helicopters appear like that.

The problem of identifying the position of these vortices in space leads to a dedicated algorithm (Klinge 2003), being able to determine the position of a light ray deviation and therefore the density object by correlating two different views of the density objects. This algorithm, called VRIEDER, was especially designed to detect the location of a vortex filament. In pre-tests it has been proven, that the algorithm works very satisfying. A brief explanation follows.

The correlation of two camera views to calculate the position of a vortex filament is done by starting with an arbitrary point of this structure as identified in the image as captured by one camera and checking all the points of the structure as identified in the image captured by the second camera whether they match in space. The complete description is given in Klinge (2003).

The algorithm has been applied successfully to data from wind tunnel experiments, where it was possible to determine the position of wing tip vortices very accurately even for low speed applications with Mach = 0.2 (Klinge *et al.* 2004).

3. EXPERIMENT AND VORTEX FILAMENT RECONSTRUCTION

Objectives

The position of blade tip vortices with respect to the rotor blade is of great interest for the development of new techniques to avoid e.g. the noise generated by Blade Vortex Interactions (BVI), which are due to the interaction of a rotor blade and a blade tip vortex, which may appear, if the helicopter is in descending flight conditions. It was shown by Raffel *et al.* (2000) and Richard *et al.* (2000) that it is possible to visualize such helicopter blade tip vortices by means of BOS.

The aim of this experiment was the determination of the position of blade tip vortices in space coordinates utilizing stereo BOS.

The subject helicopter

Rotor blade vortices as generated by a MBB Bo 105 helicopter with a 4 blade, 9.84 m rotor diameter have been investigated. The tip speed during the measurement is about 218 m/s. The flight conditions were varied from hover flight to slow climb. A three side view of the helicopter is shown in **figure 2**.

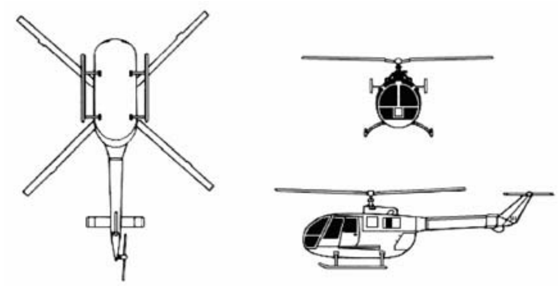

Figure 2:
Three side view of the helicopter MBB Bo 105. Rotor diameter 9.84 m

The experimental set up

The following facts were limiting the set up:

- Variations of the positions of the cameras were impossible due to the limited dimensions of the hangar and its given height.
- Variations of the positions of the backgrounds were difficult due to the limited dimensions of the apron.
- The backgrounds were preferably consisting of concrete. The contrast of it could only be varied by white color painted on it. After pre-tests it was found that the natural contrast of the concrete was sufficient.
- The given distances between the cameras and the backgrounds determine the sensitivity and accuracy of the BOS-System and the dimensions of the investigated volume.

The only choice which could be made during the experiment was with respect to the focal length of the used objectives. They were adapted in a way that finally a sufficient sensitivity of the system was given and the overlapping volume of the two views was as large as possible.

A top view of the whole experimental set up is shown in **figure 3.** The deformed shape of the background is due to the small angle between the concrete floor and the viewing direction of the cameras.

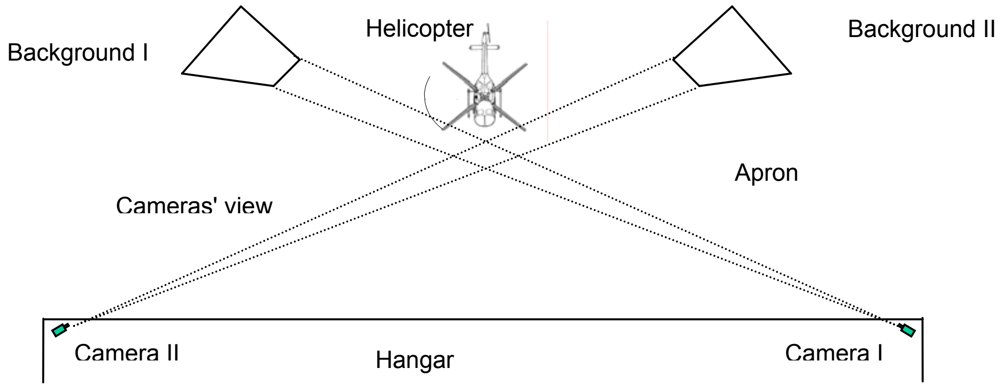

Figure 3: View of the experimental set up from top.

A side view of the set up is shown in **figure 4**. The sketch makes clear how small the investigation volume with respect to the overall dimensions is. A complicated alignment task is to arrange both cameras in a way that their view is overlapping as much as possible.

Further on, the helicopter has to be placed on the right flight track such that, while climbing, the rotor blades are passing through the investigation volume.

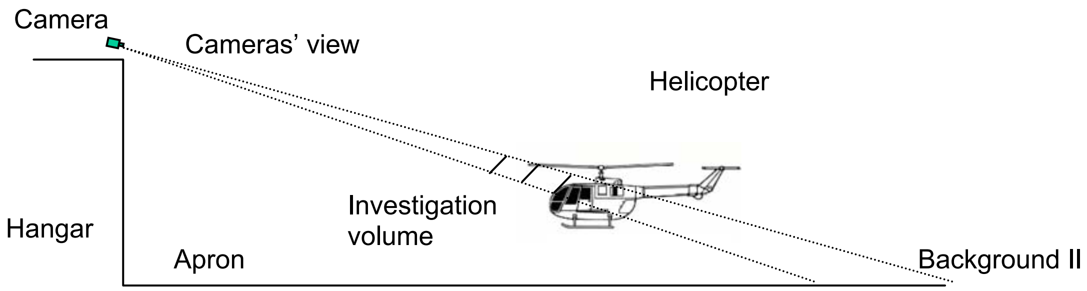

Figure 4: Side view of the set up

Figure 5: View from camera I over the helicopter towards background I, while the helicopter is hovering with its rotors into the investigation volume

Recording the images

The stereo BOS set up requires two cameras. These two cameras were equipped with a 135 mm focal length Zeiss objective. In order to obtain well focused images, it was necessary to use Scheimpflug-adapters, because the object plane was not parallel to the plane of the image plane of the CCD-sensor. The CCD sensors have a resolution of 1024 by 1280 pixels and cooling which reduces the noise and increases the dynamic range which is digitized to 12 bit resolution. The cameras were mounted on a rack, which was located outside the hangar standing on its roof. The cameras were connected to the computers by fiber optics. The BOS images were recorded with a frequency of 6 Hz.

The focus of the objectives was carefully adjusted to meet the requirements of the Scheimpflug-criteria. By help of remotely controlled focusing devices, a final adaptation of the focus was possible even during the measurements.

An illumination of the background was not necessary due to bright sunlight during these tests. Pre-tests showed that measurements without sunlight were absolutely impossible, due to low contrast of the background pattern (concrete).

The time management for the two BOS cameras was controlled by a sequencer. It was important to avoid strong brightness variations between the reference and measurement images. Thus, the reference pictures were taken just before the measurement was performed. The helicopter was parked asides the apron during that time. The investigation volume was about 1.5 m x 1.5 m x 1.5 m large. The pilot's outstanding capabilities to keep the helicopter steady on the right position were needed to have the rotor blades and their vortices inside the investigation volume during the measurement. The wind was low during the measurements; about 2 m/s. After the measurement an accurate calibration of the cameras and backgrounds took place and determined their exact location.

4. RESULTS AND DISCUSSION

The evaluation of the recorded BOS images consisted of several steps:

- The cross-correlationof the reference and the measurement image by means of standard PIV evaluation software for both cameras separately, generating a vector plot (background pattern displacement) for each camera and test case.
- Detection of vortex lines in the two vector plots.
- Calculation of the spatial coordinates of the vortices by means and visualizing the vortices in a world coordinate system.

Figure 6: Background I imaged by camera I. This image was used as a reference for the cross-correlationof one test.

Data Processing I: Cross correlation

The first step of the data processing is the cross-correlationof the reference and the measurement image of the background. The result of this step is a displacement vector map. The length and the color of the vectors correspond to the average displacement in each interrogation window of the cross-correlation algorithm. The interrogation window size was chosen to be 16 pixels x 16 pixels. An overlap of the interrogation windows of 50% was chosen.

In our evaluation the displacement vector is pointing towards the lower density, which means for the case of a vortex: towards its centerline.

The cross-correlationmust be performed for the image sets of both cameras separately, of course. If neither the camera nor the objective have been moved, the same reference image may be used for all measurement images. In case of the described experiment it was necessary to take a reference image before and after each measurement.

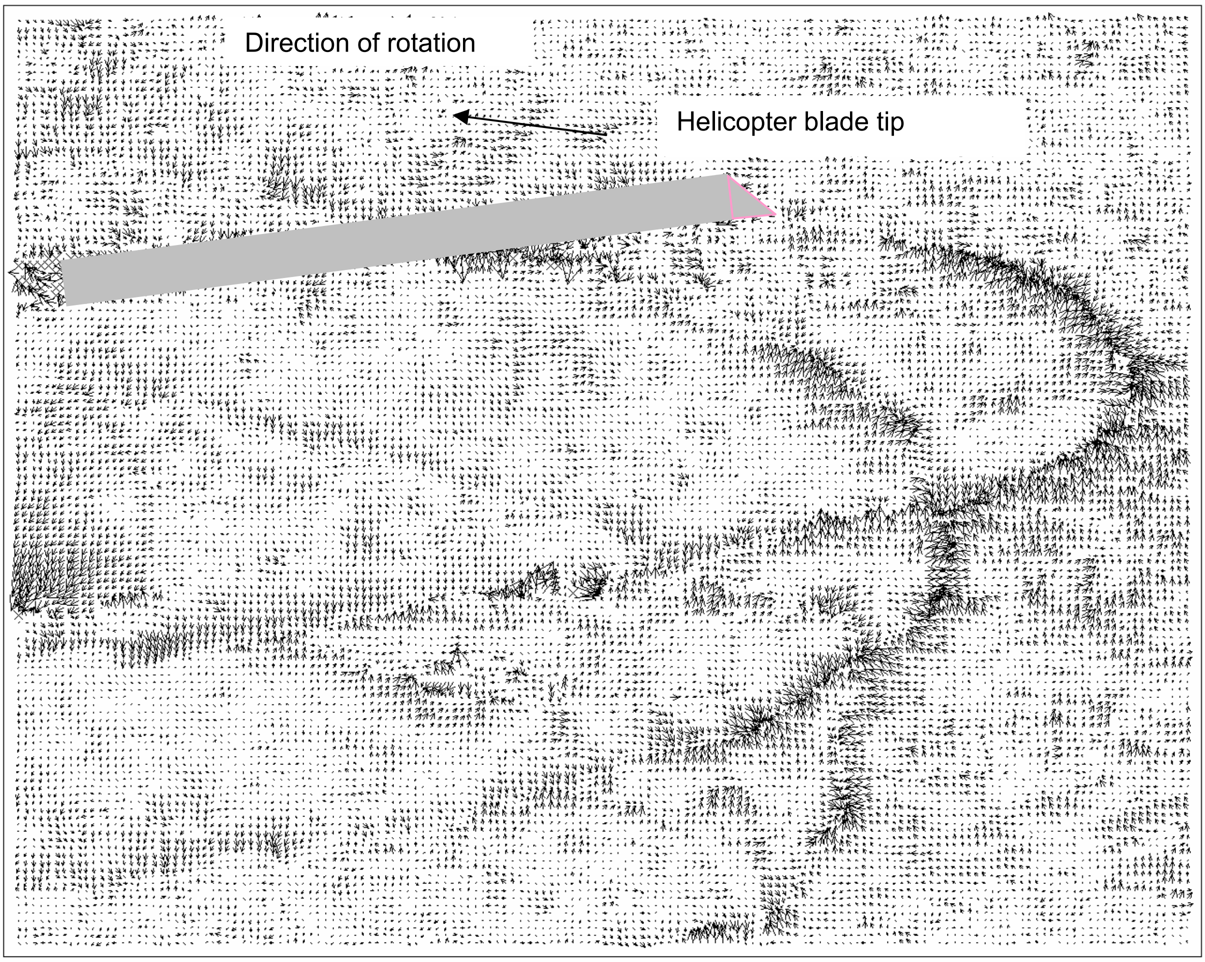

Figure 7: Vortex filaments. It can be seen that the vortex is not appearing directly at the rotor tip, since the vortex has to roll up first.

Data Processing II: Detection of the vortex filament

Already on the displacement vector plots of each camera, the vortex filaments are visible by naked eye. Their locations have to be determined on the image and their coordinates have to be stored and prepared for being used by the VRIEDER algorithm. Of course, this has to be done again separately for each camera. Since for the algorithm just a limited amount of points are used to represent the vortex filament in picture coordinates, it is important that these points are well distributed over the image. At this step it is obvious that clear vortex filament visualization by means of BOS is important for the further data processing. If problems with the detection of the vortex filament are occurring already in the 2D representation, there is no possibility to calculate the correct spatial vortex position.

Data Processing III: Calculation of the spatial vortex coordinates

By means of VRIEDER the spatial vortex position within the overlapping investigation volumes of both cameras was determined, based on the vortex positions given in picture coordinates. The output of the program includes already a spatial visualization of the vortex position. **Fig. 7 and 8** are examples of the feasibility study results of a hover flight with a slow climb rate. The tip vortex (black symbols) and the vortex generated by the leading blade (blue symbols) are clearly visible.

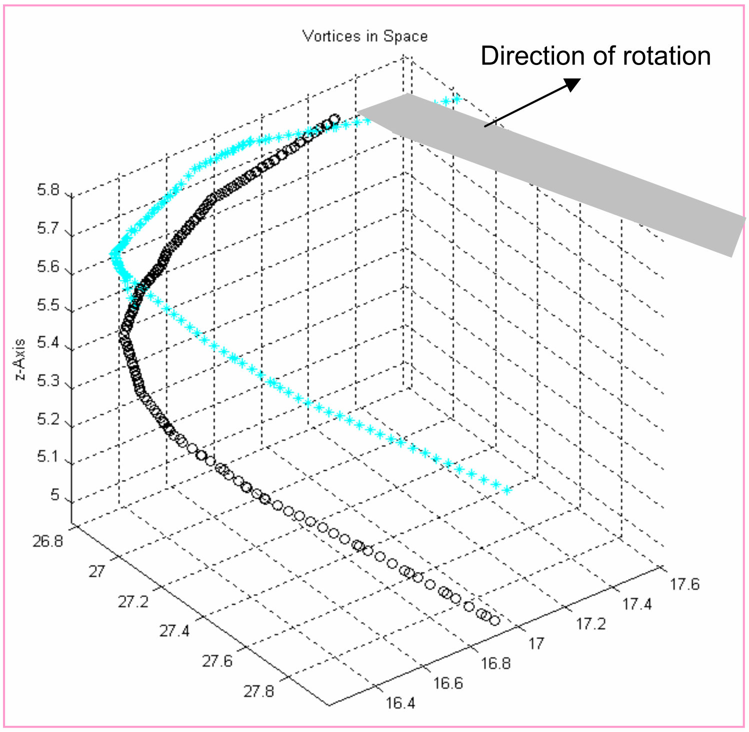

Figure 8: Result of the vortex localisation, viewed from the side.
The dimensions are meters.
The origin of the coordinate system is located at the right hangar edge on the apron ground.

Discussion

In the following the different 'lessons learned' of this experiment shall be discussed with regard to the equipment, data processing, results and the design of the experiment.

Regarding the equipment of the experiment the following remarks have to be made:

- The background pattern (concrete) was correlating very well. An evaluation was possible even if nearly no structure was visible by naked eye. An improvement can be done by additionally using white paint to brush some small scale pattern with sharp edges on the concrete. This improvement was not necessary due to the bright sun light during the experiment.
- The angle between the camera axis and the background was very small. However, due to the use of Scheimpflugadapters, it was possible to capture a well focused image with the cameras.
- The cameras used were developed for PIV, so their temporal jitter is very small. In parallel, these cameras are very light sensitive, which allowed short shutter times and a small aperture. These facts are important for a quantitative evaluation of the BOS data. A camera with an even higher resolution would be an improvement.

- The chosen objectives were appropriate for the given BOS set up and imaging conditions. A larger overlapping volume would have been desirable. But this was not possible due to the limited space on the apron and the limited height of the hangar.
- The helicopter type was extremely suited for this feasibility study as it is very maneuverable.

A remark concerning the data processing:
- The algorithms were working satisfactorily, but the evaluation is still time consuming (20 min). This will have to be changed before an industrial test can be performed. The results seem to be reasonable, a comparison to another measurement technique is not possible at present.

Some remarks concerning the results:
- The clarity of the results is remarkable.
- Small kinks in the vortex filaments are not a mistake but real physics.
- The perceptibility of the structures in the displacement plots can be improved by a better signal to noise ratio. If the BOS-results can be improved, an evaluation of the vortex strength will be possible, too.
- A larger volume of investigation is always desired.
- It will be interesting to put the vortex data in relation to flight data like climb rate, thrust, etc. in the next test.

Some remarks on design and execution of the experiment:
- Phase locked measurements will be desirable for statistical analysis (mean and fluctuations).
- Meteorological parameters like e.g. wind velocities and temperature gradients in the direct vicinity of the investigation volume should be determined, too.
- The hangar turns out to be of less influence in the vicinity of the rotor. However, better results in terms of fewer disturbances will be achievable by using open towers for the camera mounts instead of the hangar.
- Sun shine was a prerequisite for this test!
- The definition of the overlapping investigation volume as seen by both cameras is not easy and needs a good preparation in order not to confuse the pilot. Some marks on the apron are very helpful for the pilot, too.

5. CONCLUSION AND PROSPECTS

Full-scale quantitative vortex investigations under real Mach and Reynolds numbers has been performed. It was possible to detect the position of helicopter blade tip vortices in 3D space coordinates by means of the stereoscopic BOS during a free-flight full-scale out-door test.

The result of this experiment encourages the start a systematic investigation of the vortex trajectories and vortex interactions. Any systematic approach on full-scale helicopter vortex position measurements can help to understand the underlying principles. Additionally, experimental results applied to helicopters can support CFD calculations. Further on, the helicopter with its repeatable vortex structure represents a simple vortex system, which is easily to be measured due to the fixed position of its origin. This may lead to an understanding of the large variation in the results of measured vortex movements. Even the influence of meteorological effects may be possible to be investigated.

Further experiments will be performed with a larger distance between the helicopter and the background and a larger focal length. By help of a trigger signal from the helicopter, the measurements will be performed with fixed rotor angle.

Acknowledgement

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